Theoretically supported scalable algorithms for contact problems

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We first briefly review the TFETI (total finite element tearing and interconnecting) based domain decomposition methodology adapted to the solution of 2D and 3D multibody contact problems of elasticity, including classical optimal estimates. Recall that TFETI differs from the classical FETI or FETI2 by imposing the prescribed displacements by the Lagrange multipliers and treating all subdomains as "floating".

Then we present our in a sense optimal algorithms for the solution of the resulting quadratic programming problems. The unique feature of these algorithms is their capability to solve the class of convex quadratic programming problems with homogeneous equality constraints and relevant separable inequality constraints in $O(1)$ iterations provided the spectrum of the Hessian of the cost function is in a given positive interval.

Finally we put together the above results to develop scalable algorithms for the solution of both coercive and semi-coercive variational inequalities [1]. A special attention is paid to the construction of an initial approximation which is not far from the solution, so that the above results guarantee that the cost of the solution increases nearly proportionally with the dimension of the discretized problem. We illustrate the results by numerical experiments and by the solution of difficult real world problems. Our discussion will cover problems bot frictionless problems and problems with friction. We conclude by a brief discussion of possible improvements and generalizations, including the solution of dynamic problems.

References

[1] Z. Dostál, T. Kozubek, V. Vondrák, T. Brzobohatý, and A. Markopoulos, *Scalable TFETI algorithm for the solution of multibody contact problems of elasticity*, Int. J. Numer. Methods Eng., DOI: 10.1002/nme.2807, 2009.