Institut für
Kontinuumsmechanik

Leibniz
Universität
Hannover

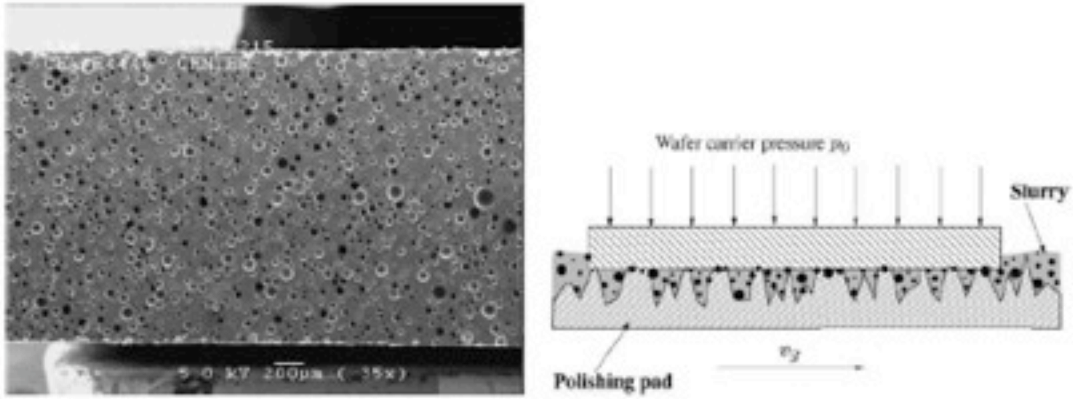
Palmanova
2010

On Multiscale Methods in Contact Mechanics

7th Colloquium:
Unilateral Problems in Structural Mechanics

Peter Wriggers & Ilker Temizer

Contact Interfaces with Third Bodies



[Yi, 2005]

Chemical-Mechanical Planarization (CMP)

- IC manufacturing: polishing of a semiconductor wafer
- perfectly matching interfaces required between successive layers of the IC
- porous polymeric pad + abrasive slurry

[Klein-Paste, 2007]

Traction Performance

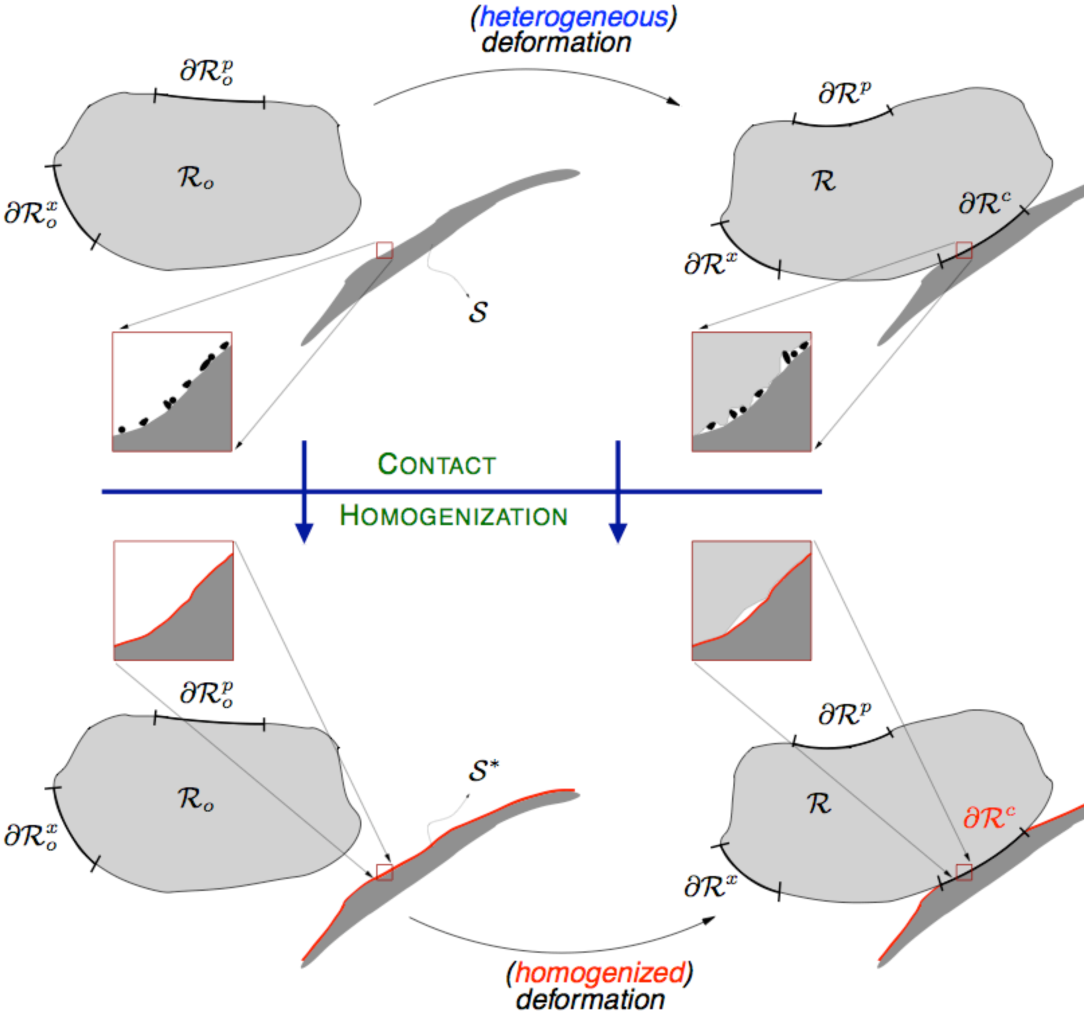
- tire traction on granular surfaces
- wheel-rail interaction in the presence of TBs

[Flannery et al., 2008]

Joint Implants

- *Intrinsic* TBs: wear particles
- *Extrinsic* TBs: cement contaminants from bone-implant interface

Analysis Tool: Contact Homogenization



- Multiscale modeling of third bodies in the contact interface
- Aim: simplifying the analysis of a heterogenous macro-system
- Highly non-uniform heterogeneous surface topographies
- Particles in motion
  - ⇒ evolving contact surface
  - ⇒ coupled problem
- Tool: contact homogenization
  - ⇒ macroscopic behavior
  - ⇒ construct a new surface $\mathcal{S}^*$
  - ⇒ *effective friction coefficient* $\mathbf{k}^*$

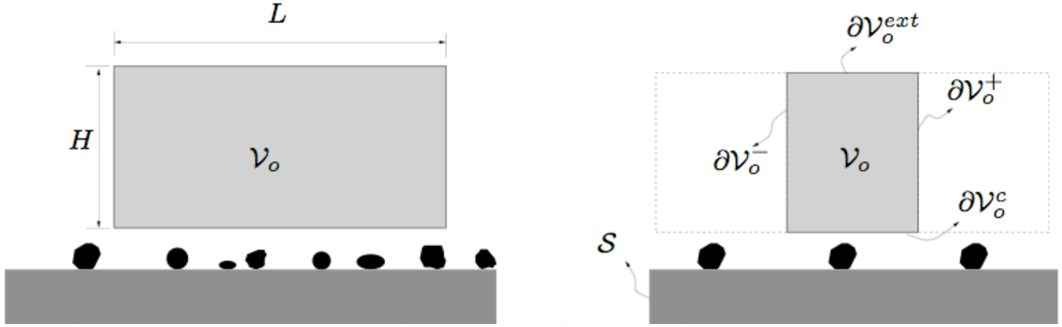
Literature Overview: Computational Approaches

