Seventh Meeting
UNILATERAL PROBLEMS IN STRUCTURAL ANALYSIS
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ROLLING/SLIDING CONTACT OF COATED BODIES

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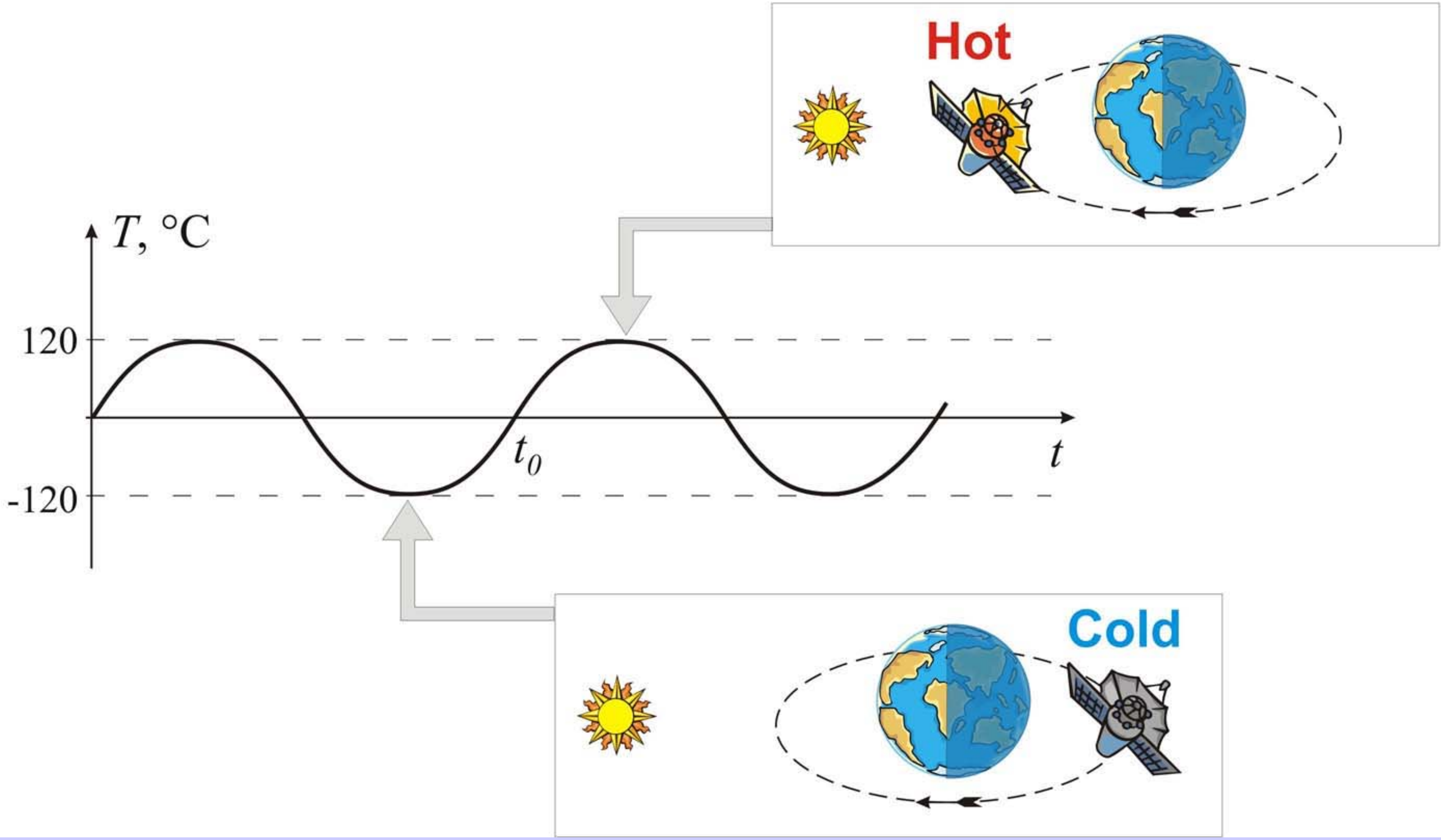
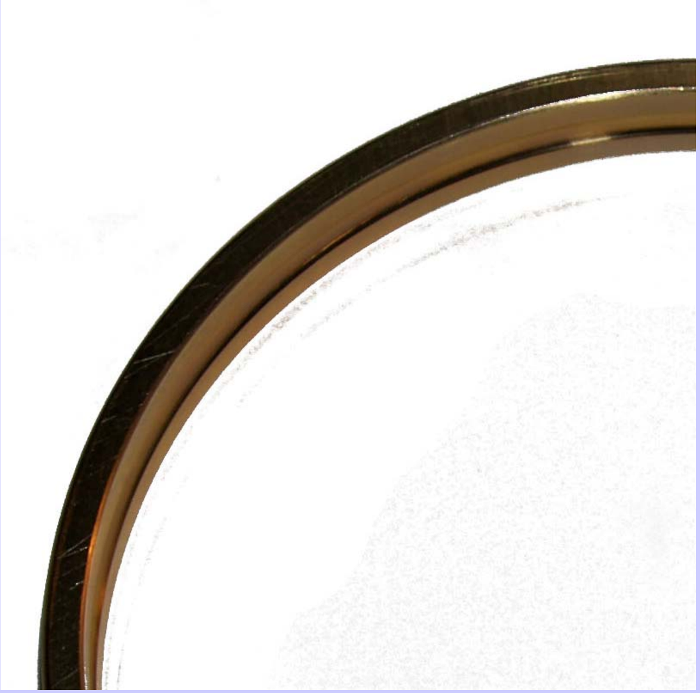
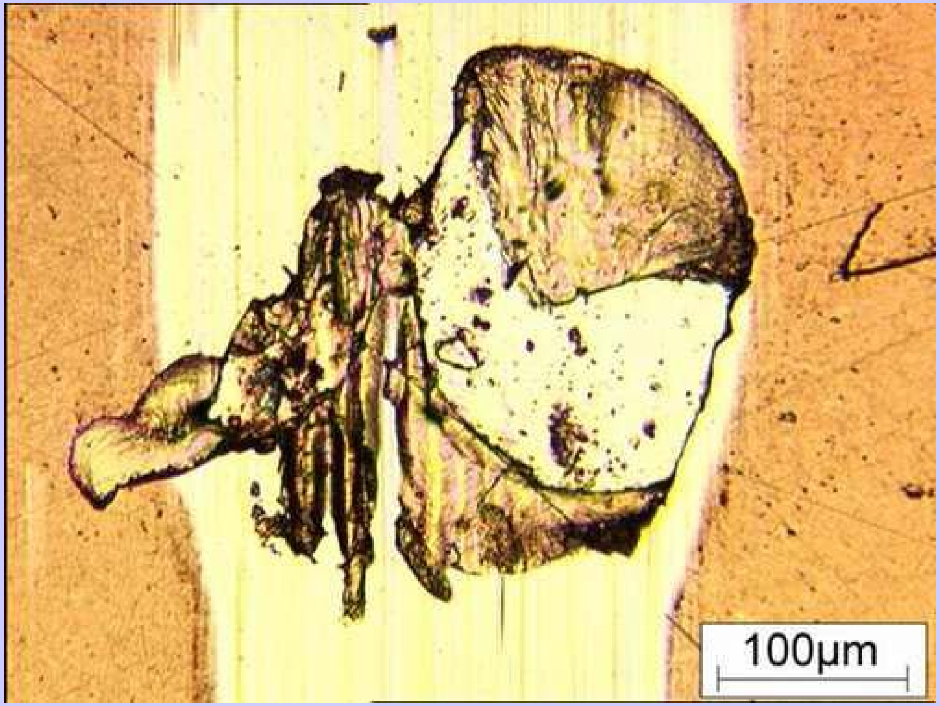
General conditions of tribological experiments in open space

