Unilateral contact analysis for masonry bridges: Direct and inverse problems

G.A. Drosopoulos, M.E. Stavroulaki, G.E. Stavroulakis,

Institute of Computational Mechanics and Optimization
www.comeco.tuc.gr
Technical University of Crete, Chania, Greece and
Depatment of Civil Engineering,
T.U. Braunschweig, Germany

Olympia: a monument with contact interfaces

Masonry bridges in Epirus

Motivation

- Strength of masonry – unilateral interfaces
- Contact analysis – for service and collapse
- Collapse – failure mode and load
- Classical approaches – rigid-contact models
- Bridges – ideal structures for comparison
- The method is obviously applicable to other masonry (3-D, shells, cathedrals ..)

Motivation

- Inverse analysis: from a given crack pattern, find the cause of damage
- Trial-and-error approach
- MPEC approach (incomplete)
- Post mortem investigation

Outline

- Unilateral contact and friction (nonsmooth mechanics)
- Solvability related to collapse
- Failure analysis of bridges
- Effect of fill, settlement of support etc
- Parametric investigation (shape of bridge)
- Strengthening (FRP reinforcement, delamination)

GENERAL ATTRIBUTES OF STONE ARCH BRIDGES

- Non homogenous material consisted of

a) Stones (often positioned as blocks in a segmental shape)

- high strength in compression
- low strength in tension ⟶ No tension material

b) Mortar joints (frictional joints)

- generally low strength

- Geometrical form + self weight: An issue of great importance for the stability of the structure

- Usage of non – linear mechanics

CATEGORIES OF NON LINEAR MODELS

I. Discrete models

- The structure is divided into large discrete parts
- Unilateral law possibly with friction is used for the interfaces

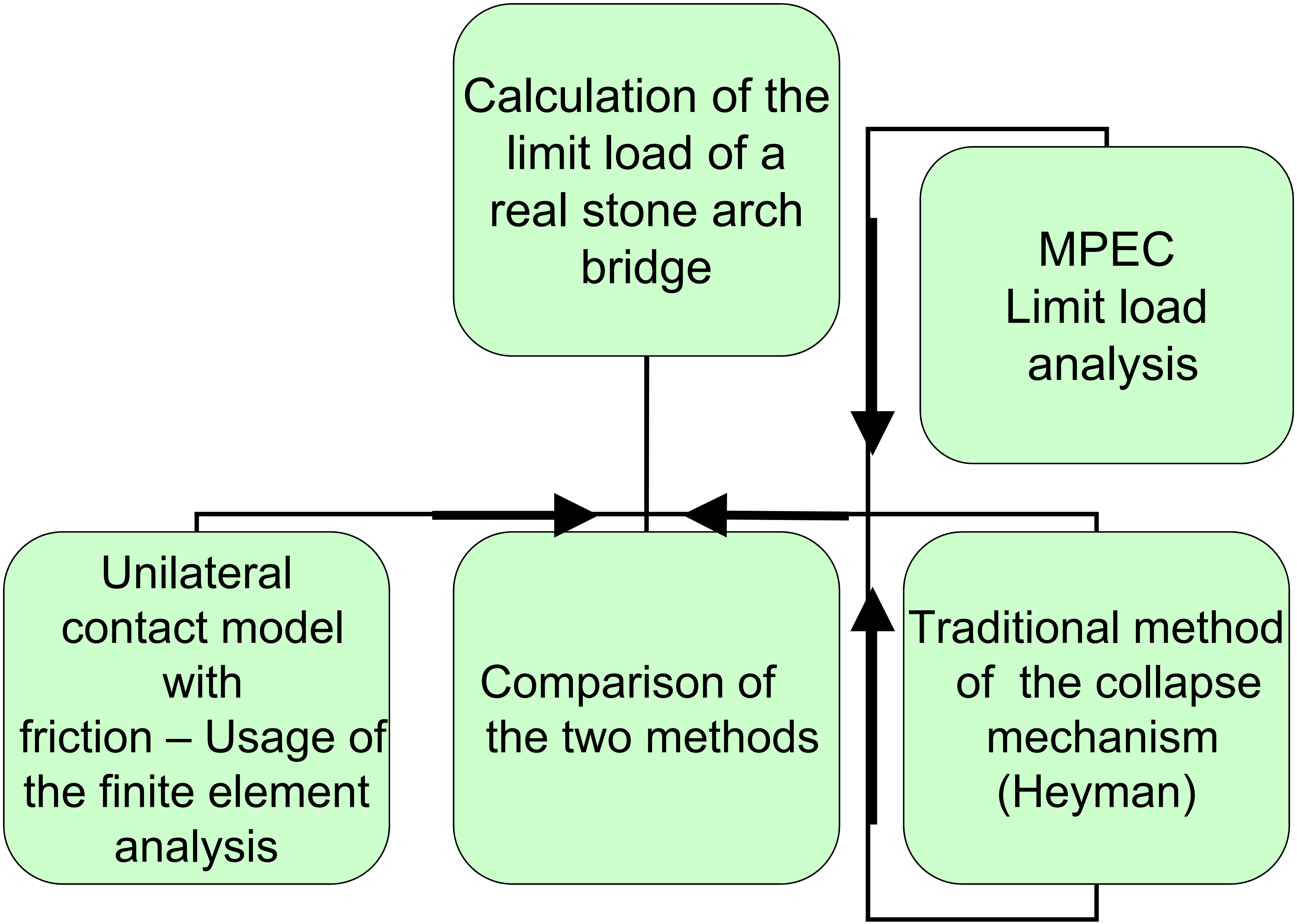
II. Continuum models

- Non-linear constitutive law where either

a) Adoption of a single material → Inelastic theory

b) Anisotropy induced by stones-mortar → Homogenization

In the present study



LIMIT LOAD ANALYSIS IN THE FRAMEWORK OF THE MATHEMATICAL PROGRAMMING

Problem related to lower bound theorem

maximize λ

- Equilibrium equations: $\mathbf{Ku} + \mathbf{N}^T\mathbf{t}^n = \mathbf{P}_o + \lambda\mathbf{P}$
- Contact constraints:

$$\begin{cases} -\mathbf{t}^n \geq 0 \\ \mathbf{Nu} - g \leq 0 \\ (\mathbf{Nu} - g)^T\, \mathbf{t}^n = 0 \end{cases}$$

- $\mathbf{P}_0$, $\mathbf{P}$: The self weight and the live load

- **K**: The stiffness matrix – **N:** geometric transformation matrix

- **u**: The displacement vector - $\mathbf{t}^n$: normal pressure

- λ: is a scalar loading factor ($0 \leq \lambda \leq \lambda_{failure}$)

- g: initial gap

Non smooth
Parametric
LCP problem

Problem related to upper bound theorem

minimize λ

- <u>Equilibrium equations</u>: $\mathbf{K}\mathbf{u} + \mathbf{N}^T\mathbf{t}^n = \mathbf{P}_o + \lambda\mathbf{P}$

- Collapse condition:
(positive energy dissipation) $\mathbf{P}\mathbf{u} > 0$ ($\mathbf{P}\mathbf{u} = 1$)

- <u>Contact constraints</u>:

$$\begin{cases} -\mathbf{t}^n \geq 0 \\ \mathbf{N}\mathbf{u} - g \leq 0 \\ (\mathbf{N}\mathbf{u} - g)^T \mathbf{t}^n = 0 \end{cases}$$

Non convex MPEC - Limit load problem

M.C. Ferris, F. Tin-Loi, 2001

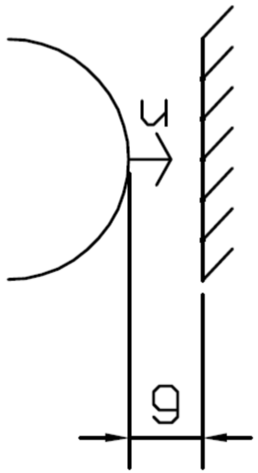
Explanation of our approach

- Discrete model in the framework of the finite element analysis

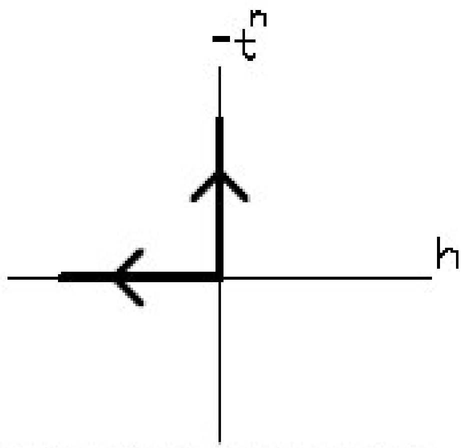
- Contact interfaces simulating potential cracks are considered in the geometry of the bridge
- Opening or sliding of a number of potential interfaces indicates crack initiation / propagation
- Zero tensile resistance in the normal direction of the interfaces

- Validation by the classical collapse mechanism method introduced by Heyman

Unilateral contact problem



Bodies in contact



Graphical representation of the complementarity relation

$u - g \leq 0 \Longrightarrow h \leq 0$ ⟶

- Nonpenetration relation

$-t^n \geq 0$ ⟶

- Only compressive stresses are allowed (contact pressure)

$t^n(u - g) = 0.$ ⟶

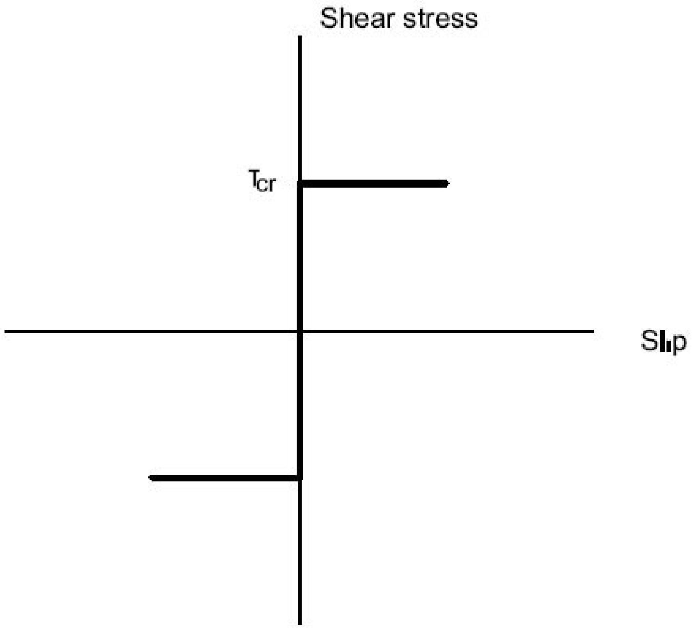
- Complementarity relation

Frictional Stick-Slip problem
(Tangential direction of the interfaces)

Coulomb friction model

- Two contacting surfaces start sliding when $\tau_{cr} = \mu \cdot t^n$
- Stick conditions: No sliding when $\tau < \tau_{cr}$
- τ_{cr}: Critical shear stress
- μ: Friction coefficient
- t^n: Contact pressure

- Graphical representation



Solution of frictional contact probolem (1)

- Equilibrium equations by the Principle of the Virtual Work in a general form

$$\int_V \boldsymbol{\tau} : \delta\boldsymbol{\epsilon} dV = \int_S \delta\mathbf{u} \cdot \mathbf{t} dS + \int_V \delta\mathbf{u} \cdot \mathbf{f} dV + \int_{S'} \delta\mathbf{u} \cdot \mathbf{t}^{\mathrm{n}} dS + \int_{S'} \delta\mathbf{u} \cdot \mathbf{t}^{\mathrm{t}} dS$$

- Normal direction

- Lagrange multipliers (contact pressure) introduced in the Principle of the Virtual Work for the contact constraint

- Consideration of the Complementarity relation

Solution of frictional contact probolem (2)

