Gradient damage models and their use to approximate brittle fracture

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Abstract

In its numerical implementation, the variational approach to brittle fracture approximates the crack evolution in an elastic solid through the use of gradient damage models. We formulate the quasi-static evolution problem for a general class of such damage models. Then, we introduce a stability criterion in terms of the positivity of the second derivative of the total energy under the unilateral constraint induced by the irreversibility of damage. These concepts are applied in the particular setting of a one-dimensional traction test. The variational framework allows us to exhibit analytical solutions of homogeneous as well as localized damage states and to illustrate the concepts of loss of stability, of scale effects, of damage localization, and of structural failure. Considering several specific constitutive models, stress-displacement curves, stability diagrams, and energy dissipation provide identification criteria for the relevant material parameters, such as the internal length. The one-dimensional analytical results are compared with the numerical solution of the evolution problem in a two dimensional setting. Finally, we show some examples of large-scale computations performed to simulate the fracture of brittle solids.

Keywords: Fracture, energy methods, gradient damage, stability, variational inequalities