- low residual pressure (10^{-7} Pa);
- cosmic radiation;
- temperature variation (from -120°C up to 120°C).







An example of thin hard coating fracture after cyclic loading

K.L. Dahm, E. Torskaya, I. Goryacheva & P.A. Dearnley. Tribological effects on sub-surface interfaces.
Proceedings of the Institution of Mehcanical Engineers J: Journal of Engineering Tribology, V. 211 (2007), pp. 345-353.

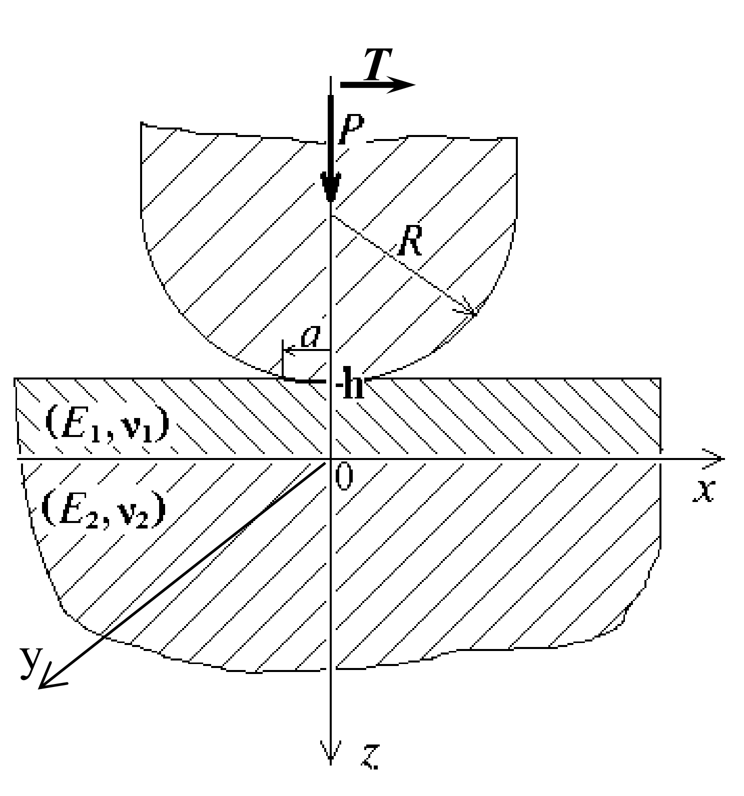
The assumptions used for the problem solution:

- for lightly load bearing an elastic contact of a ball and a trench is considered to be not conform (a probable inaccuracy is evaluated);
- the surface tangential stresses and displacements don’t influence on normal contact characteristics, and contact problem is axisymmetric.

Steps of the problem solution:

- Calculation of contact characteristics (pressure distribution and the size of contact zone) for ball-trench contact in assumption that tangential forces don’t change normal surface displacements.
- Calculation of tangential stress distribution inside the contact zone from the value of friction moment
- Calculation of internal stresses

Contact problem formulation



Boundary conditions at the interface:

$$w^{(1)} = w^{(2)}; \quad u^{(1)} = u^{(2)};$$

$$\sigma_z^{(1)} = \sigma_z^{(2)}; \quad \tau_{rz}^{(1)} = \tau_{rz}^{(2)},$$

Boundary conditions at the surface:

$$w^{(1)}(r) + w^{(3)}(r) = f(r) + D, \qquad 0 < r < a;$$

$$\sigma_z^{(1)} = 0, \qquad a < r < +\infty;$$

$$\tau_{rz}^{(1)} = 0, \quad \tau_{\theta z}^{(1)} = 0. \qquad 0 \le r < +\infty,$$

Equilibrium condition:

$$P = \int_0^a \int_0^{2\pi} p(r) r \, dr d\varphi$$

Torskaya E. V., Goryacheva I. G. The effect of interface imperfection and external loading on the axisymmetric contact with a coated solid // Wear. 2003. Vol 254. Pp. 538–545.

Axisymmetric contact pressure calculation

$$p_i = \int_0^{\infty} Q_i(\alpha) J_0(\alpha r)\alpha\, d\alpha$$

$$\varphi^{(1)}(r,z) = \int_0^{\infty} [(A_1(\alpha)+zB_1(\alpha))e^{\alpha z}+(C_1(\alpha)+zD_1(\alpha))e^{-\alpha z}]\, J_0(\alpha r)\, dr$$

$$\varphi^{(2)}(r,z) = \int_0^{\infty} (A_2(\alpha)+zB_2(\alpha))e^{-\alpha z}\, J_0(\alpha r)\, dr$$

$$G_j(A_1(\alpha), B_1(\alpha), C_1(\alpha), D_1(\alpha), A_2(\alpha), B_2(\alpha), z) = Y_j(\alpha)$$