- Natural problems involving particles trapped in a contact interface: tire traction [Hornet et al. (2001), Nakashima (2004), Iordanoff et al. (2002a)], wheel/rail interaction [Berthier et al., 2004], powder lubricants [Iordanoff et al., 2002b], surface wear [Fillot et al., 2004a,b]
- Direct resolution of the granular interface is prohibitively expensive
- Contact homogenization: Orlik (2004), Stupkiewicz (2007)
  $\Rightarrow$ micromechanically motivated effective contact constitutive formulations
- Asperity based small deformation contact: inelastic random surfaces with frictional interaction [Tworzydlo et al. (1998), Haraldsson and Wriggers (2000), Bandeira et al. (2004), Orlik et al. (2003)]
- Large deformation contact with a fractal surface: rubber friction [Hornet et al. (2001), Wriggers and Reinelt (2009)], mesoscale tire tread pattern analysis [Hofstetter et al., 2006]

Present approach: volumetric homogenization ideas are employed

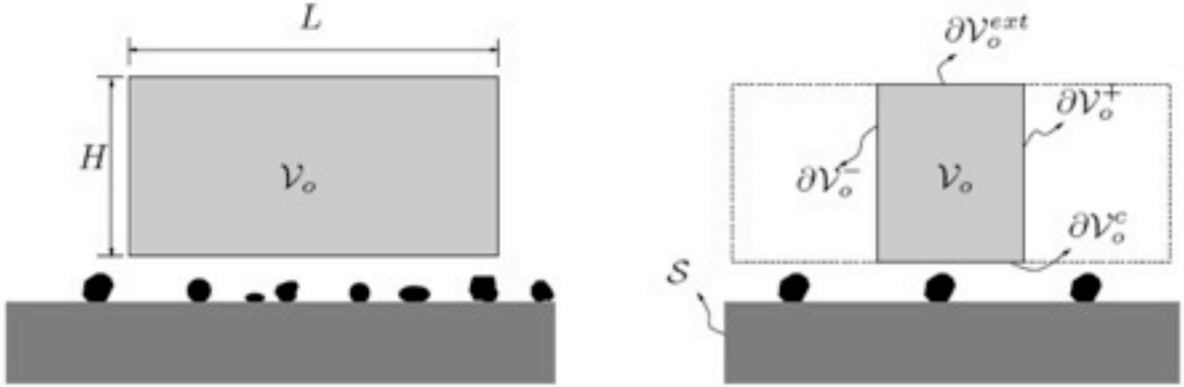
- A single layer of particles on a smooth rigid contact surface
- *effective friction coefficient* [Mack and Rubenstein, 1958]
- purely elastic materials: Temizer and Wriggers (2008)
- inelastic effects: damage and viscosity

Model for Third Bodies in the Contact Interface



- Sole ingredient of the homogenized contact law: $\mathbb{k}^*$
- Conduct a micromechanical sample test: (1) compress and (2) drag
- Identify the macroscopic friction coefficient: $\bar{\mathbb{k}}$
- $\bar{\mathbb{k}} \to \mathbb{k}^*$ *if* the sample is statistically representative:
  $\Rightarrow$ RCE (Representative Contact Element)
- Contact microstructure: area fraction and particle morphology
- Nature of the particle distribution: random or periodic
- Motivated by the periodic case, employ periodic BCs on lateral surfaces:
  $\Rightarrow \boldsymbol{x}^+ - \boldsymbol{x}^- = \boldsymbol{X}^+ - \boldsymbol{X}^-$ and $\boldsymbol{p}^- = -\boldsymbol{p}^+$ on $\partial\mathcal{V}_o^{int} \overset{\text{def}}{=} \partial\mathcal{V}_o^+ \cup \partial\mathcal{V}_o^-$
  $\Rightarrow$ must be coupled with periodicity BCs on particle positions
- FEM mesh must be coupled to the particle dynamics

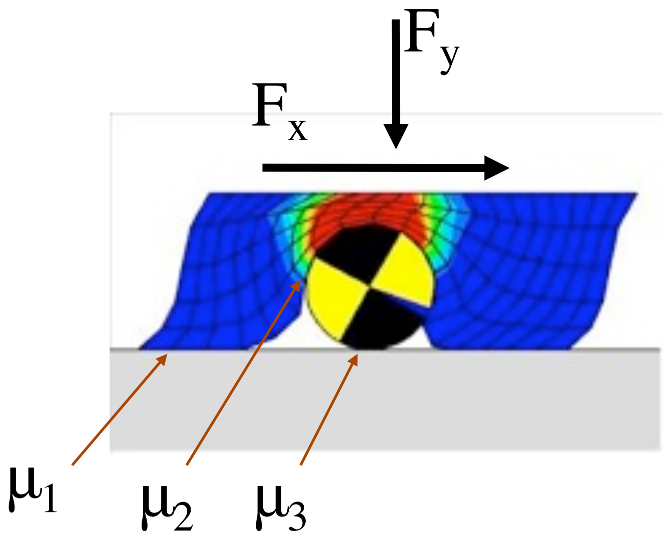
Micro–Macro–Transition



- Averaging over the *observable* test surface: $\langle \mathcal{Q} \rangle \overset{\text{def}}{=} \dfrac{1}{|\partial\mathcal{V}_o^{ext}|} \displaystyle\int_{\partial\mathcal{V}_o^{ext}} \mathcal{Q} \, da$
- Identification of macroscopic variables:
  $\bar{\boldsymbol{t}} \overset{\text{def}}{=} \langle \boldsymbol{t} \rangle \quad , \quad \bar{\boldsymbol{v}} \overset{\text{def}}{=} \langle \boldsymbol{v} \rangle \rightarrow \boldsymbol{A}_T$ (macroscopic slip direction)
- Definition of the macroscopic coefficient of friction: $\bar{\mathbb{k}} \overset{\text{def}}{=} \dfrac{|\bar{\boldsymbol{t}} \cdot \boldsymbol{A}_T|}{|\bar{\boldsymbol{t}} \cdot \boldsymbol{A}_N|}$
- Appropriate BCs on $\partial\mathcal{V}_o^{ext}$
  $\Rightarrow$ *Work Criterion (Frictional Dissipation)*: $\langle \boldsymbol{t} \cdot \boldsymbol{v} \rangle \equiv \bar{\boldsymbol{t}} \cdot \bar{\boldsymbol{v}} = \langle \boldsymbol{t} \rangle \cdot \langle \boldsymbol{v} \rangle$
  $\Rightarrow$ e.g. $\boldsymbol{v}$: constant (or, $\boldsymbol{t}$: constant)

Model for Third Bodies in the Contact Interface

- Assume periodic distribution of third bodies in the contact interface
  - consider a unit-cell as the RVE
  - periodic boundary conditions on the vertical edges
- Dynamic simulation to extract μ*
  - time step adaptivity
  - time averaging

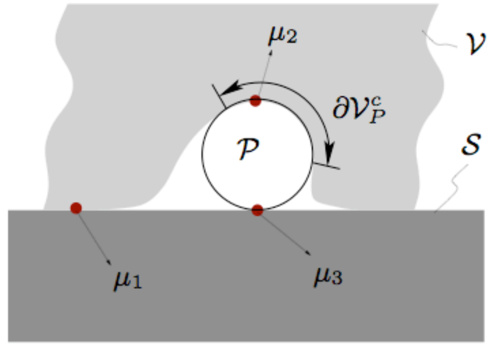


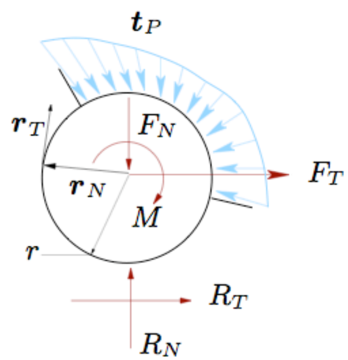
$(\mu_1=0.3, \mu_2=0.3, \mu_3=1.5)$

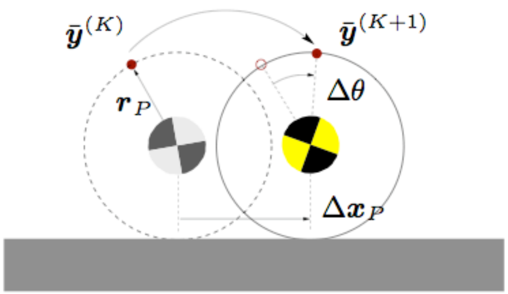
Model for Third Bodies in the Contact Interface

