Rigid Body Dynamics with Quaternions and Perfect Constraints

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ABSTRACT

The formulation of the dynamics of a rigid body requires a parameterization of finite rotations to represent the orientation of the body. A commonly used parameterization is the unit quaternion, also known as Euler parameters. Usually one starts with the equations of motion of a rigid body in terms of the linear and angular velocity. Subsequently, the derivative of the unit quaternion is related to the angular velocity, yielding a kinematic constraint for the unity of the quaternion on velocity level (cf. [1]). To prevent constraint drift in numerical simulations, the quaternion has therefore to be resized to unit length after each integration step or, alternatively, the description has to be extended to a differential algebraic equation (DAE) formulation, where the unit constraint is explicitly contained in the set of equations. Unfortunately, the extension of the equations of motion of a rigid body to a DAE yields a cumbersome formulation for which the Lagrange multiplier and the constraint equation have no direct physical meaning and the resulting mass matrix is singular.

In the present work a different approach is taken. First, the infinite dimensional dynamics of the continuum is reduced, by using perfect bilateral constraints, to a body with three translational, three rotational and one uniform scaling degree of freedom. The displacement of the center of gravity and an unconstrained quaternion are used as generalized coordinates. By introducing an additional perfect bilateral constraint one can force the 7 degrees of freedom body to become a rigid body. Without reducing the set of coordinates, this yields naturally a DAE description of the dynamics of a rigid body with a positive definite 7×7 mass matrix and a Lagrange multiplier for the constraint force. Finally, the derivative of the quaternion can be transformed to a scaling velocity and a generalized angular velocity, giving the DAE a form which can be directly linked to the classical equations of motion of a rigid body.

The unit constraint of the quaternion formulated as a perfect constraint can be stated as a normal cone inclusion and solved together with those of the unilateral contacts. This DAE formulation of a rigid body is useful for the construction of energy consistent integrators for non-smooth mechanical systems.

References

[1] P. E. Nikravesh, *Computer-aided analysis of mechanical systems.* Prentice-Hall International, London, 1988.