$j=1..6$

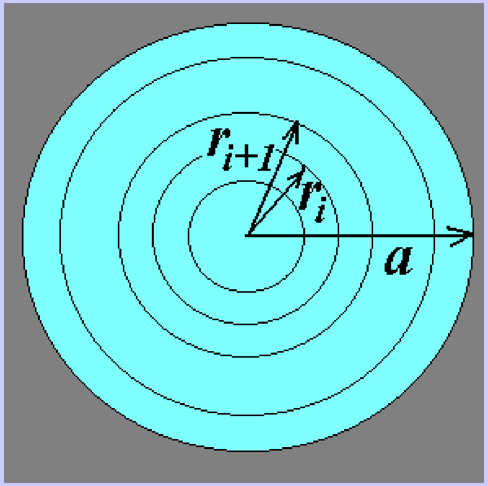
$w^{(1)}, w^{(2)};\ u_r^{(1)}, u_r^{(2)};\ \sigma_z^{(1)}, \sigma_z^{(2)};\ \sigma_r^{(1)}, \sigma_r^{(2)};\ \tau_{rz}^{(1)}, \tau_{rz}^{(2)}$

$$p_1 k_1^{(i)} + p_2 k_2^{(i)} + \ldots + p_N k_N^{(i)} = f(r_i)$$

$i=1,2,\ldots,N-1$

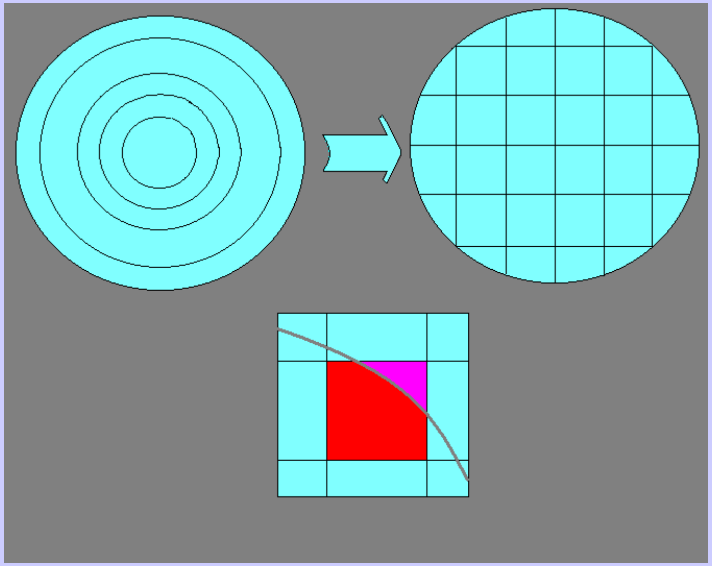
$$\pi \sum_{i=1}^{N} p_i (r_i^2 - r_{i-1}^2) = P$$

$p(r)$



Calculation of internal stresses

$$\sigma_z(x,y) = -p(x,y), \quad \tau_{xz}^{(1)} = -\mu p(x,y), \quad 0 < r < a;$$
$$\sigma_z^{(1)} = 0, \quad \tau_{xz}^{(1)} = 0, \quad a < r < \infty;$$
$$\tau_{yz}^{(1)} = 0, \quad 0 < r < \infty.$$



$$\sigma_z^{(1)} = -\gamma_j^k p_0, \quad -a + \frac{a}{m}(j-1) < x < -a + \frac{a}{m}j, \quad -a + \frac{a}{m}(k-1) < y < -a + \frac{a}{m}k;$$
$$\sigma_z^{(1)} = 0, \quad \tau_{xz}^{(1)} = 0, \quad a < r < \infty;$$
$$\tau_{xz}^{(1)} = -\mu\gamma_j^k p_0, \quad -a + \frac{a}{m}(j-1) < x < -a + \frac{a}{m}j, \quad -a + \frac{a}{m}(k-1) < y < -a + \frac{a}{m}k;$$
$$\tau_{yz}^{(1)} = 0, \quad 0 < r < \infty.$$

Torskaya, E. V. Analysis of friction effect on stress stateof coated bodies. *J. Frict.Wear*, 2002, **23**(2), 16–23.