- Tangential direction

- Lagrange multipliers are used in the Principle of the Virtual Work to enforce sticking conditions

a) Introduction of the relation : $\tau_i = q_i + k_0 \Delta\gamma_i$ where

q_i: Lagrange multipliers

$\Delta\gamma_i$: Slip

k_0 (stiffness term in the tangential direction): To eliminate zero terms on the diagonal of the stiffness matrix that could cause numerical problems when slip occurs

Solution of frictional contact probolem (3)

b) Substitution of the relation $\tau_i = q_i + k_0\Delta\gamma_i$ in the Principle of the Virtual Work

c) $\tau_i < \tau_{cr}$ ⟶ Sticking conditions ⟶ $\Delta\gamma_i = 0$

($\tau_i = q_i$ = shear stress)

$\tau_i = \tau_{cr}$ ⟶ Slipping conditions ⟶ $q_i = 0$

Solution of frictional contact probolem (4)

- Adoption of the Newton – Rapson and load incrementation iterative method for the solution of the non – linear equilibrium equations

- Force convergence: 0.005

- Displacements convergence: 0.01

A path - following method

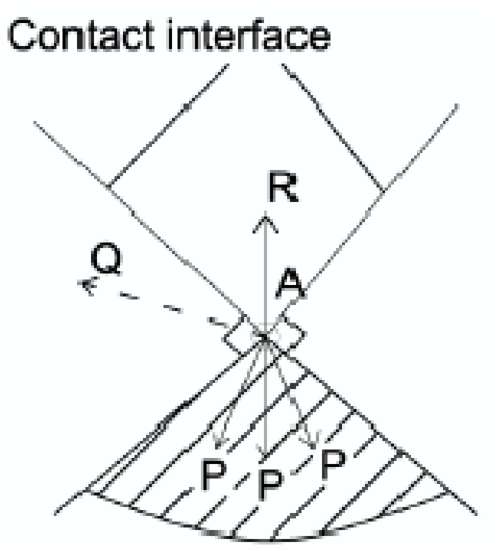
Solvability Conditions

- A part of the bridge between two interfaces may develop rigid body displacements
- Semidefinite stiffness matrices arise
- Equilibrium configuration may or may not exist depending on the geometry of the structure and the direction of the applied loading
- No solution exists if rigid body displacements are not compatible with the constraints of the contact problem

Panagiotopoulos 1985, Stavroulakis et. Al. 2001

Solvability for contact problems

- Signorini-Fichera conditions for variational inequalities



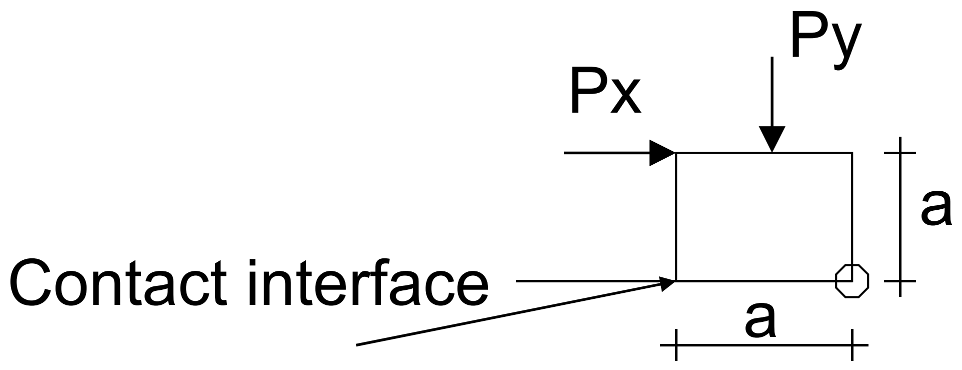
In robotics:

Al-Fahed, Stavroulakis, Panagiotopoulos 1991

In masonry:

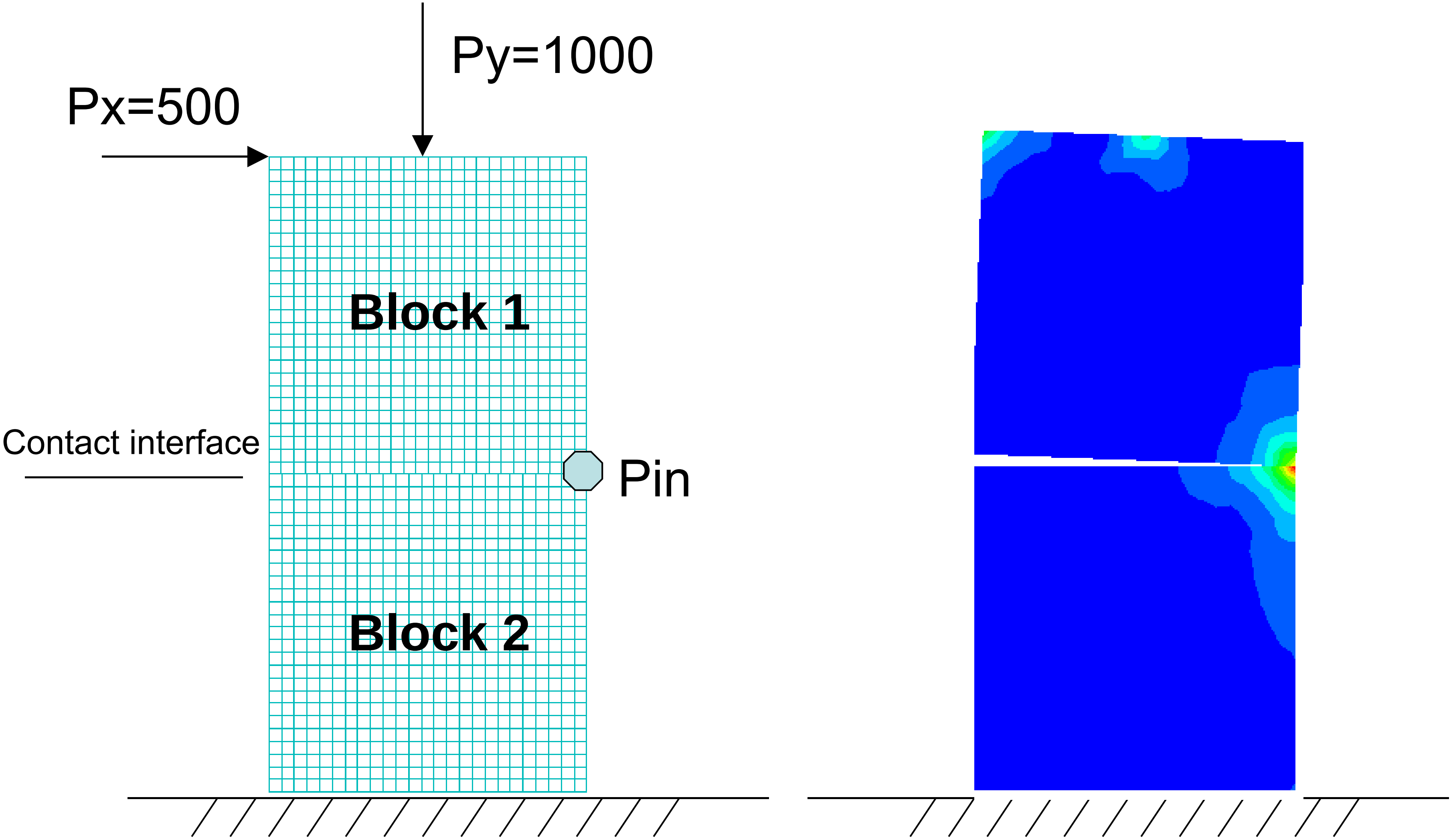
Ferris, Tin-Loi 2001, Lourenco, Ordunia 2003

Solvability conditions: a simple example



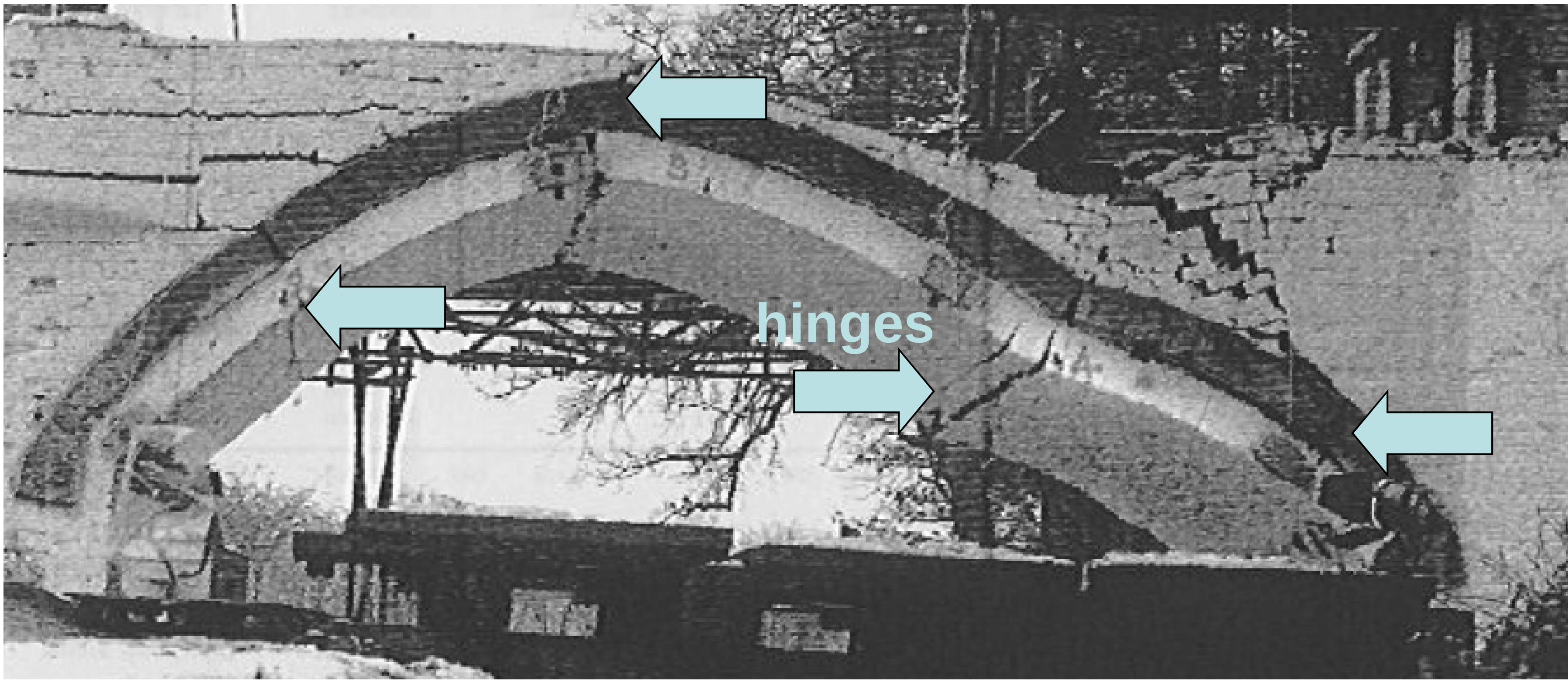
- Overbalance begins when Px = Py/2
- Follows a numerical example with a contact constraint

- Semidefinite or negative definite stiffness matrices arise when the gap appears
- A first numerical validation



A REAL BRIDGE TESTED TO COLLAPSE - 4 HINGES MECHANISM

Quarter span loading



Page J. Load tests to collapse on two arch bridges at Preston, Shropshire and Prestwood Staffordshire. TRL, 1987

A FIRST APPLICATION – STRATHMASHIE BRIDGE

It will be shown that:

- Identical failure mechanism arises in comparison with the classical collapse mechanism method

- Convergence in the value of the failure load in comparison with the classical collapse mechanism method

