# From yielding function and plastic potential to pseudo-potentials and bipotential : a constructive procedure

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Constitutive laws for materials with rate independent flow rules are usually defined using the generalized normality assumption : an elastic domain ($f \leq 0$) is defined and, in the simplest cases, the direction of the flow is given by a normality condition to the elastic domain. For non-associative flow rules, the flow is not assumed to be normal to the elastic domain and its direction is defined as the gradient, with respect to the thermodynamic forces, of a function $g$ called plastic potential.
In order to be sure that the dissipation is positive, an elegant procedure is to introduce pseudo-potentials. But the definition of such pseudo-potentials is not straightforward in the non-associated cases. Here, we propose a constructive method to obtain, for a given model defined by the functions $f$ and $g$ , the two dual pseudo-potentials : they are characterized by an additional dependence on the state variables.Then a bipotential, i.e. a function depending on the dual variables, the thermodynamic forces and the flow, separately convex with respect to both of them, is deduced. The notion of bipotential was first by introduced by de Saxcé, for the Drucker-Prager model and for unilateral contact in particular, and it permits to define the physical behavior as extremal couples of the bipotential..
Our constructive procedure is based on a careful distinction between dissipative and non dissipative forces and on a suitable use of their relationship. The method is illustrated by three examples : the Drucker-Prager model, the non linear kinematic model and also the Bouc-Wen Endochronic theory. Work is on progress to apply the same procedure to other constitutive laws, in particular to unilateral-contact, to recover the bipotential already proposed by de Saxcé.

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