SOLUTION ALGORITHM FOR THE MICROMECHANICAL TESTING PROCEDURE

- STEP 1: Start with a converged configuration: $\boldsymbol{x}$, $\boldsymbol{x}_P$, θ
- STEP 2: Increment BCs on $\partial\mathcal{V}^{ext}$ at time t through Δt.
- STEP 3: Compute $\boldsymbol{F}$ and M for each particle
- STEP 4: Update the particle configurations ($\boldsymbol{x}_P$ and θ)
- STEP 5: Solve the implicit FEM equations $\boldsymbol{x} = \boldsymbol{f}(\boldsymbol{x})$
- STEP 6: Check particle configuration convergence
  – If CONVERGENCE then DONE
  – Otherwise New iteration



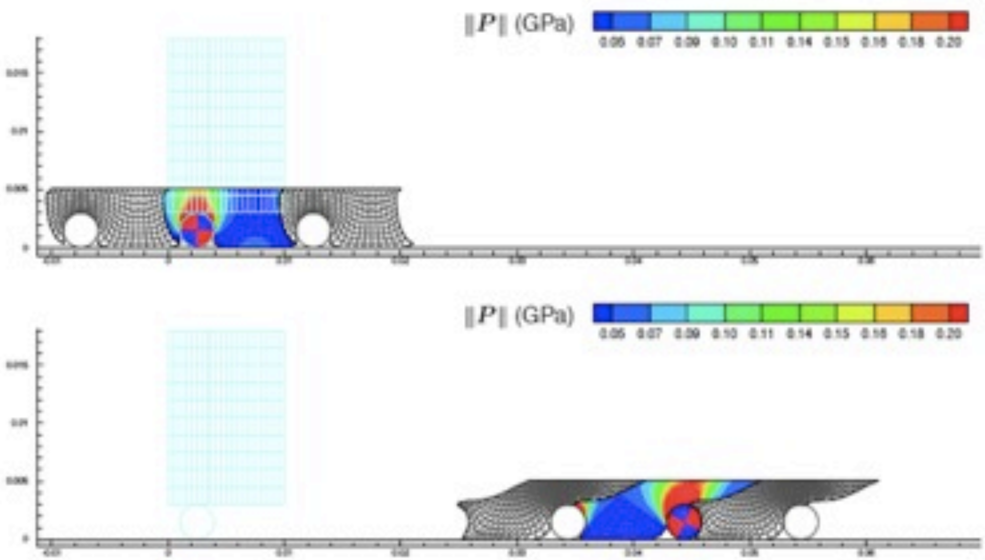


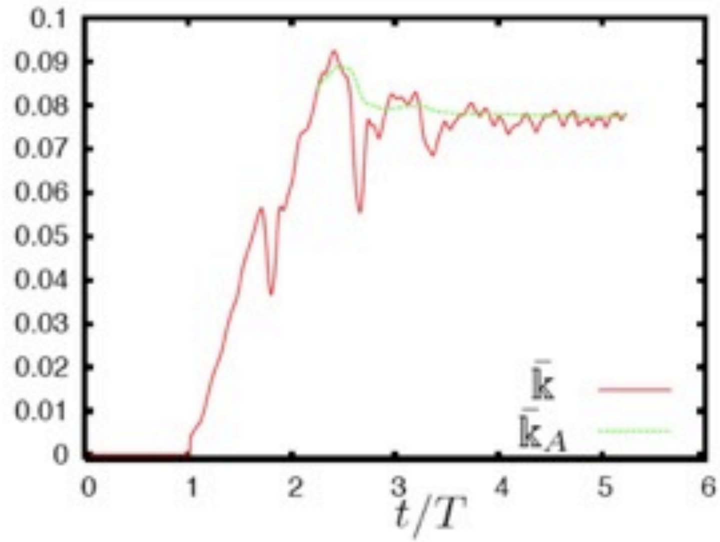


$\Longrightarrow$ Convect the point of contact: $\bar{\boldsymbol{y}} = \boldsymbol{x}_P + \Delta\boldsymbol{x}_P + \boldsymbol{R}\,\boldsymbol{r}_P$

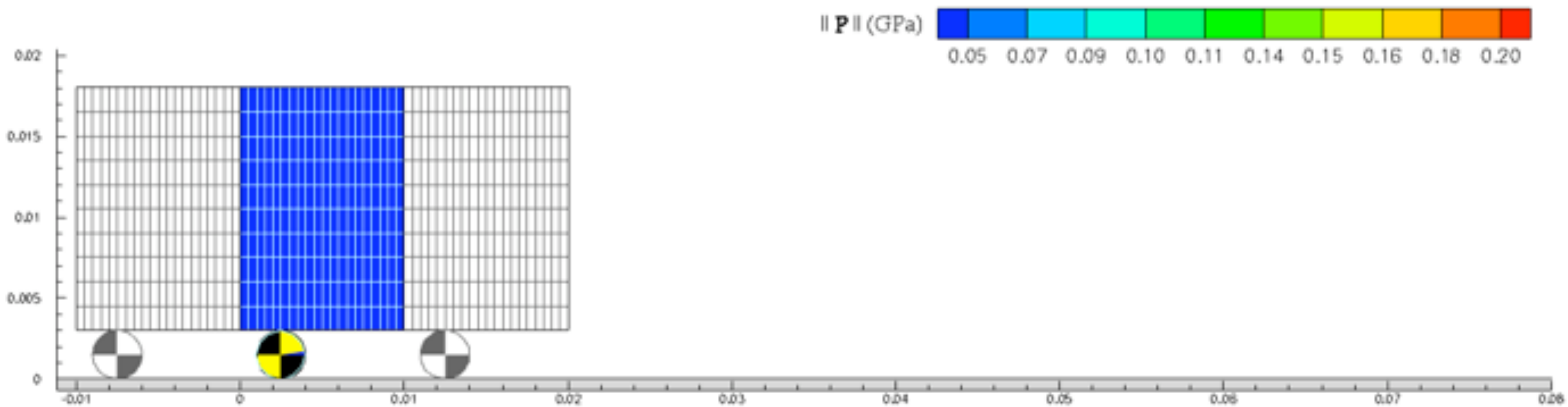
Micromechanical Test (Periodicity)

Time Averaging alleviates dynamic effects





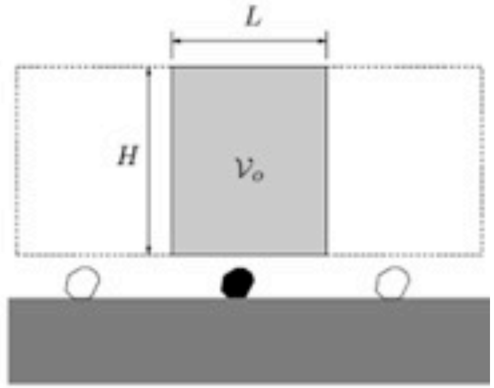
- Quasistatic Compression $\Rightarrow$ control $\bar{p} = \bar{\boldsymbol{t}} \cdot \boldsymbol{A}_N$
- Dynamic Drag (time-adaptive setting): $\bar{\mathbb{k}} = \bar{\mathbb{k}}(t)$
- Moving time average: $\mathcal{Q}_A = \frac{1}{t_F - t_A} \int_{t_A}^{t_F} \mathcal{Q}(t)dt$
- Averaged macroscopic CF: $\bar{\mathbb{k}} \longrightarrow \bar{\mathbb{k}}_A$