- Meaningless the exact number of the interfaces along bridge’ s geometry in case many interfaces are used

- Comparison between quarter – middle span loading demonstrates smaller failure load in quarter loading

MODELING INFORMATION

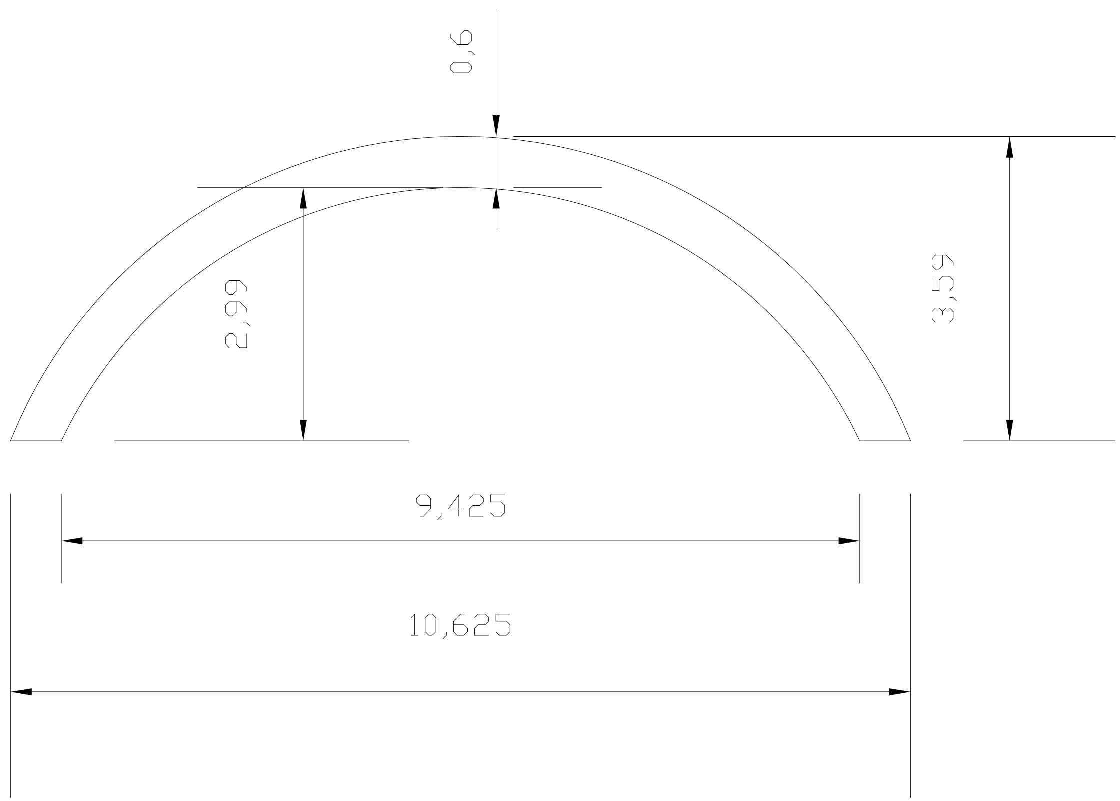
- Mechanical properties

YOUNG MODULUS (Gpa)	5
POISSON RATIO	0.49
DENSITY (Kg/m3)	2200
FRICTION COEFFICIENT	0.6

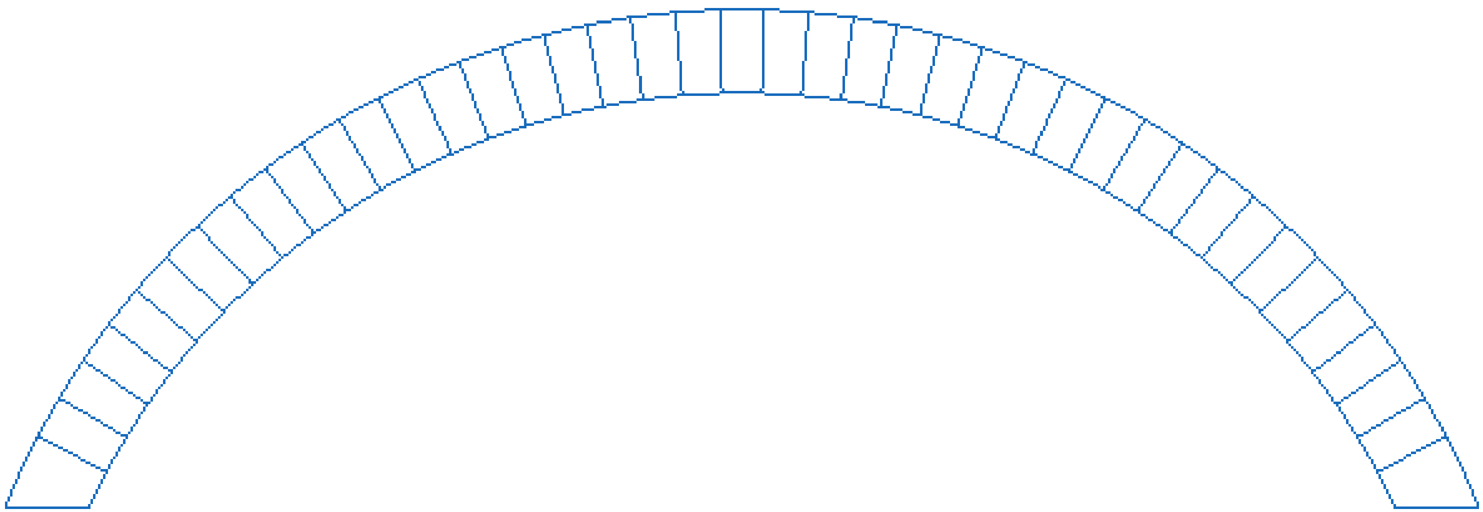
- Finite element modeling

- Increasing number of unilateral contact interfaces
- Perpendicular interfaces to the center line of the arch
- 3036 quadrilateral four - node plain strain elements are used
- No fill is regarded
- 0.02KN load increment is used in the N-R iterative solution
- Large displacement effects are neglected
- friction coefficient 0.6 -> no slip