$$\overline{\gamma_j^k}(\alpha,\beta)=\int_0^\infty\int_0^\infty(\mu\gamma_j^k p_0 x)e^{-i(\alpha x+\beta y)}\,dx\,dy$$

$$-a+\frac{a}{\Delta s}(j-1)<x<-a+\frac{a}{\Delta s}j,\quad -a+\frac{a}{\Delta s}(k-1)<y<-a+\frac{a}{\Delta s}k$$

$$\tau_{xz}^{(1)}(x,y)=\frac{\partial}{\partial x}(-\mu\gamma_j^k p_0 x),\quad \tau_{yz}^{(1)}(x,y)=\frac{\partial}{\partial y}(-\mu\gamma_j^k p_0 x)=0$$

$$-a+\frac{a}{\Delta s}(j-1)<x<-a+\frac{a}{\Delta s}j,\quad -a+\frac{a}{\Delta s}(k-1)<y<-a+\frac{a}{\Delta s}k$$

Tangential stresses calculation inside the contact zone

Stick zone : $$\left|\boldsymbol{s}(x,y)\right| = 0, \left|\boldsymbol{\tau}(x,y)\right| \le \mu p(x,y), (x,y) \in \Omega_{st},$$

Slip zone:

$$\left|\boldsymbol{s}(x,y)\right| > 0, \boldsymbol{\tau}(x,y) = \mu p(x,y)\boldsymbol{s}/\left|\boldsymbol{s}\right|, \quad (x,y) \in \Omega_{sl}$$

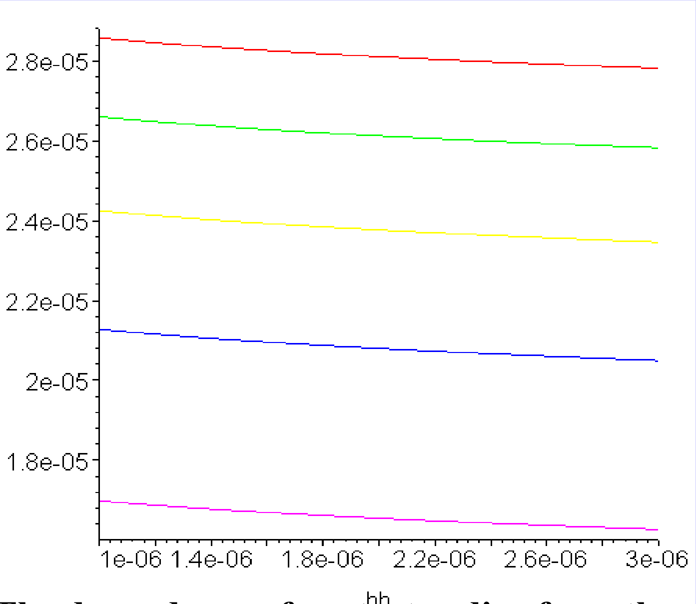
Variation method of solution:

$$\min_{|\boldsymbol{\tau}| \le \mu p} \{F[\boldsymbol{\tau}, \boldsymbol{s}(\boldsymbol{\tau})] = \int_{\Omega} [\mu p \left|\boldsymbol{s}(\boldsymbol{\tau})\right| - \boldsymbol{\tau} \cdot \boldsymbol{s}(\boldsymbol{\tau})] dxdy\},$$

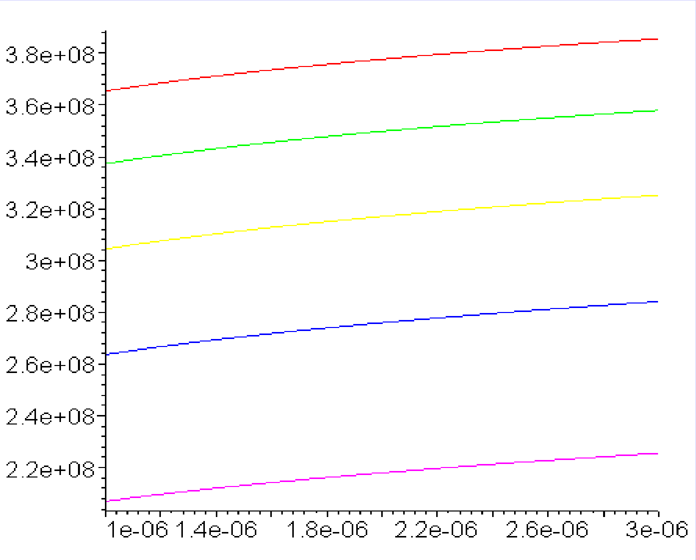
$$\boldsymbol{s} = \boldsymbol{s}[\boldsymbol{\tau}] = -V\boldsymbol{B}(\boldsymbol{\tau}) + \delta \boldsymbol{V}$$

$$\boldsymbol{B}(\boldsymbol{\tau}) = -V \int_{\Omega} B(x-x', y-y') \boldsymbol{\tau}(x', y') dx' dy'.$$

Goldstein R.V., Zazovsky A.F., Spektor A.A., Fedorenko R.P. Solution of space rolling contact problems with steak and slip by variation method // (in Russian) “Uspekhi mekhaniki”, 1982. V. 5 № ¾. Pp. 62-102.