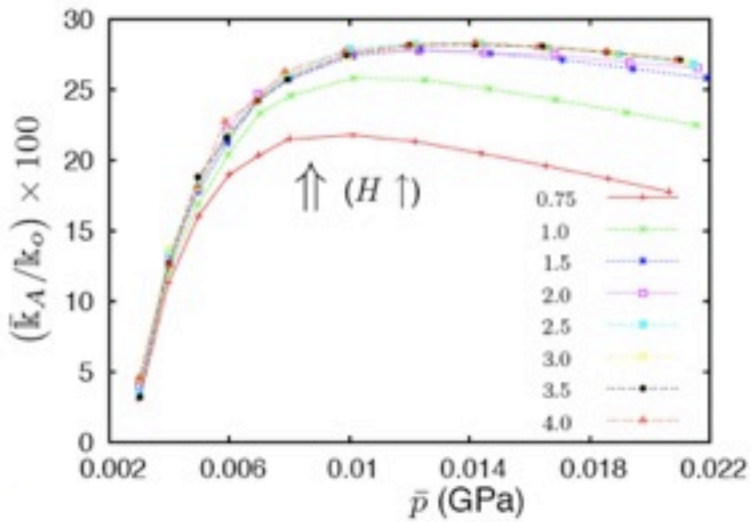

|| P || (GPa)
0.05 0.07 0.09 0.10 0.11 0.14 0.15 0.16 0.18 0.20
0.02
0.015
0.01
0.005
0
-0.01
0
0.01
0.02
0.03
0.04
0.05
0.06
0.07
0.08

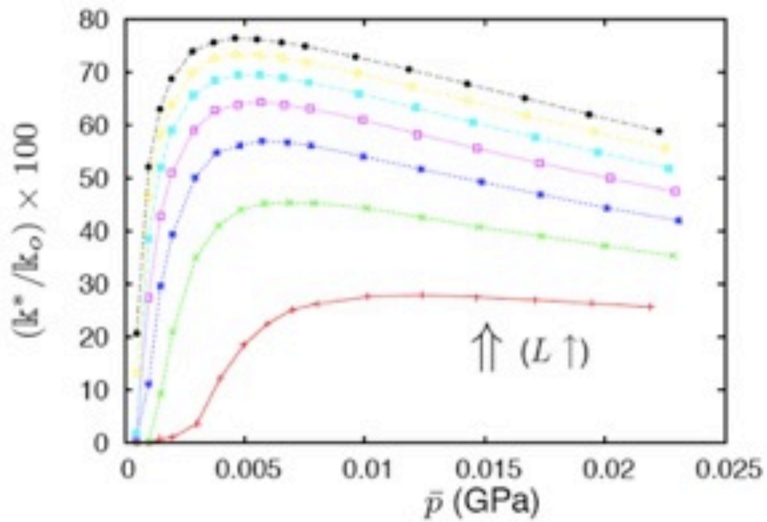
Effect of the RVE (unit cell) size and the particle density

(Elastic) Granular Interfaces Display a CF < Nominal Value: $\bar{\mathrm{k}}_A < \mathrm{k}_o$

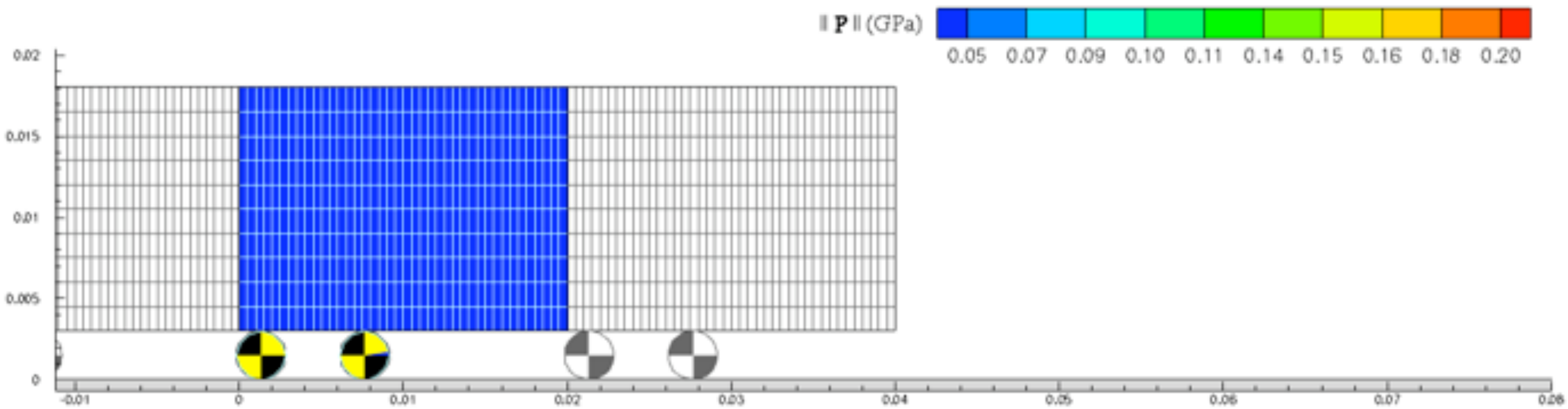
- *Pressure dependency*: *non*-Amontons type friction
- For a given area fraction (L) $\Rightarrow$ sample height (H) is a variable
- $\bar{\mathrm{k}}_A$ saturates with $H| \uparrow \propto$ RCE identification
- $\lim_{H\to\infty} \bar{\mathrm{k}}_A = \mathrm{k}^*$ (in practice: $H \approx 1.5$ cm)
- Negligible particle effect at low area fractions:
  $\Rightarrow \lim_{r/L\to\infty} \mathrm{k}^* = \mathrm{k}_o$
- Negligible velocity dependence (elastic sample)