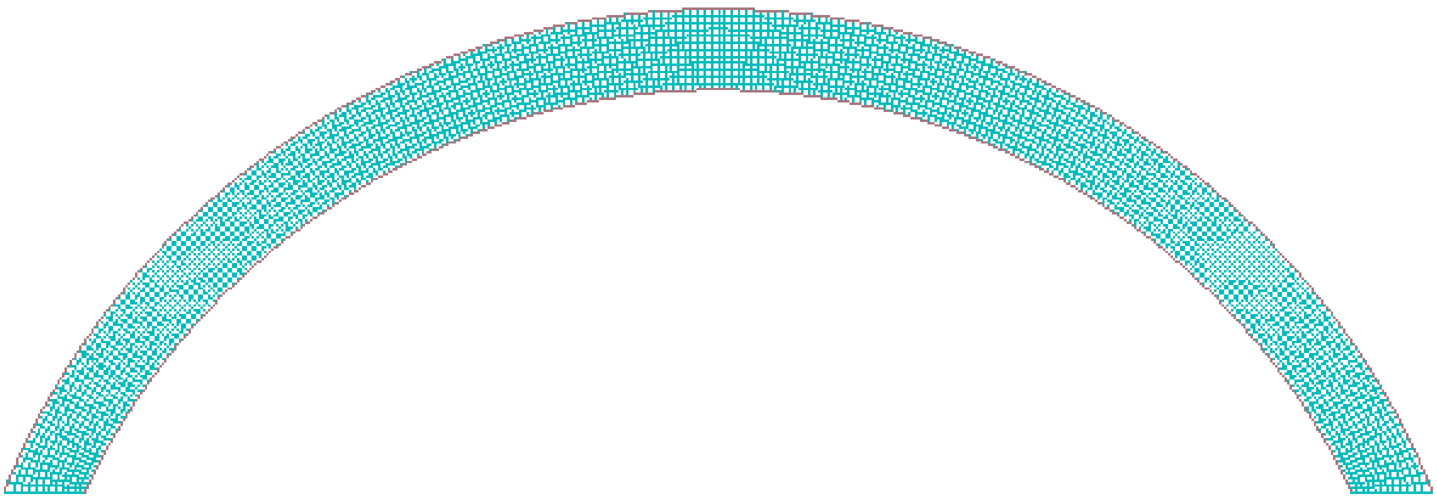
GEOMETRY OF THE STRATHMASHIE BRIDGE



MODEL WITH 40 INTERFACES

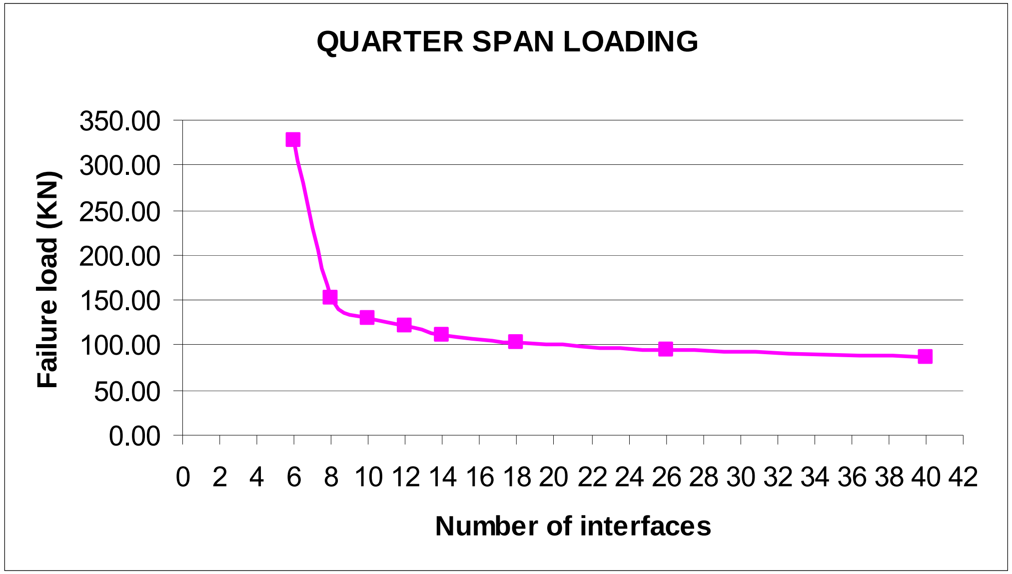


MESH OF THE MODEL

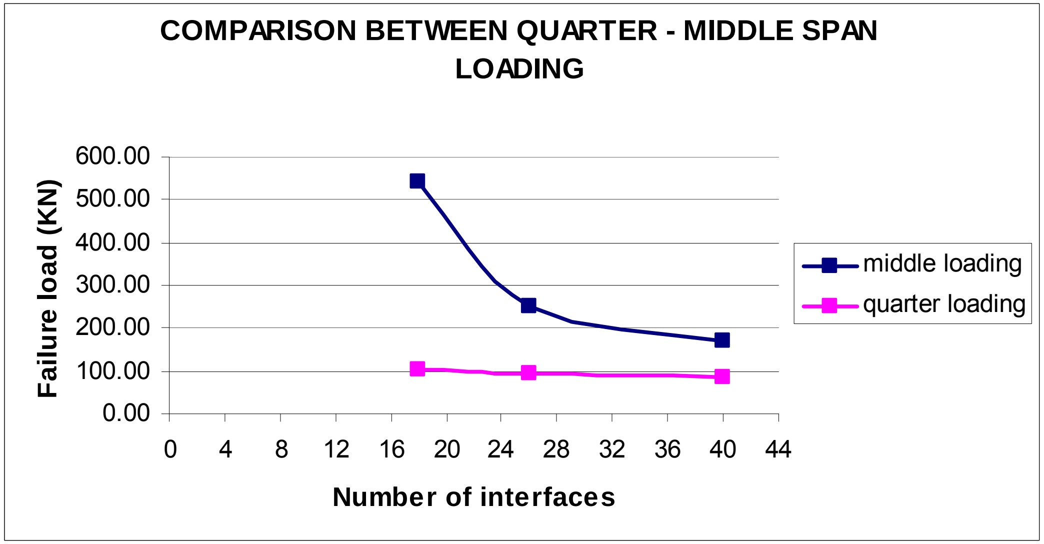


RESULTS

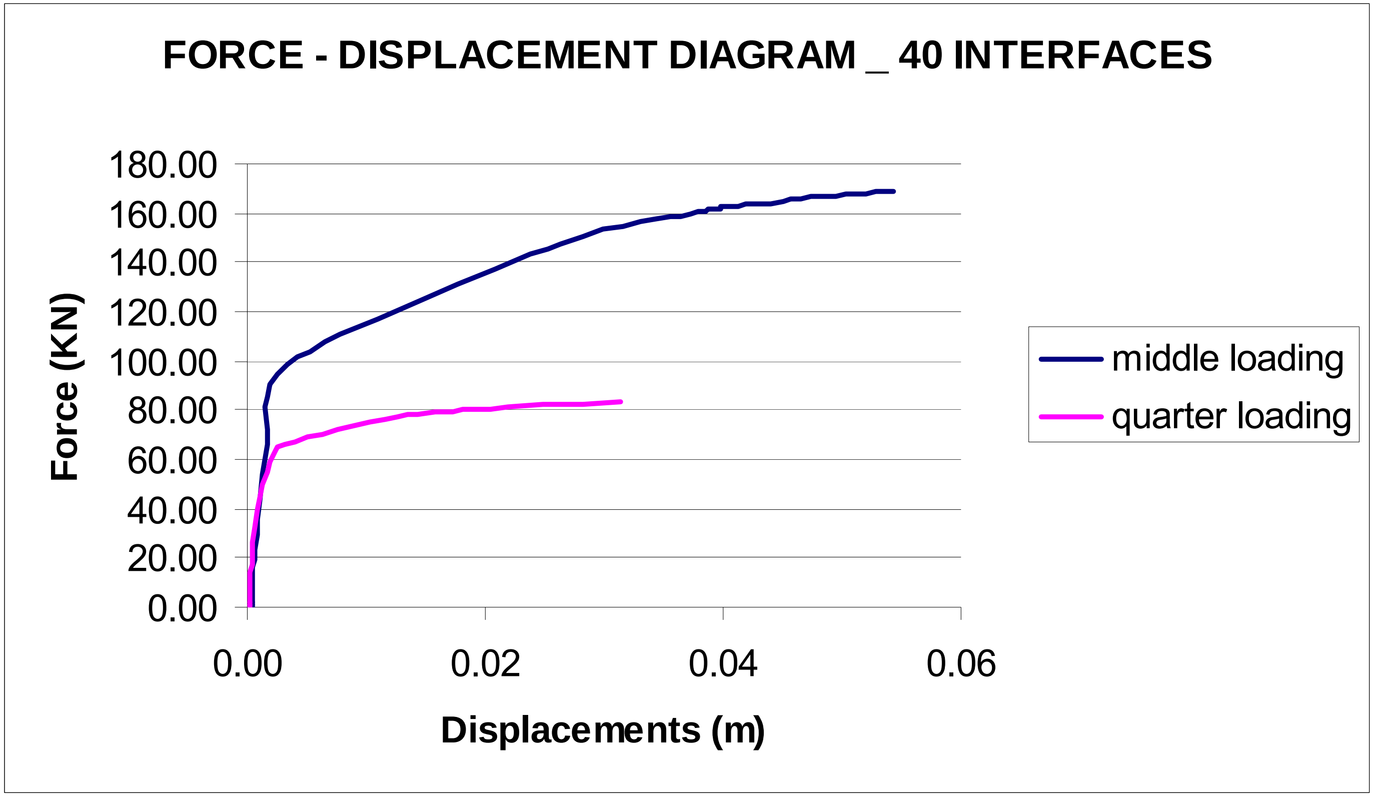
- Almost horizontal branch denoting convergence of the failure load prediction for a large number of interfaces



- Greater failure load for a middle span load



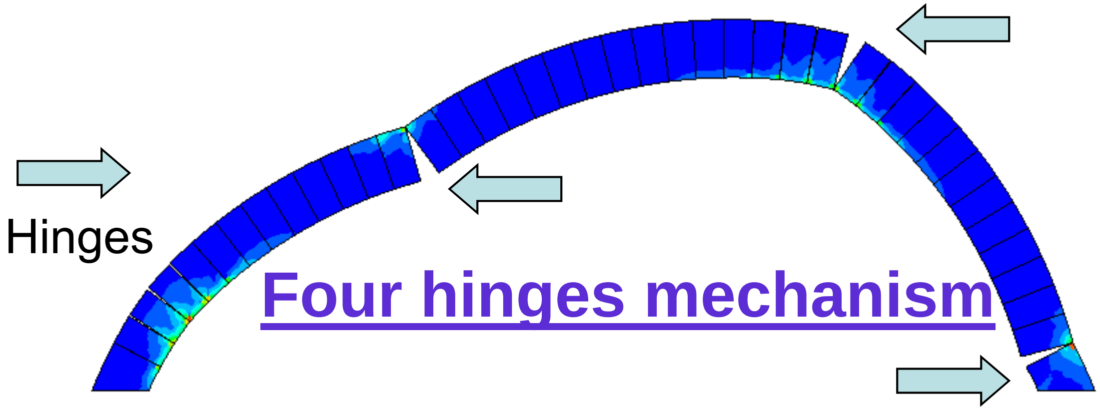
- Force – displacement diagram in the model of 40 interfaces
- Different failure load for middle vs quarter span loading



UNILATERAL CONTACT FRICTION MODEL - FEA

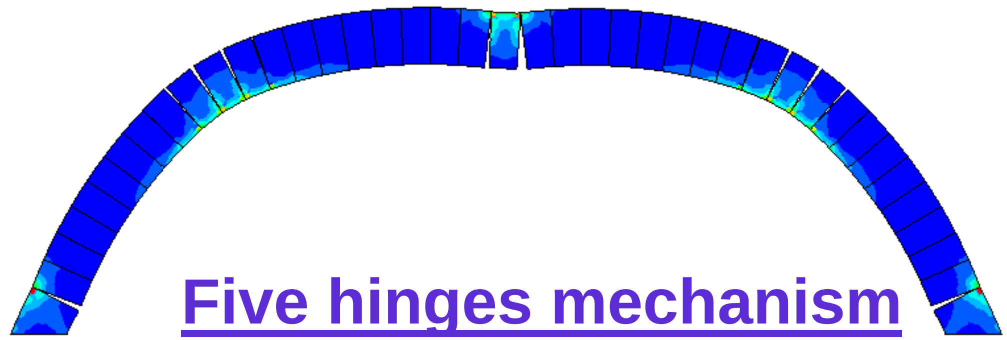
- Failure mechanism – Quarter span loading

Failure load = 87.14 KN



- Failure mechanism – Middle span loading

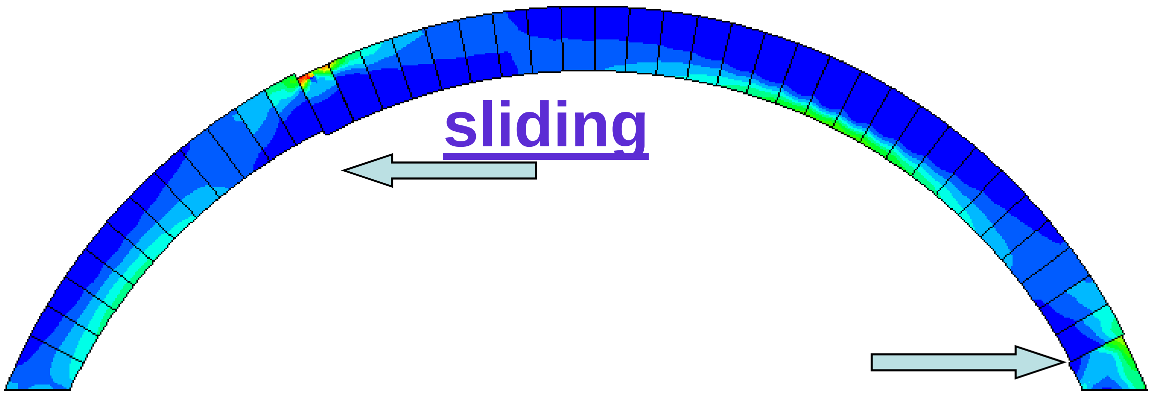
Failure load = 168.95KN



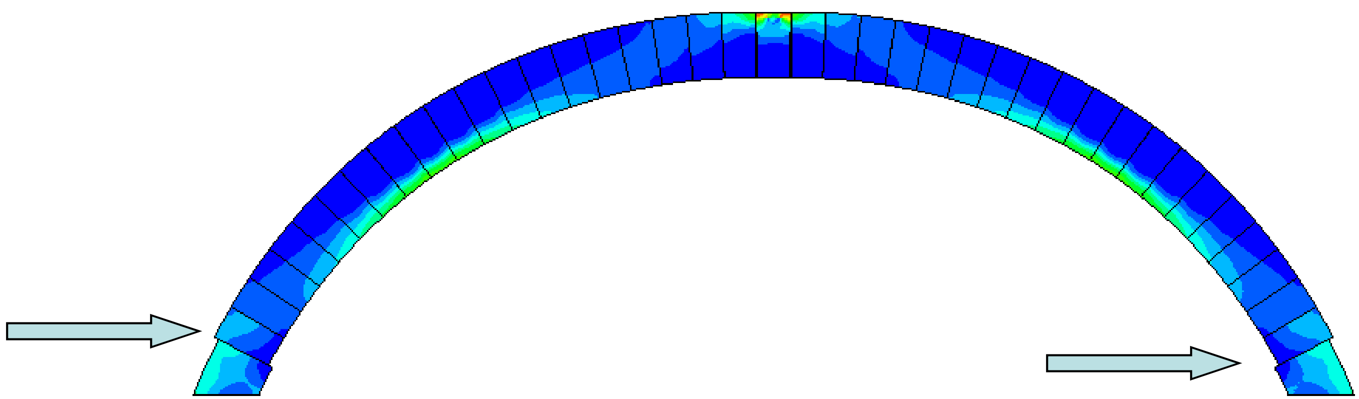
AN ADDITIONAL CAPABILITY OF THE PROPOSED METHOD

- **Sliding mode failure**
- (sliding could be crucial for the behavior of the bridge)

Quarter span loading



Middle span loading



CLASSICAL COLLAPSE MECHANISM METHOD
(Introduced by Heyman)

Overview of the method

- Thrust line: A funicular polygon whose position in a cross section defines the resultant force in this section when the structure is in equilibrium

- When the thrust line in a cross section is adjacent to the ring of the arch a hinge is opened at that point

- Three - pin arch: A statically determinate structure
  Opening of a fourth hinge generally leads to a mechanism

- Thrust line → Equilibrium equations → Hinges' opening → A mechanism + Failure load

<u>Main assumptions of the collapse mechanism method</u>

I. Sliding failure cannot occur

II. No tensile strength of the masonry

III. Infinite compressive strength of the masonry

MODERN IMPLEMENTATION OF THE COLLAPSE MECHANISM METHOD WITH LINEAR PROGRAMMING

Ring:

- A program developed by M. Gilbert, H. Ahmed, A. Sollis

- Based on the classical collapse mechanism method

- The stucture is divided into a number of rigid blocks

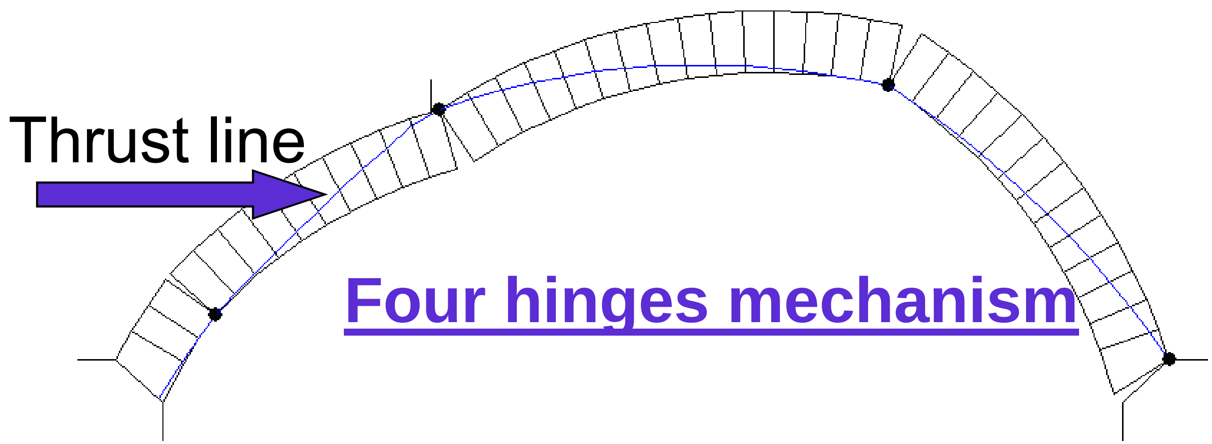
- Interfaces allow opening or sliding among blocks

- A linear programming formulation related to upper bound theorem is used for the solution of the limit load analysis

COLLAPSE MECHANISM METHOD – LINEAR PROGRAMMING

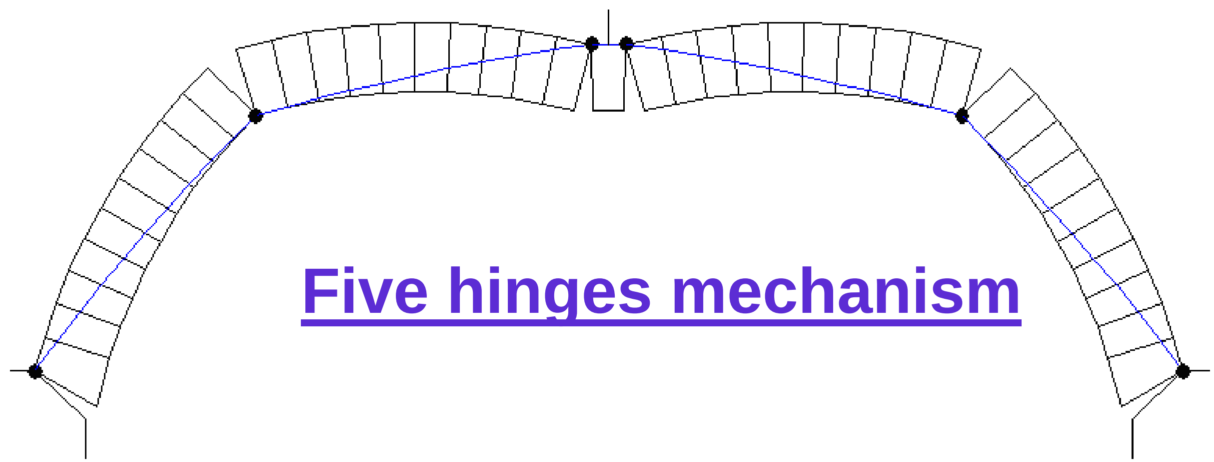
- Failure mechanism – Quarter span loading

Failure load = 88.64 KN



- Failure mechanism – Middle span loading

Failure load = 174,83 KN



Unilateral contact friction Model - FEA

FILL

AN IMPORTANT PARAMETER IN STONE BRIDGES STUDY

- Ground material over the arch barrel

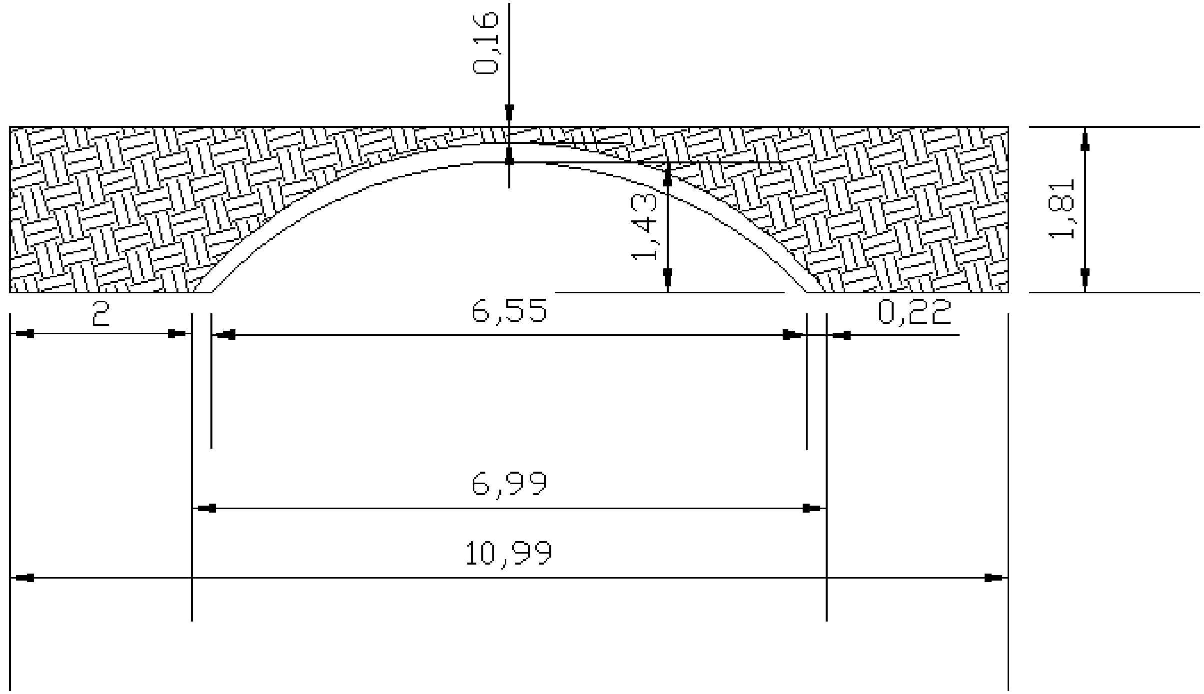
- Generally improves the behavior of the bridge:

- Distributes concentrated load through the mass of the bridge

- Increases stability by inducing initial compression in the arch prior to live loading

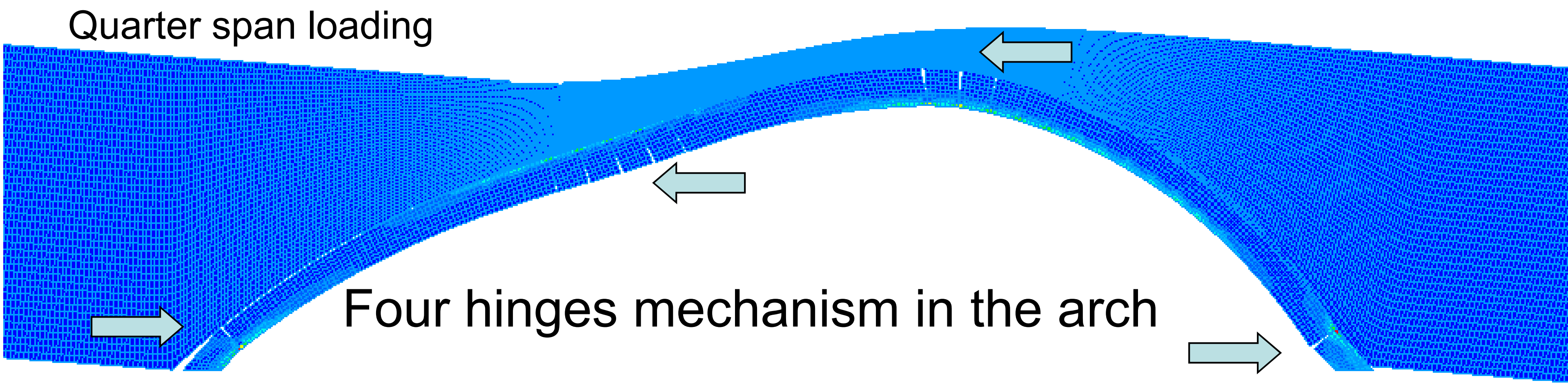
A SECOND APPLICATION – PRESTWOOD BRIDGE

- Unilateral contact friction interfaces for the arch
- Unilateral contact friction interface arch – fill
- Linear elastic material for the fill



MECHANICAL PROPERTIES			
	MASONRY	FILL	ARCH - FILL INTERFACE
YOUNG MODULUS (Gpa)	15	0.3	Unilateral contact friction law Friction coefficient 0.6
POISSON RATIO	0.3	0.3	
DENSITY (Kg/m^3)	2000	2000	
FRICTION COEFFICIENT	0.6		

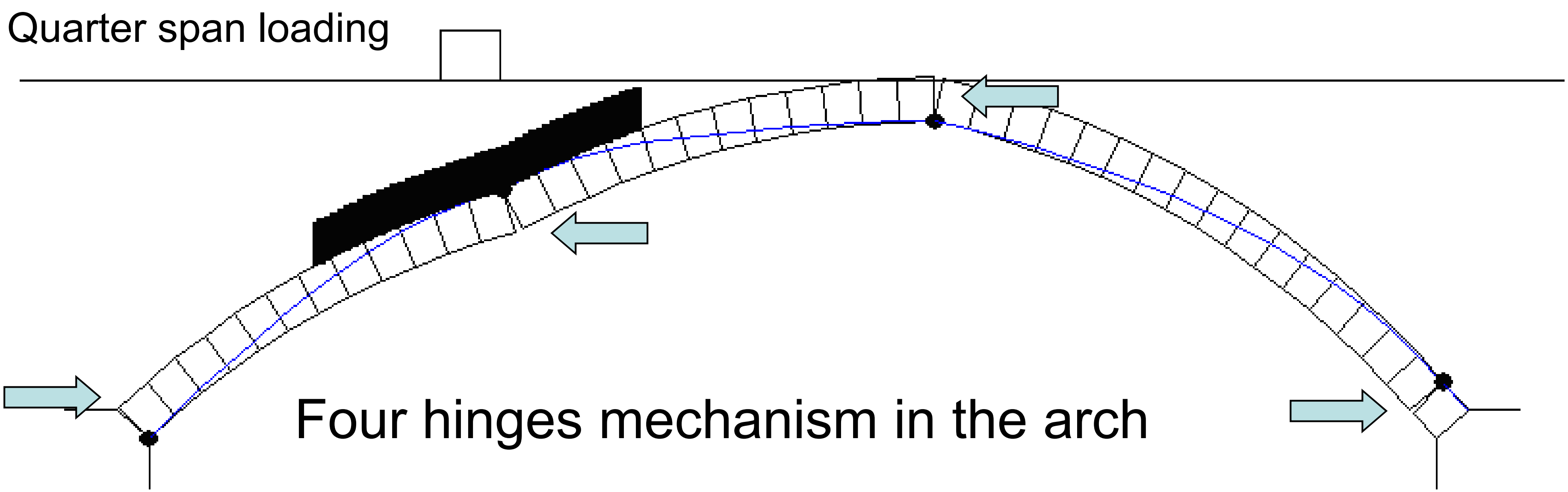
RESULTS – UNILATERAL CONTACT FRICTION MODEL