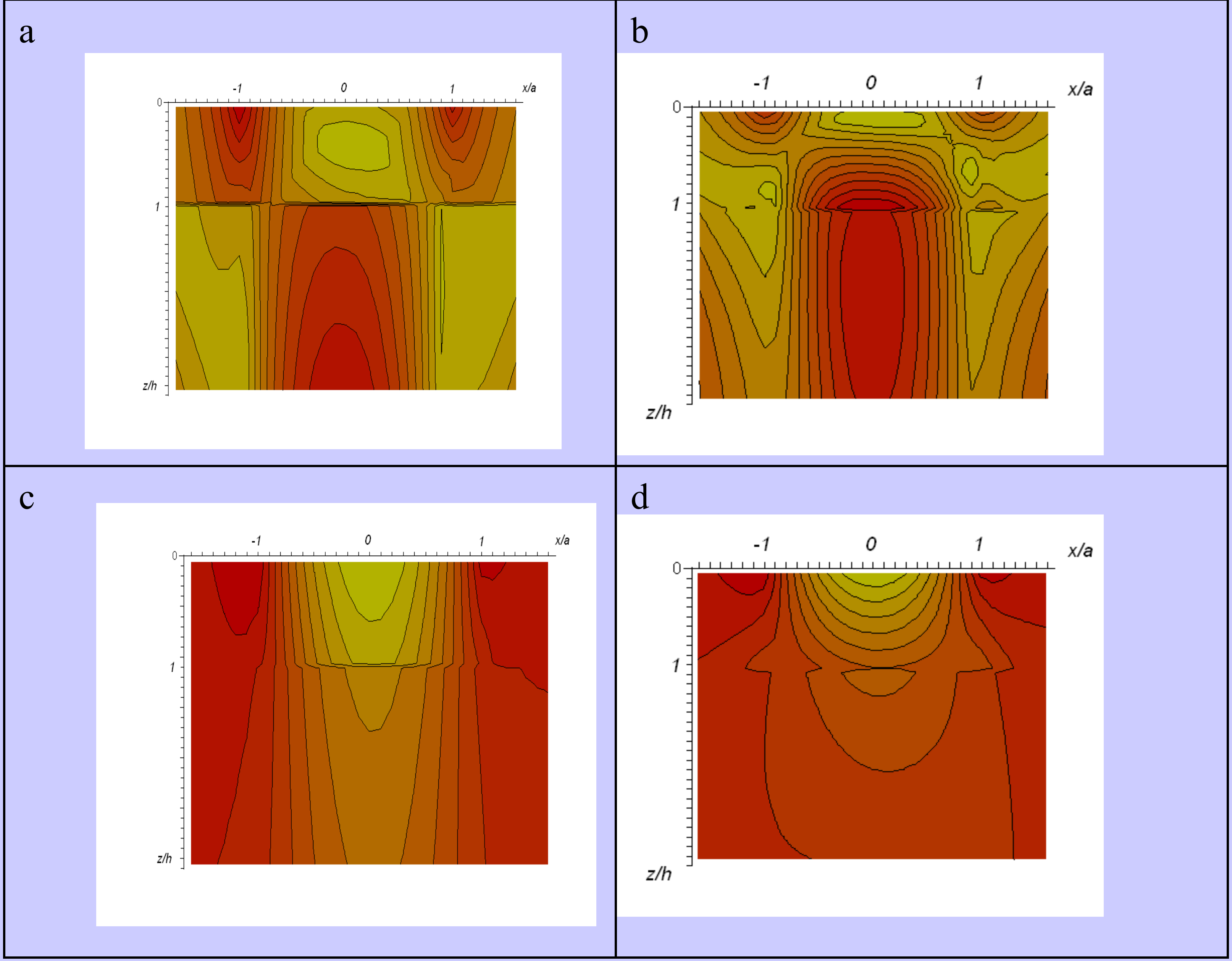


The dependence of contact radius from the coating thickness.