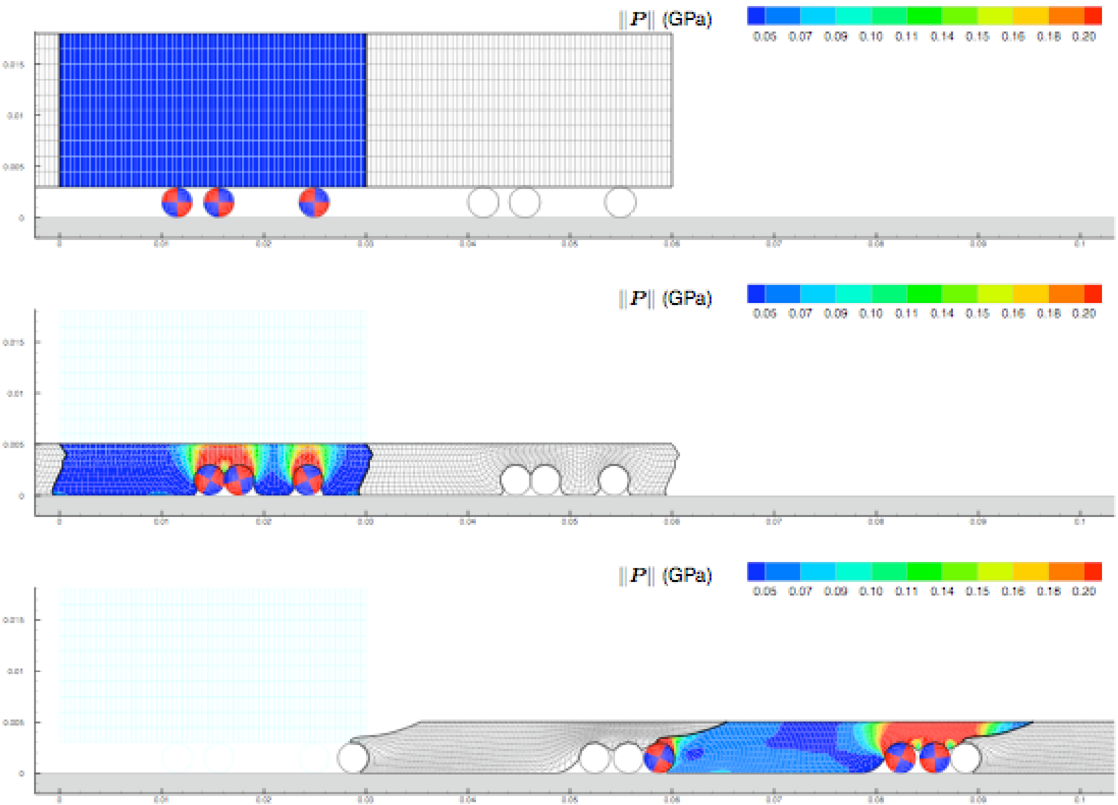


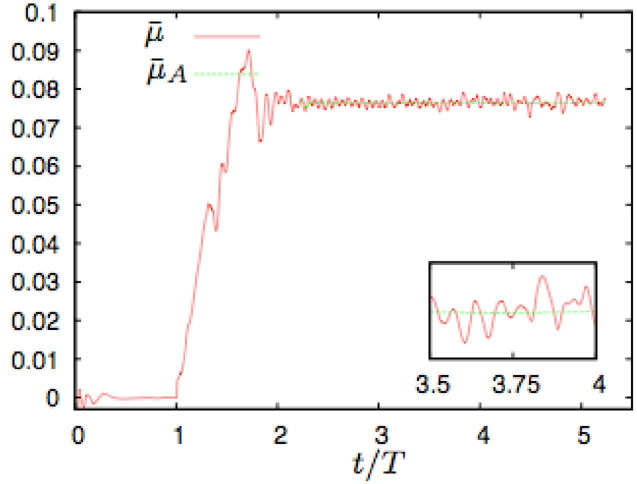


Effect of a different particle distribution



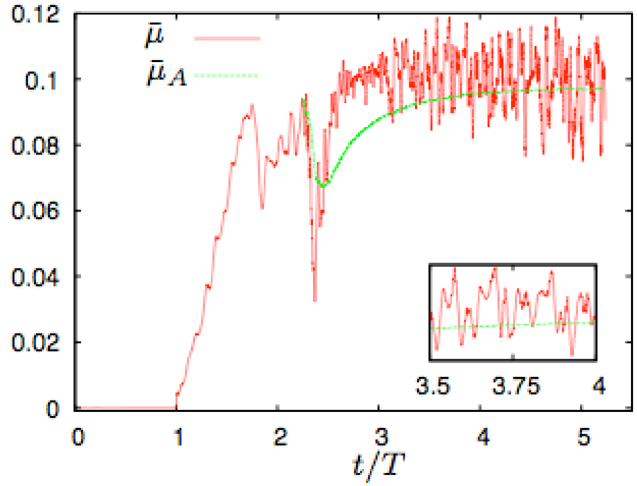
Effect of a different particle distribution





(a) No Clutter ($N_P = 2$)

- Disordered particle distribution
- Particle clutter → interaction : penalty method
- No frictional interaction among the particles
- Implicit particle-FEM interaction required

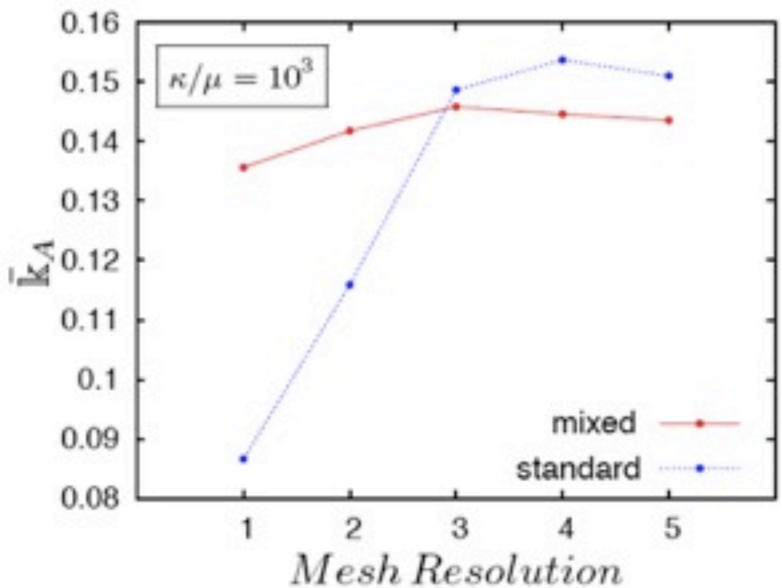


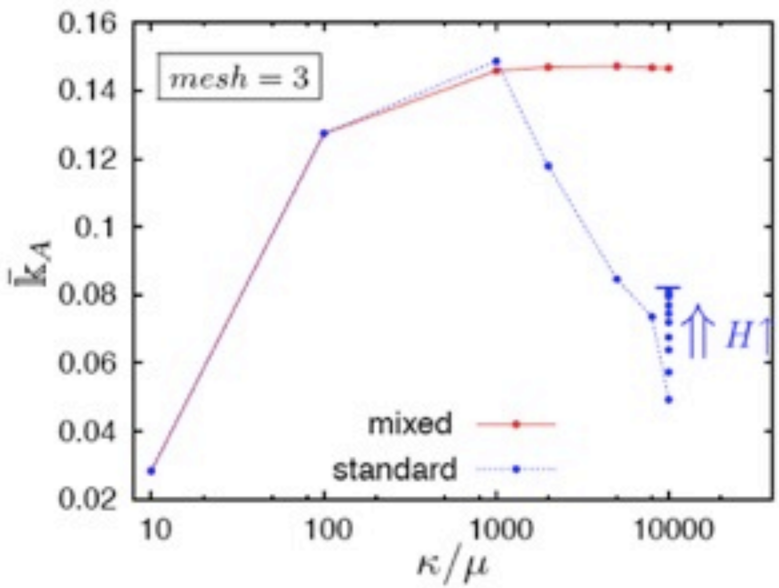
(b) Clutter ($N_P = 2$)

Accounting for Sample Incompressibility

Typical materials of interest display incompressible response

- Three-field mixed variational formulation (Veubeke-Hu-Washizu Principle):
  - Define: $\boldsymbol{F} = \frac{\partial \boldsymbol{x}}{\partial \boldsymbol{X}} \rightarrow \quad \bar{\boldsymbol{F}} := \left(\frac{\bar{J}}{J}\right)^{\frac{1}{3}} \boldsymbol{F}$ where $\bar{J} = \exp(\vartheta) > 0$
  - Construct: $\mathcal{G}(\boldsymbol{x}, \pi, \vartheta) = \int_{\mathcal{R}_o} \Psi(\bar{\boldsymbol{F}})\, dV + \int_{\mathcal{R}_o} \pi(\ln(J) - \vartheta)\, dV - \mathcal{G}_{ext}$
  - Solve: $\delta\mathcal{G} = 0 \longleftarrow Q1P0$ element
- Frictional response is more stable with the $Q1P0$-formulation even at low κ/μ