• Significant increase of the failure load in comparison with the case without fill – Confirmation by the collapse mechanism method

•Overestimation of the failure load due to the consideration of linear elastic material for the fill

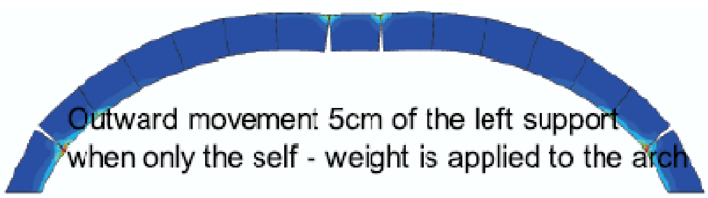
RESULTS – COLLAPSE MECHANISM METHOD

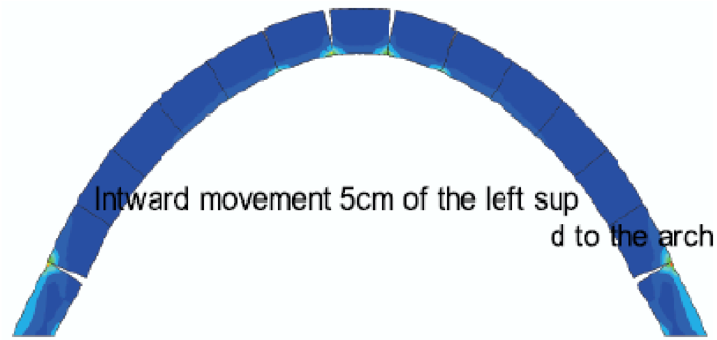


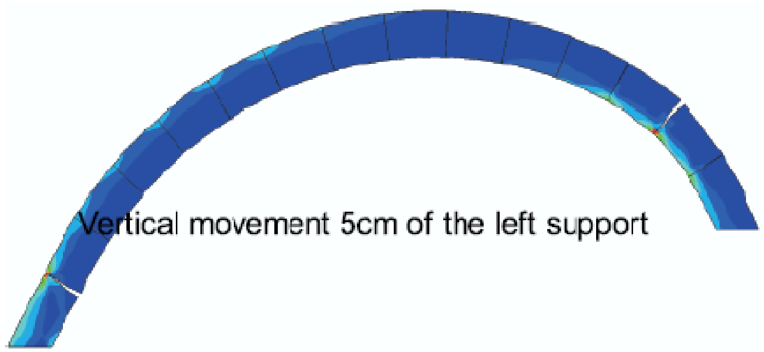
- Confirmation of the significant increase of the failure load in comparison with the case without fill

- Significant influence of the fill parameters on the failure load

Parametric Investigation





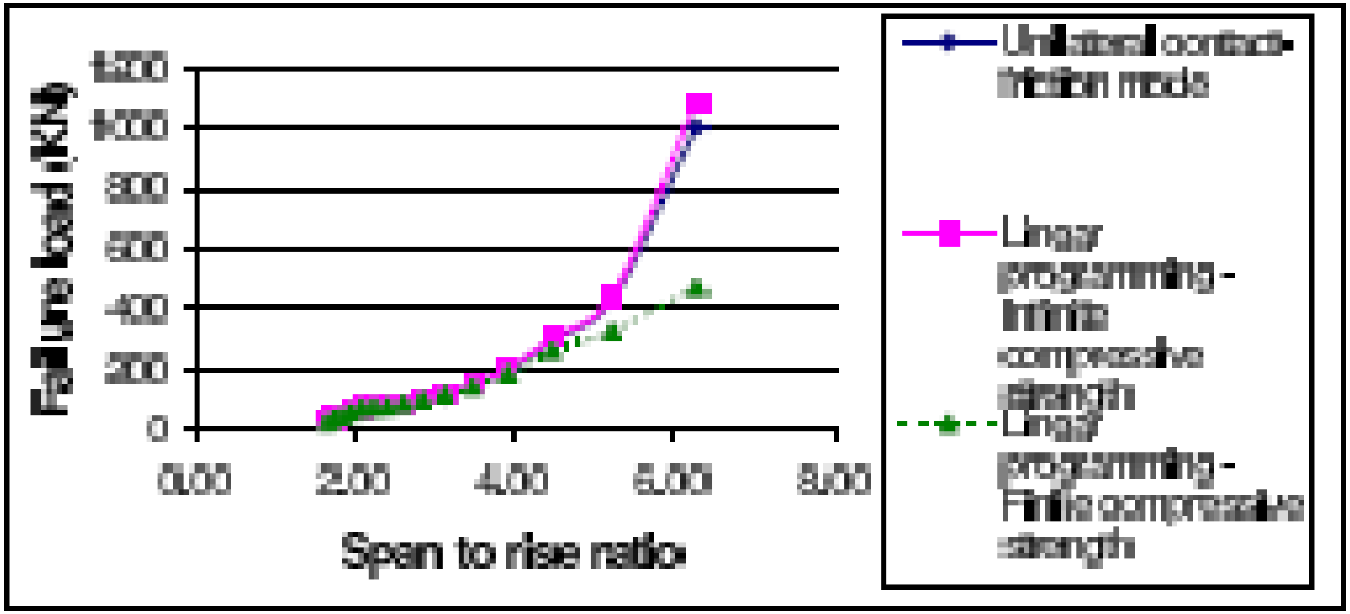


Parametric Investigation

- Reduction of the rise (for same span) causes an increase of the ultimate load, until an ‘optimum geometry’
- Deep arches -> four hinges collapes mechanism
- Shallow arches -> compressive failure of the masonry

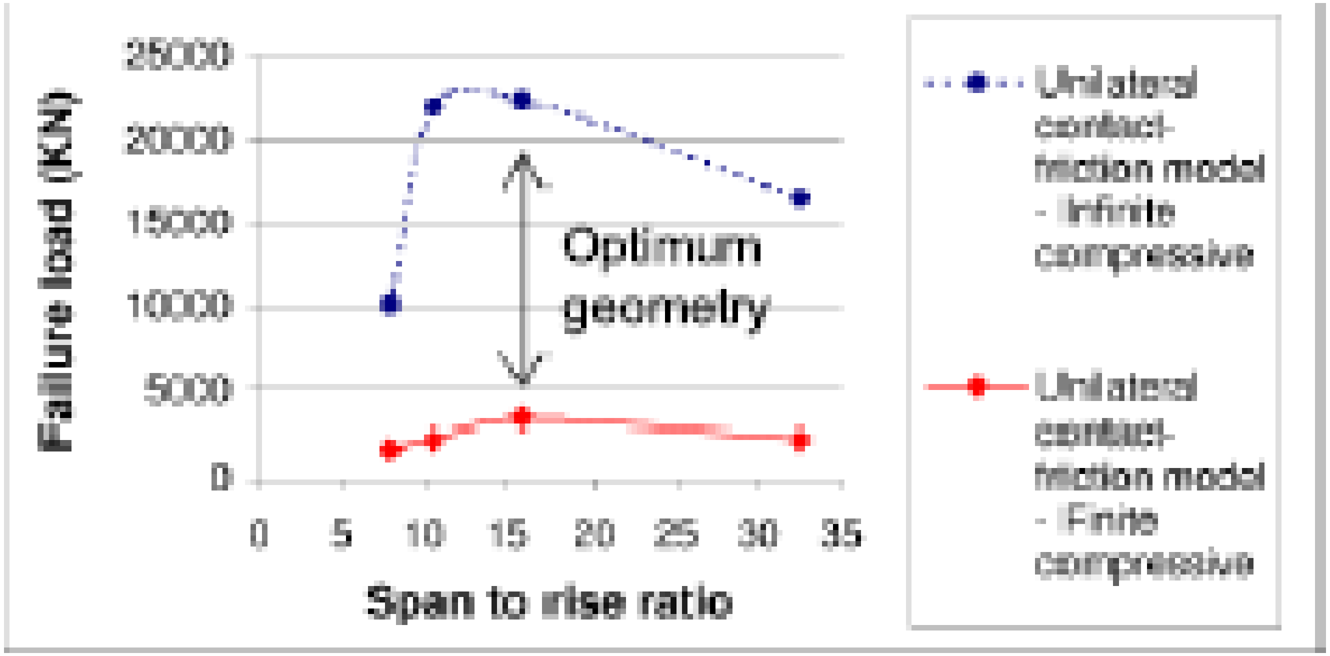
Parametric Investigation

- Limit load – span to rise ratio diagrams



Parametric Investigation

- Limit load – span to rise ratio diagrams



FRP Reinforcement

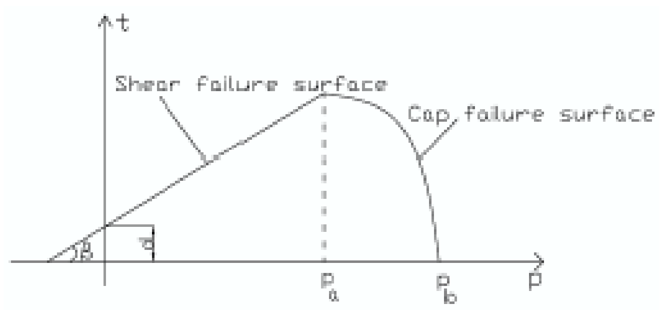
- Fiber Reinforced Polymers strips
- Polymeric matrix + different fibers (glass, carbon, etc)
- High tensile strength, negligible self-weight and corrosion
- Brittle behaviour, possibility of poor bond with the masonry (debonding)

FRP: modelling details

- Von Mises yield criterion
- No-compression
- Contact between FRP and masonry
- Delamination between FRP and masonry

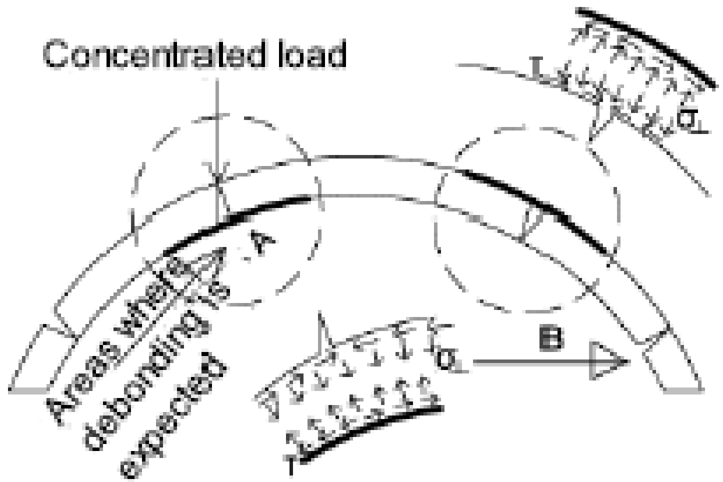
Compressive failure of arch

- Due to reinforcement (for higher loads)
- A Drucker-Prager plasticity with a cap
- The cap yield is a bound for the hydrostatic compression
- Bilinear hardening law between hydrostatic compression yield stress and the plastic strain



Debonding

- For reinforcement placed on the intrados
- Occurs in areas of the intrados where a hinge of the collapse mechanism tends to open



