Self-contact, self-collisions and large deformations

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In large deformations distinct parts of the surface of a solid may be in contact: this is the case when rubbing hands in cold weather. Self contact may result from collision: this is the case when clapping hands for a diva at the Scala.

We address this problem of mechanics involving velocities either continuous (smooth contact) or discontinuous (collision or non smooth contact) with respect to time. We investigate the equations of motion and the constitutive laws, [1].

We prove that there exists equilibrium positions of a solid in large deformations with the possibility of self-contact, [2]. The equilibrium positions may be non-unique. We prove also that there exist unique velocities after a self-collision, [3].

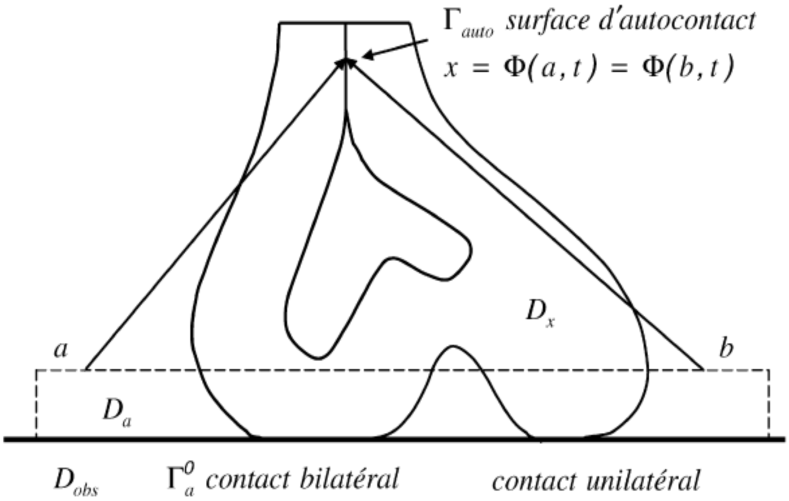

Figure 1: A point of the solid which was at initial position $\Phi(a, 0) = a$ in domain D_a at time 0 has position $\Phi(a, t) = x$ at time t in domain D_x. It is fixed on rigid obstacle D_{obs} on part Γ_a^0 (bilateral contact). It can be in unilateral contact with the obstacle. On part Γ^{auto}, there is self-contact: points a and b which are different have the same position $x = \Phi(a, t) = \Phi(b, t)$.

In the predictive theory we take into account the spatial variation, $grad\boldsymbol{R}$ of the rotation matrix $\boldsymbol{R}$ of polar decomposition $\boldsymbol{F} = \boldsymbol{RW}$ of matrix gradient of position function $x = \Phi(a)$

$$\boldsymbol{F} = (\boldsymbol{F}_{i,\alpha}) = \left(\frac{\partial \Phi_i}{\partial a_\alpha}\right).$$

The principle of virtual power involves the gradient of the angular velocity. It gives the equations of motion introducing a new interior force, a torque flux.

In large deformations, self-contact or self collision may occur, see Figure 1. Self-contact introduces contact forces requiring constitutive laws. Self-collision introduces contact percussions which are investigated in [3]. The local impenetrability condition we choose requires that the eigenvalues of the extension matrix $\boldsymbol{W}$ are non negative. Our parti pris is that the material may flatten into a surface, $rank\boldsymbol{W} = 2$ (for instance when a structure is flatten by a power hammer), a curve, $rank\boldsymbol{W} = 1$ (for instance when an ingot is transformed into a wire in an extruder), even into a point, $rank\boldsymbol{W} = 0$. Free energy $\Psi(\boldsymbol{W}, grad\boldsymbol{R})$ involves the indicator function of convex cone C of the symmetric semi definite positive matrices. The constitutive laws are given by the derivatives and the subgradients of the free energy. They introduce impenetrability reactions. For instance, when the solid is flattened into a plate, the reaction is a pressure maintaining the flattening. Equilibrium positions may be found by minimization of the potential energy. Potential energy depends on two quantities: the position $x = \Phi(a)$ and the rotation matrix $\boldsymbol{R}(a)$ because this matrix is not uniquely given by function Φ if $rank\boldsymbol{W} < 2$. The cinematically admissible couples $(\Phi, \boldsymbol{R})$, those which may be reached from position $\Phi(a) = a$ and rotation matrix $\boldsymbol{R} = \boldsymbol{I}$, satisfy unilateral and bilateral boundary conditions and a global non interpenetration condition. If the free energy satisfies coercivity and sequential weak lower semi continuity simple properties and if the exterior forces are smooth functions, we prove that there exist equilibrium couples $(\Phi, \boldsymbol{R})$ which minimize the potential energy. Free energy $\Psi(\boldsymbol{W}, grad\boldsymbol{R})$ may be a convex function, [4], of $(\boldsymbol{W}, grad\boldsymbol{R})$ without any mechanical restriction. The solutions are not unique because the set of the cinematically admissible couples $(\Phi, \boldsymbol{R})$ is not convex.

In a self collision, assumed to be instantaneous, the basic idea of the theory is to take into account the relative velocities of the material points of the surface of the solid and the velocities of the material points of the solid and of the obstacle. New velocities of deformations and new interior forces are introduced: surface percussion and volume percussion stresses, which satisfy new equations of motions. Constitutive laws for the interior forces resulting from a pseudo-potential of dissipation take into account the impenetrability conditions. The velocities after the collisions are unique, [3].

Preliminary numerical applications will be given.

References

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