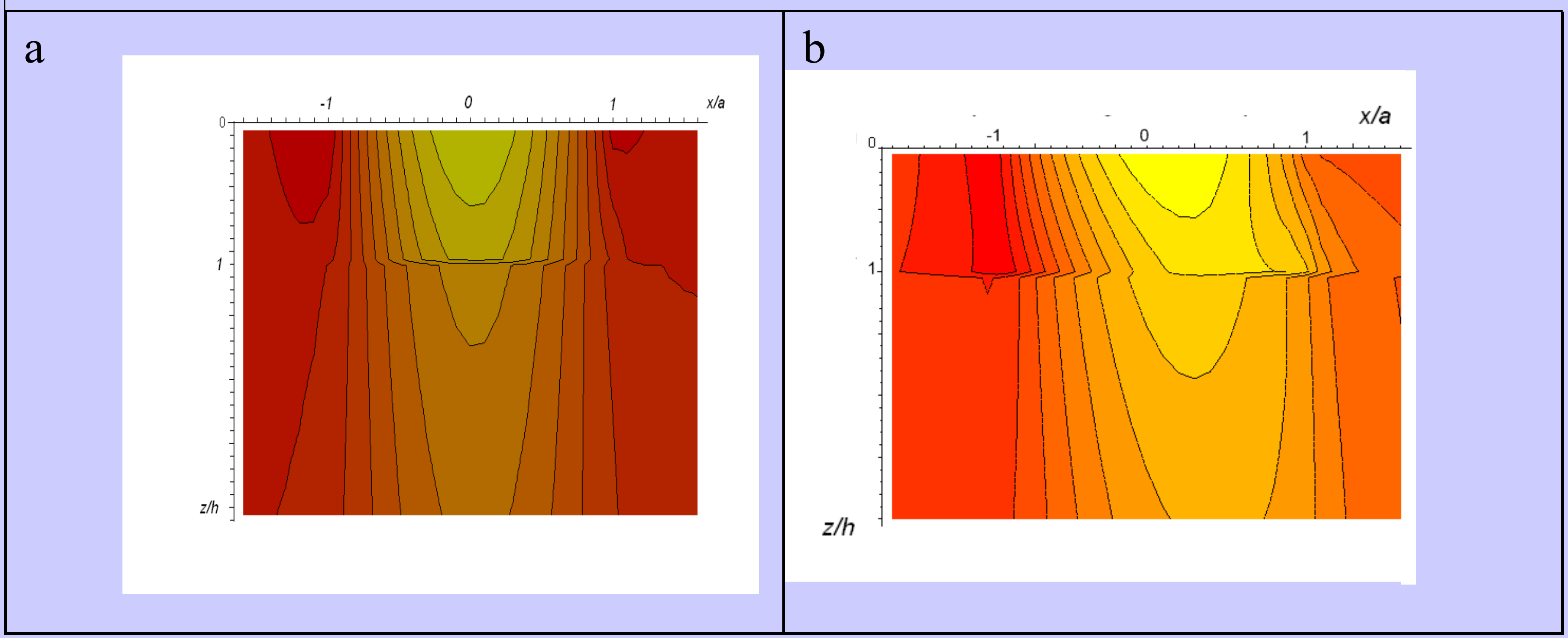
The dependence of maximum contact pressure from the coating thickness.

