Mathematical and numerical treatments of elastic rods with frictionless self contact

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In the last three decades, the theory of elastic rods has witnessed great development because of its various industrial and biomechanical applications. Among the types of elastic rods used in practical applications we mention beams in civil constructions, cables in marine industries, pipelines in the oil industries, and fragments of the DNA molecule in the modeling of live sciences. Several models of elastic rods were employed for the study of the deformations and supercoiling of fragments of the DNA molecules. The problem of self contact in elastic rod has attracted, in at least the last two decades, a continuous attention. The classical theories of elastic rods are local ones and do not account for the global behavior of the configuration of the rod. In particular, the equations resulting from these theories cannot foresee nor avoid the self-penetration of different parts of the rod.

In this talk, we will describe a uni-dimensional model for the treatment of selfcontact in elastic Cosserat rods. The model we propose is based on the introduction of a signed contact distance that takes into account the geometric description of the rod. By minimizing the elastic energy under the constraint that the contact distance is nonpositive we avoid self-penetration of different parts of the rod. We describe a penalty method for the treatment of the mutli-valued and non-differentiable contact law. We then give the details of the finite-element discretization of the elastic rod self-contact problem as well as some numerical examples.