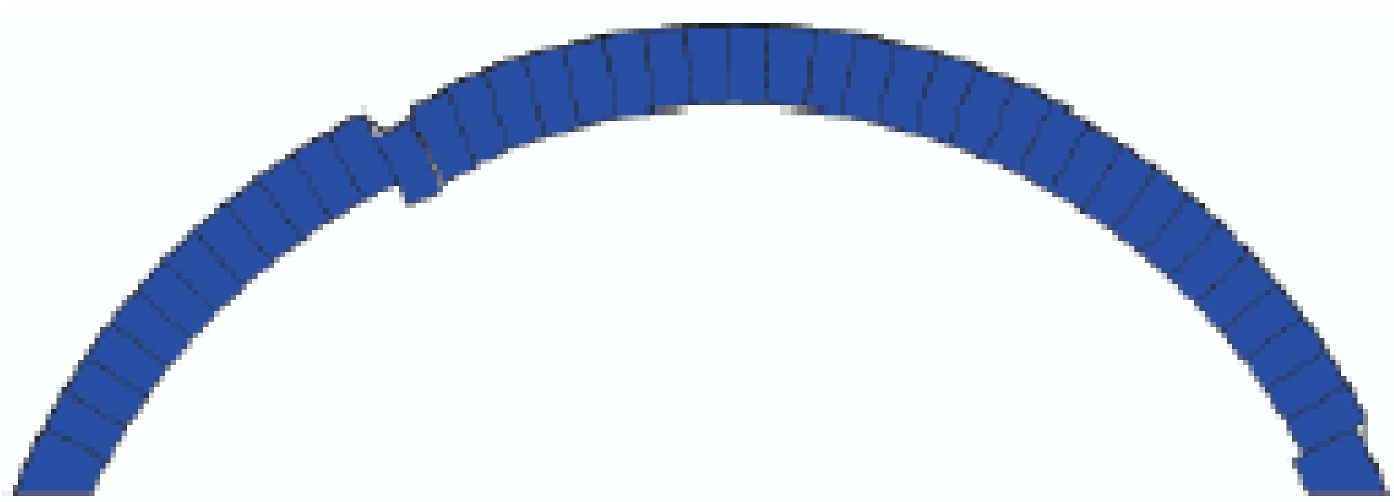
FRP reinforcement

- On whole length of extrados → Sliding of masonry at the springing and point of ext. loading
- On whole length of intrados → Compressive failure (for strong masonry-FRP connection) or debonding
- Both on extrados and intrados of the arch → Debonding of the FRP or the four hinges mechanism

FRP attached to the whole extrados

- Collapse due to masonry sliding at the springing and at the point of loading, similar to experiments
- Six times higher ultimate load
- FRP yields at the areas where sliding with masonry takes place, no compressive failure of masonry

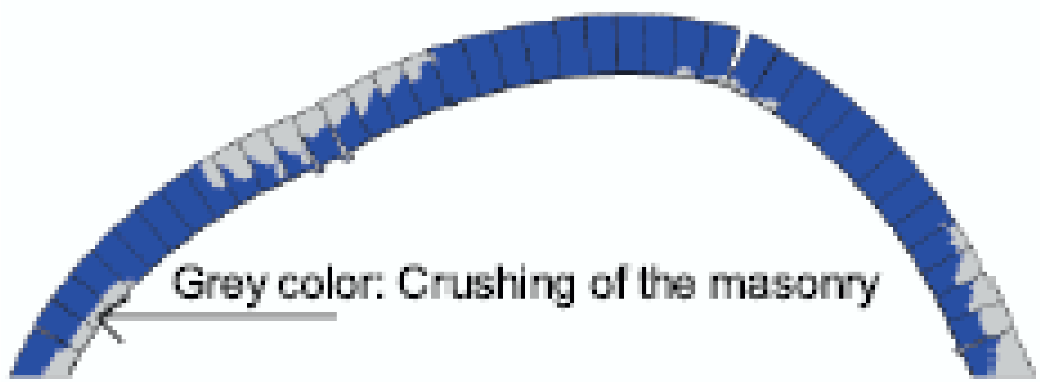
FRP attached to the whole extrados

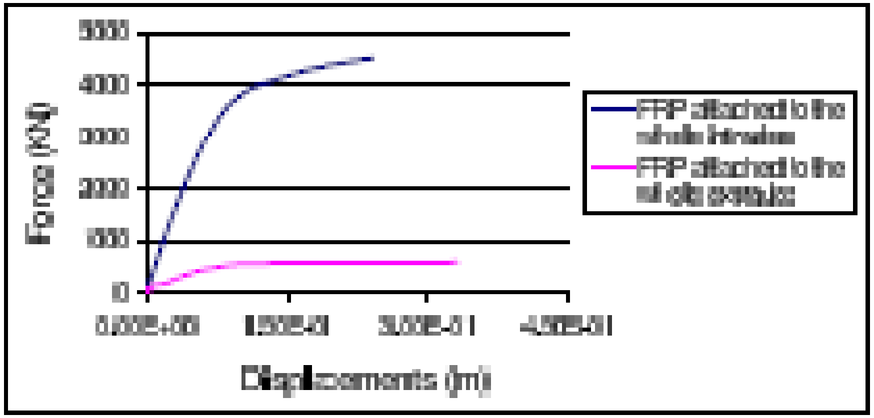


FRP attached to the whole intrados (a: crushing of masonry)

- For strong masonry-FRP connection
- At higher ultimate loading
- Occurs in places where a hinge of the collapse mechanism opens, in the boundary of the ring opposite to the crack
- FRP yielding takes place near the point of loading as well as in the springing opposite to the loading
- FRP at intrados is more effective in comparison to exterior FRP (for no debonding)

FRP attached to the whole intrados (a: crushing of masonry)

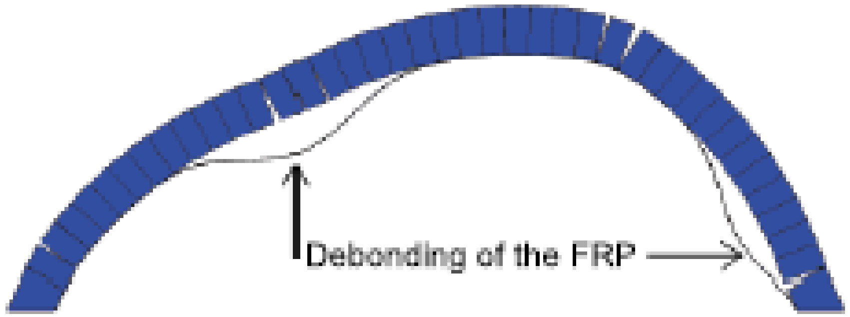


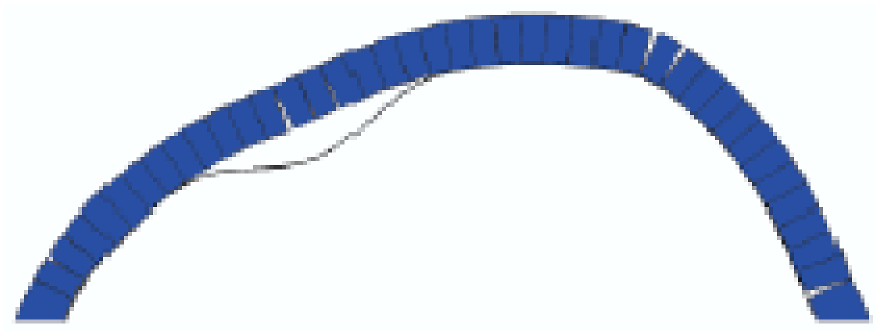


FRP attached to the whole intrados (b: debonding)

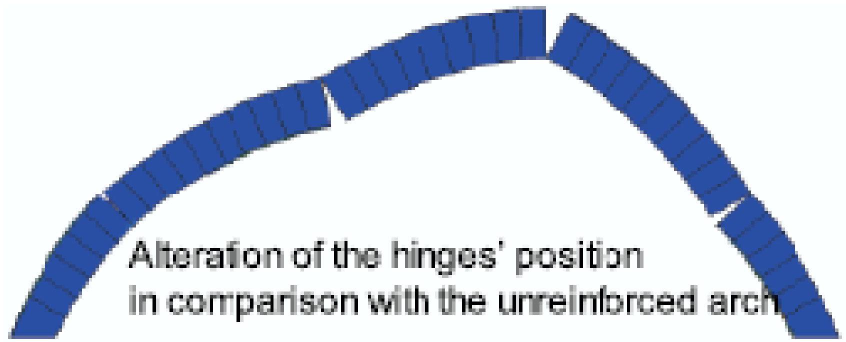
- For weaker masonry-FRP connection
- Detachment occurs near the point of loading and at the springing opposite to loading
- Is accompanied by the four hinges collapse mechanism

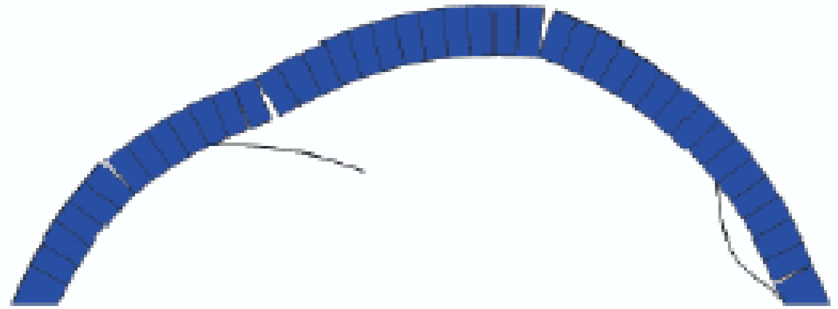
FRP attached to the whole intrados (b: debonding)