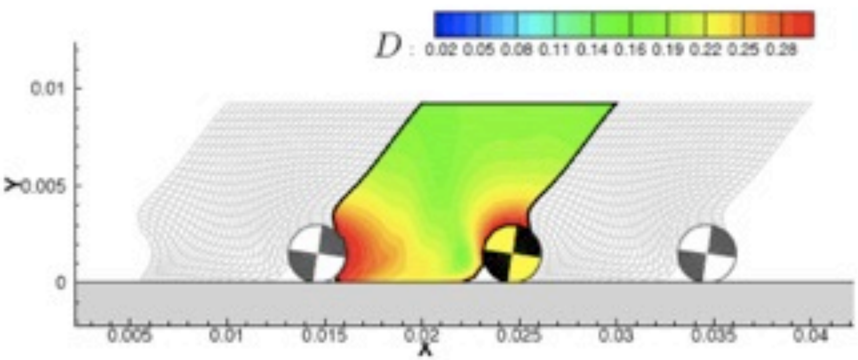
Inelastic Response of Sample: Damage

Example: Mullins' effect for rubber-like materials (Qualitative Behavior)

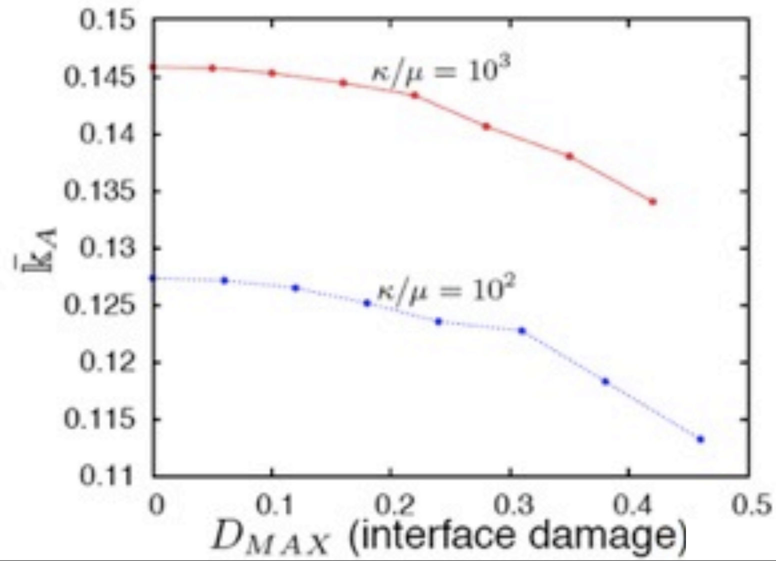
- Damage based on the maximum strain (Ξ_{MAX}) attained [SIMO, 1987]
  - Isotropic damage: $\Psi(\boldsymbol{E}, D) = (1 - D)\,\Psi_o(\boldsymbol{E}) \quad \Rightarrow \quad G = 1 - D$
  - Equivalent strain: $\Xi(\boldsymbol{E}) := \sqrt{2\,\Psi_o(\boldsymbol{E})}$
  - Evolution law:

$$G_{NEW} = G(\Xi) = \begin{cases} G_{OLD} & \text{if} \quad \Xi \leq \Xi_{MAX} \\ G_\infty + (1 - G_\infty)\dfrac{1 - \exp(-\Xi/\alpha)}{\Xi/\alpha} & \text{if} \quad \Xi > \Xi_{MAX} \end{cases}$$

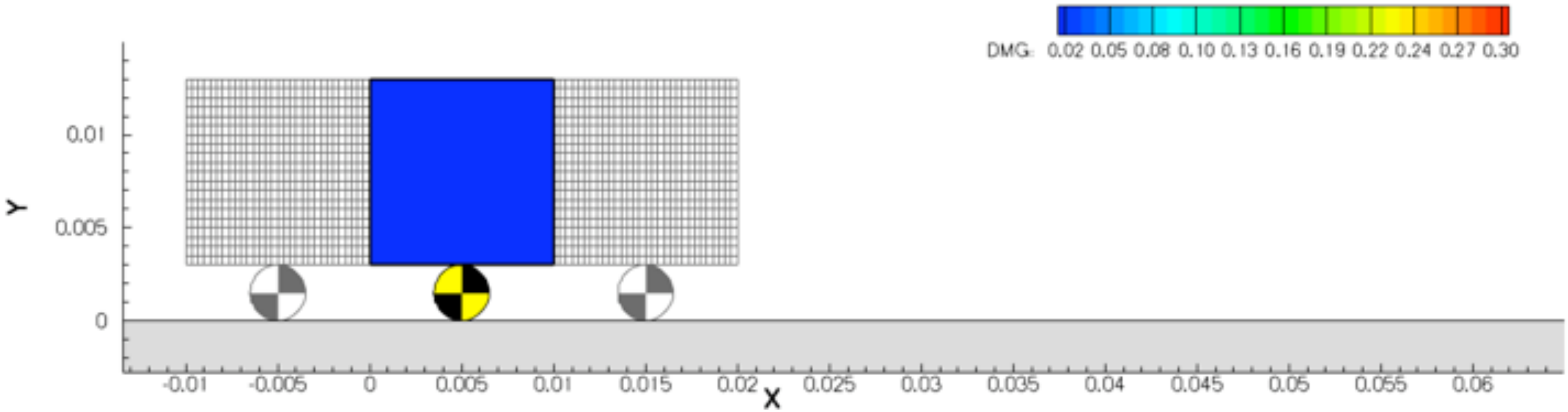
- Fully developed damage layer is necessary for the saturation of $\bar{\mathrm{k}}_A$