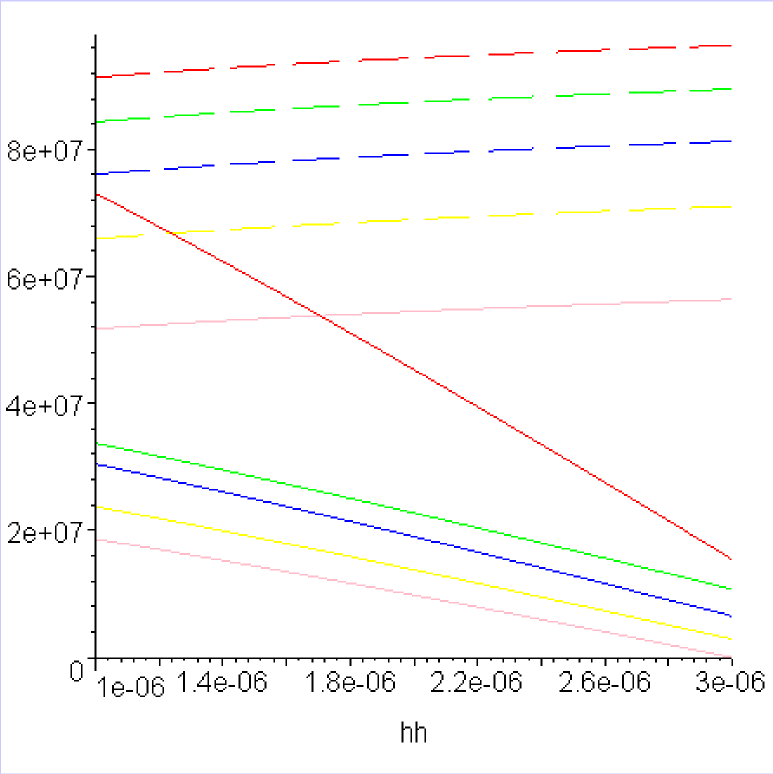
- Rolling bearing: 32 balls (diameter 4.763mm) are in contact with trench. The trench average diameter is 90mm. The load is distributed uniformly over the balls. Maximum load is 100N. The rotation maximum is 1000 rotations per minute.
- Steel trench (E=204GPa) is modified by the coating (E=500GPa), which thickness is varied from 1 to 3 µm.
- The following 5 values of load are considered: 20N, 40N, 60N, 80N, 100N.

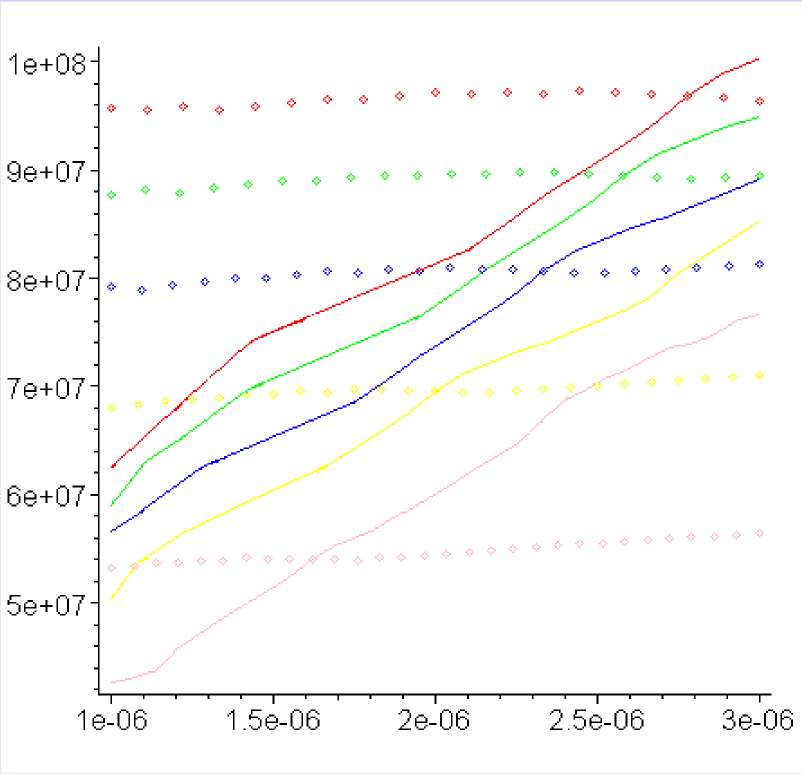


Distribution of principal shear stresses (a, b) and tensile (compressive) stresses (c, d) inside the layer and the substrate for coating thickness 1 μkm and load 100N (a, c), and for the coating thickness 3 μkm and load 20N (b, d).

Distribution of tensile (compressive) stresses inside the layer and the substrate for coating thickness 1 µkm and load 100N for rolling friction (a) and sliding friction (b). Sliding friction coefficient is 0.225.







The dependence of maximum values of tensile stresses from the coating thickness at the surface and at the interface.

The dependence of maximum values of principal shear stresses from the coating thickness at the surface and at the interface.

Conclusion

- For relatively low load (20-40 N) the optimal thickness is 1.0-1.5 μm
- For relatively high loads (40-100 N) the thickness is 1.5-2.0 μm.