(partially developed damage layer)
$\kappa/\mu = 10^2,\ D_{MAX} = 0.31$

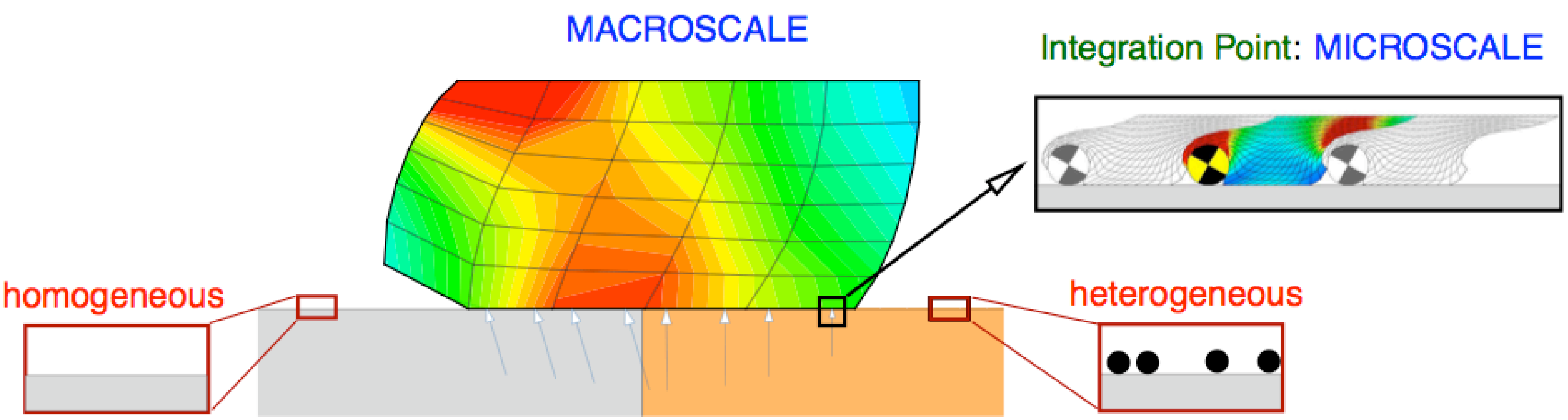


Inelastic Response of Sample: Damage

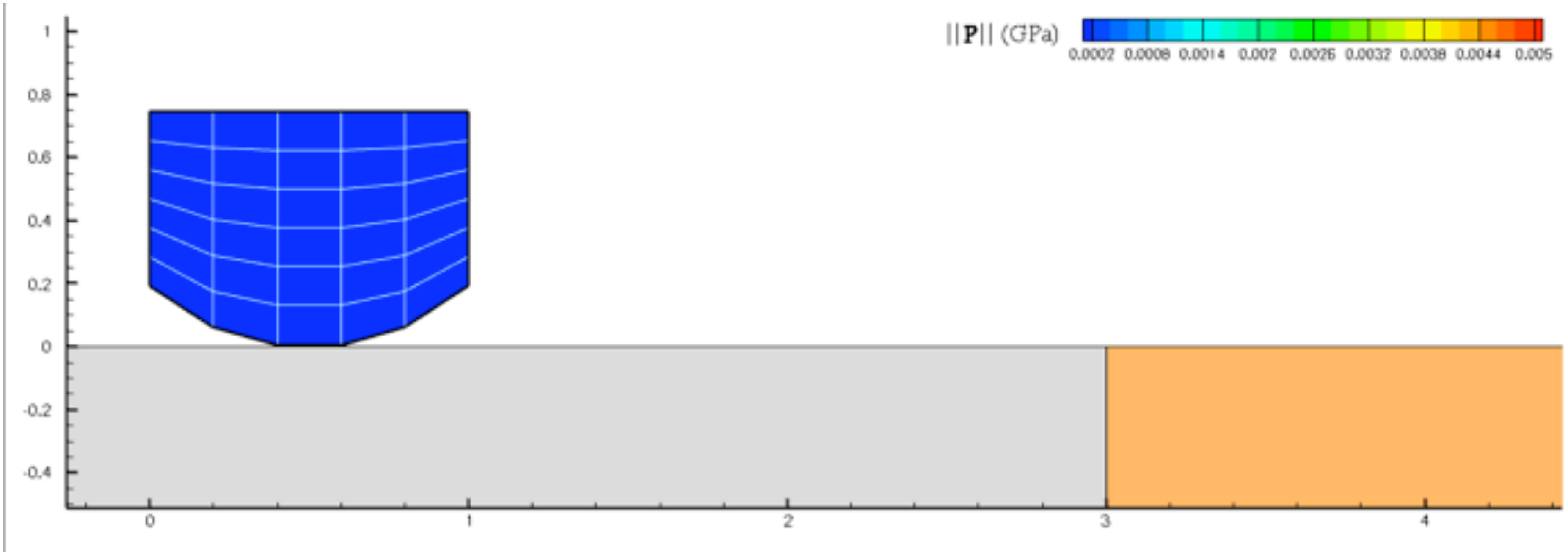


Multiscale Analysis

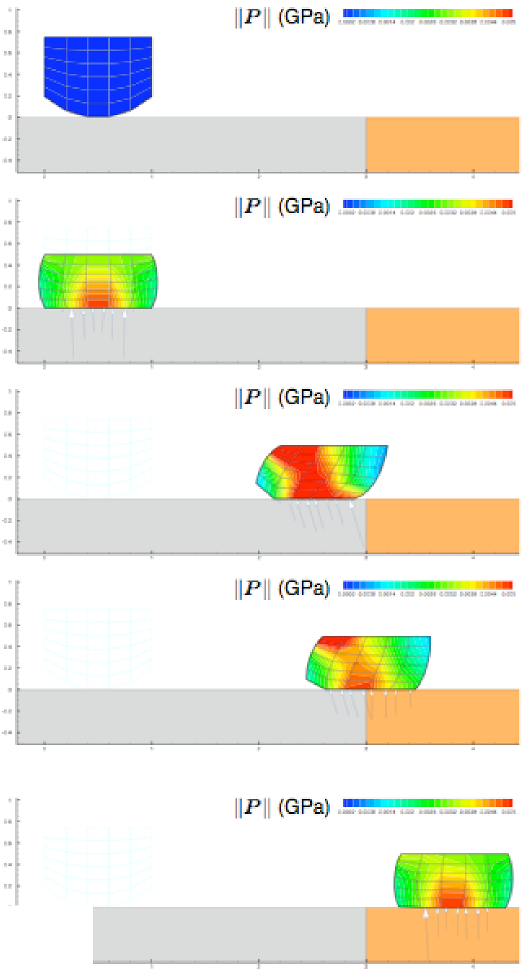
- Final aim is to solve a macroscale problem: $\mu^* = \mu^*(p^*)$ is unknown
- Implementation in a fully-coupled micro-macro simulation
- During the macroscopic check for slip in the contact algorithm:
  $\Rightarrow$ impose p^* as a BC on a microstructural sample
  $\Rightarrow$ extract μ^* from an embedded microscale simulation
- Underlying assumption: particle $\ll$ macrostructure
- Example (dynamic) problem: slip into a heterogeneous region



Multiscale Analysis

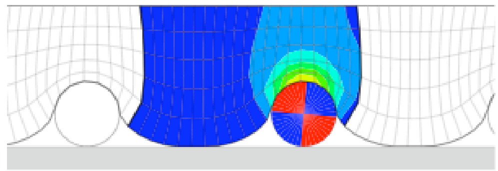


Multiscale Analysis

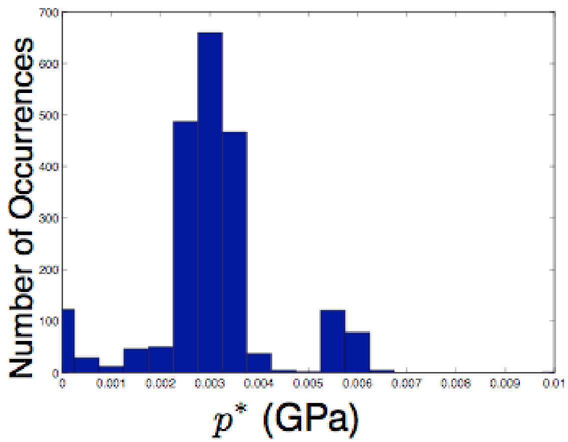


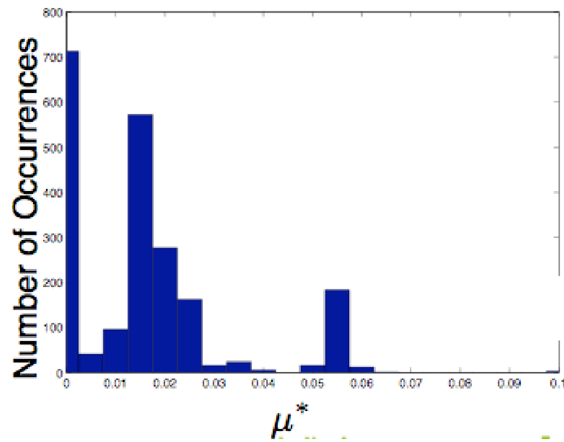
MACROSCALE

$\Longleftarrow$ MICROSCALE $(A = 0.3)$ $\Longrightarrow$

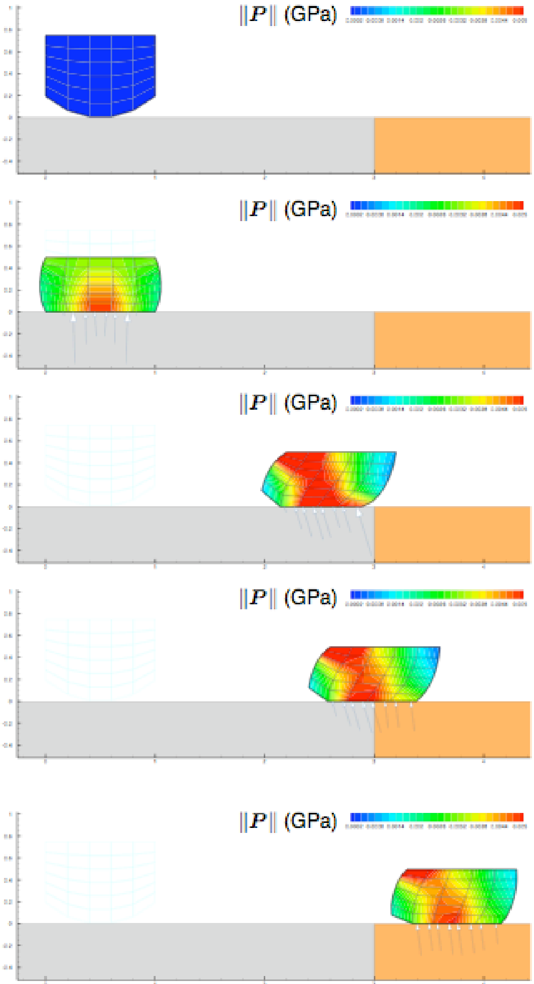


- Periodic unit cell at microscale with $L = 1$ cm
- 2123 multiscale computations (22 hours)
- Low pressure $\rightarrow$ low (but non-zero) frictional resistance: $\Rightarrow \mu_1 = 0.3\ ,\ (p^*)_{mean} \approx 3$ MPa $\rightarrow \mu^*_{mean} \approx 0.02$



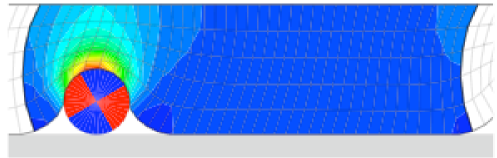


Multiscale Analysis

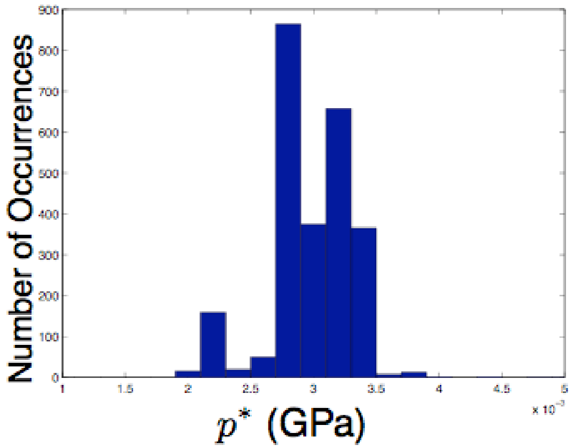


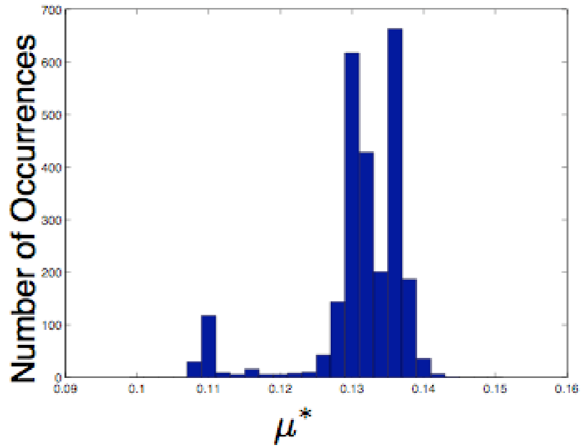
MACROSCALE

$\Longleftarrow$ MICROSCALE $(A = 0.15) \Longrightarrow$



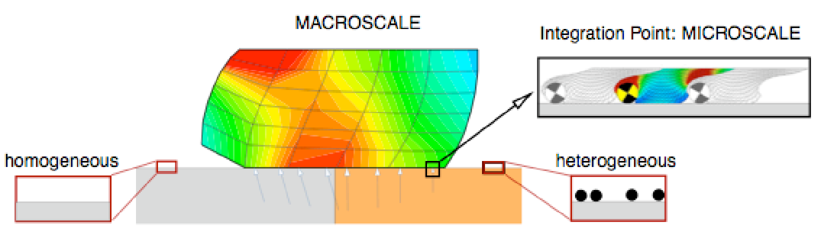
- Periodic unit cell at microscale with $L = 2$ cm
- 2536 multiscale computations (118 hours)
- Significant frictional resistance:
  $\Rightarrow \mu_1 = 0.3\ ,\ (p^*)_{mean} \approx 3$ MPa $\rightarrow \mu^*_{mean} \approx 0.13$





Multiscale Analysis

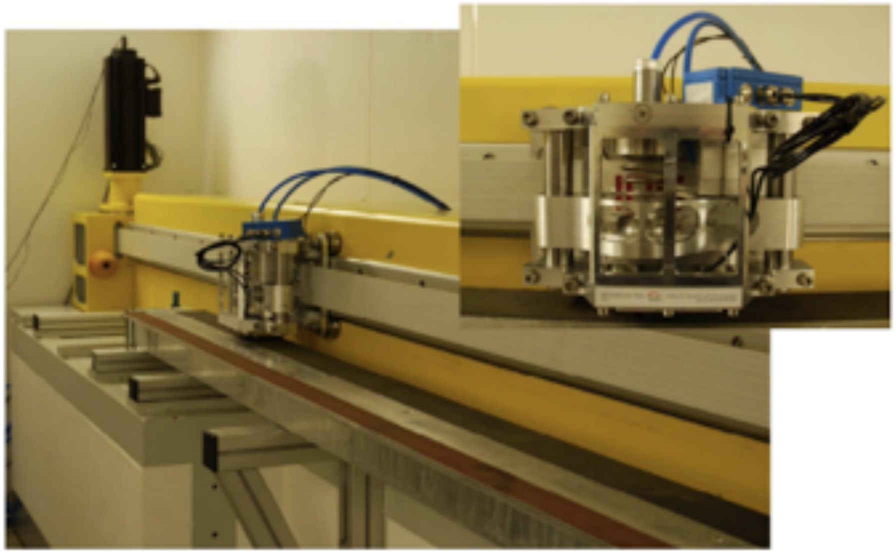
- A framework for contact homogenization in the presence of a single layer of interface inhomogeneities with the central ingredient μ^*
- Random and periodic contact microstructures considered
- Particular emphasis on the micro-to macro transition procedure and the identification of the RCE
- Strongly pressure dependent macroscopic response
- Multiscale implementation in a coupled micro-macro simulation



- *Possible extensions*:
  1. viscous material with damage for the micromechanical sample
  2. multi-layered particle distribution
  3. particle morphology
  4. extract macroscopic contact tangent from microscale computation

Outlook

- Challenges: true multi-scale analysis
- Homogenization and multi-scale analysis for three-dimensional problems
- Verification and Validation using experimental data



Laboratory Experimentation

- linear frictional test rig (LFTR)
- velocity/pressure/surface control
- 3D numerical model is needed
- particle morphology