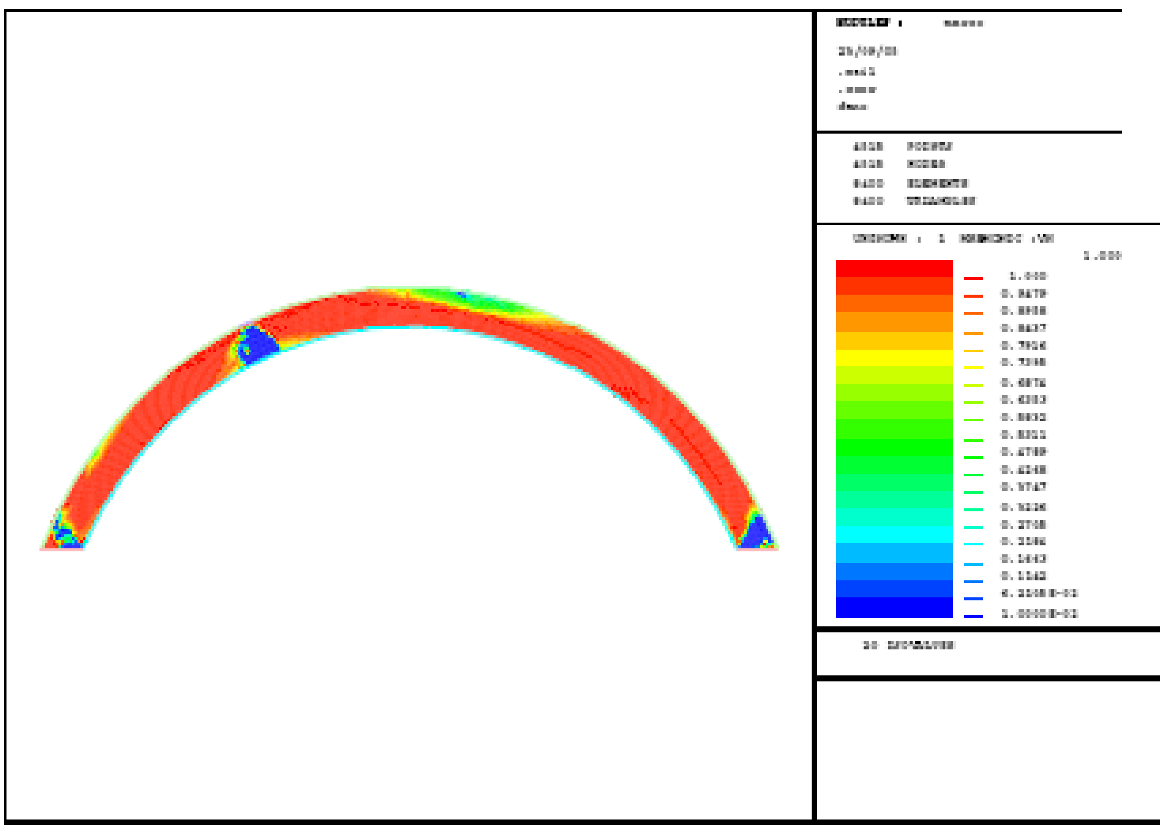


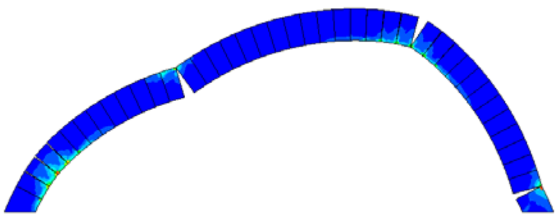
FRP both to intrados and extrados



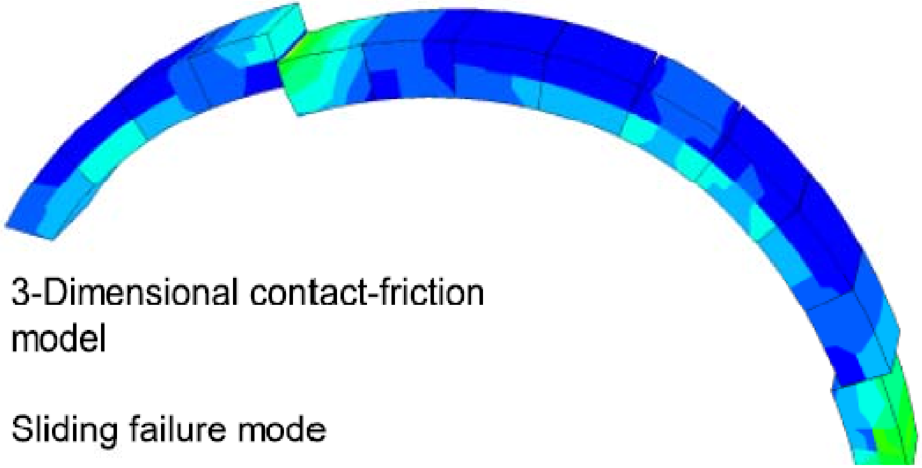


Comparison with continuous damage model



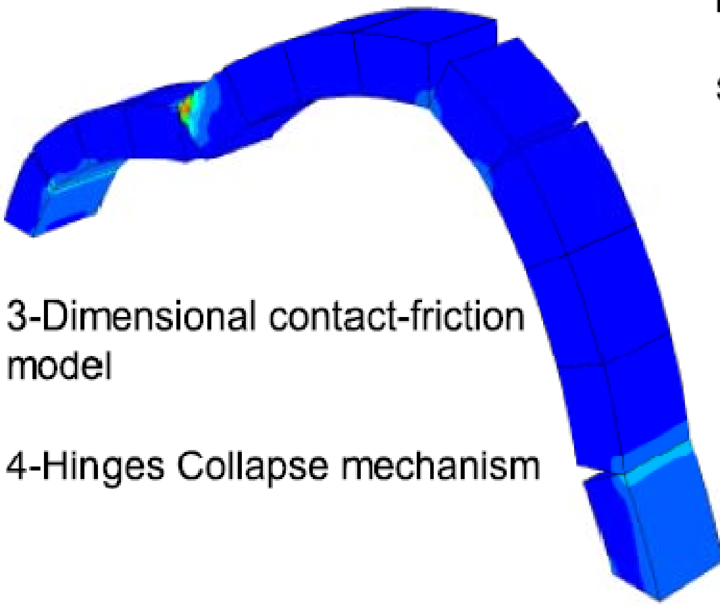


Extension to 3-D models



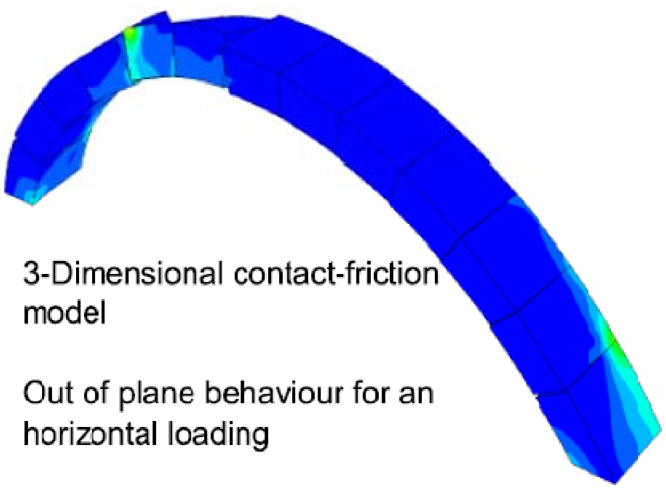
3-Dimensional contact-friction model

Sliding failure mode



3-Dimensional contact-friction model

4-Hinges Collapse mechanism



3-Dimensional contact-friction model

Out of plane behaviour for an horizontal loading

CONCLUSIONS of direct investigation

- SATISFACTORY RESULTS OBTAINED BY THE PROPOSED UNILATERAL CONTACT FRICTION MODEL:

  SATISFACTORY COMPARISON WITH THE CLASSICAL COLLAPSE MECHANISM METHOD

- BRIDGE WITHOUT FILL:

- CONVERGENCE OF THE TWO METHODS ON THE VALUES OF THE FAILURE LOAD

- EXTRACTION OF THE SAME COLLAPSE MECAHNISM

(4 HINGES MECHANISM IN QUARTER SPAN LOADING 5 HINGES MECHANISM IN MIDDLE SPAN LOADING)

- MEANINGLESS THE EXACT NUMBER OF THE INTERFACES ALONG BRIDGE’ S GEOMETRY IN CASE MANY INTERFACES ARE USED

- BETTER BEHAVIOR OF THE QUARTER SPAN LOADING MODEL IN COMPARISON WITH THE MIDDLE SPAN LOADING MODEL

- BRIDGE WITH FILL:

- SIGNIFICANT INCREASE OF THE FAILURE LOAD
- IMPACT OF THE PARAMETERS OF THE FILL ON THE FAILURE LOAD

- SLIDING MODE FAILURE CAN BE INDICATED BY THE PROPOSED MODEL
- FRP reinforcement:

INTERESTING INTERACTION BETWEEN FRP, BONDING, CRUSHING OF MASONRY

Inverse analysis: motivation

From a given crack or damage pattern, find the input that caused it.

Static problems: perform a load incrementation analysis for each possible scenario and compare the results (best fit of damage pattern leads to an MPEC formulation)

Dynamic problems: compare modal stress intensities for possible (earthquake) frequencies and correlate the results with the picture.

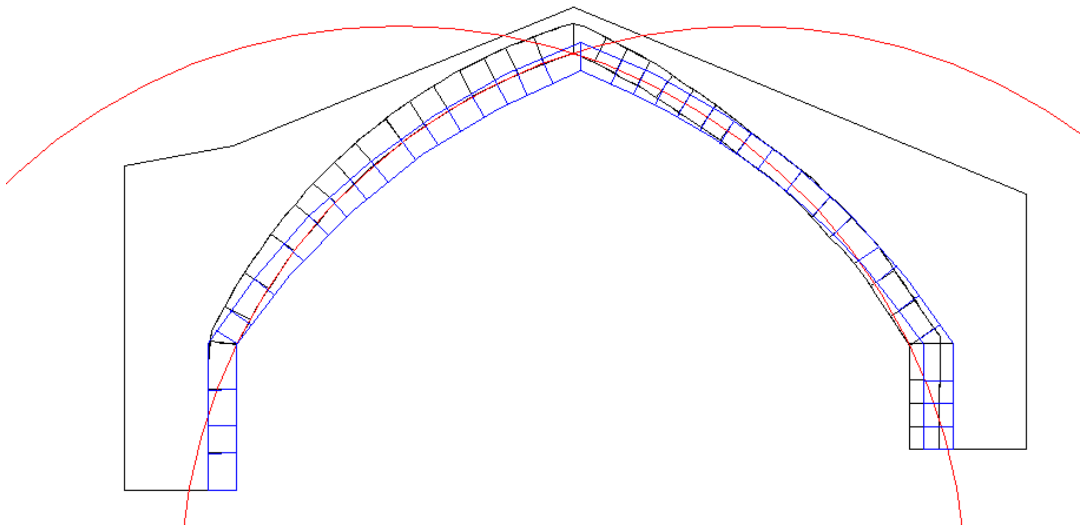
Inverse analysis: case study

Το οξυκόροφο τόξο, με εμφανείς φθορές.

Τα θεμέλια της γέφυρας, έχουν επικίνδυνα κλωνισθεί από τα νερά του ποταμού.

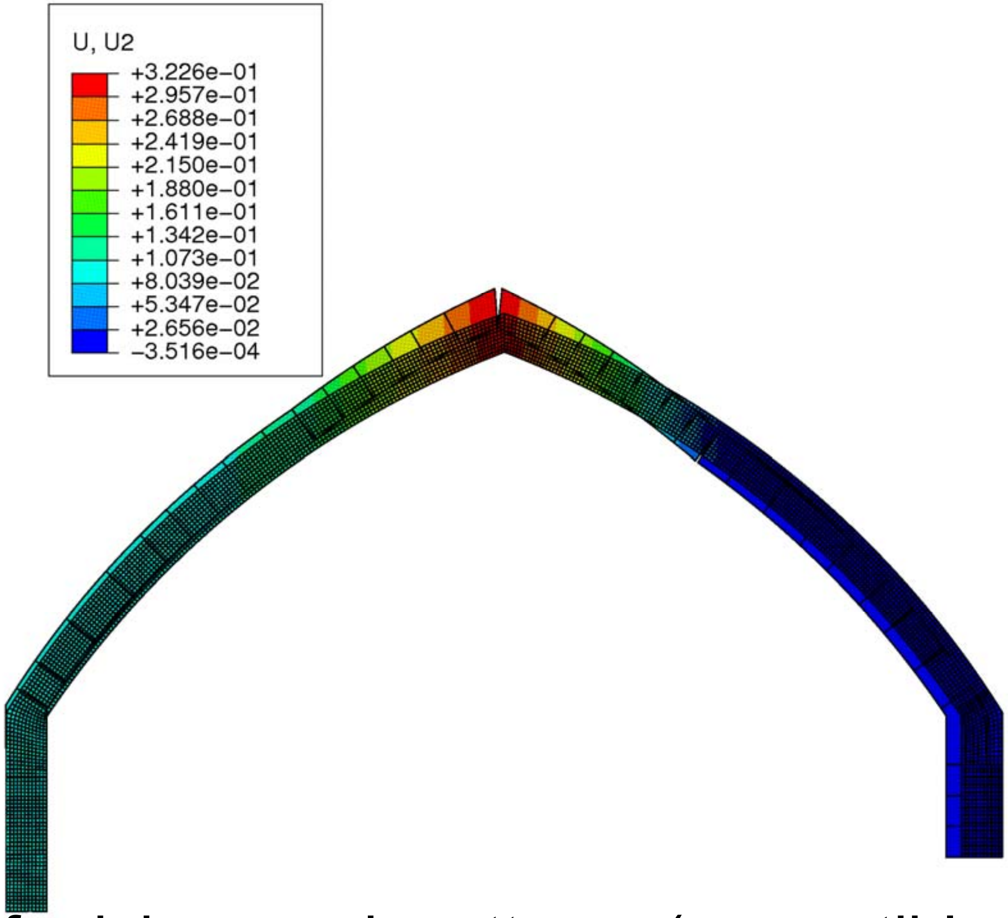
Βαθιές ρωγμές στο "σώμα" της γέφυρας, την καθιστούν επικίνδυνη για τη διέλευση των πεζών.

Inverse analysis: case study



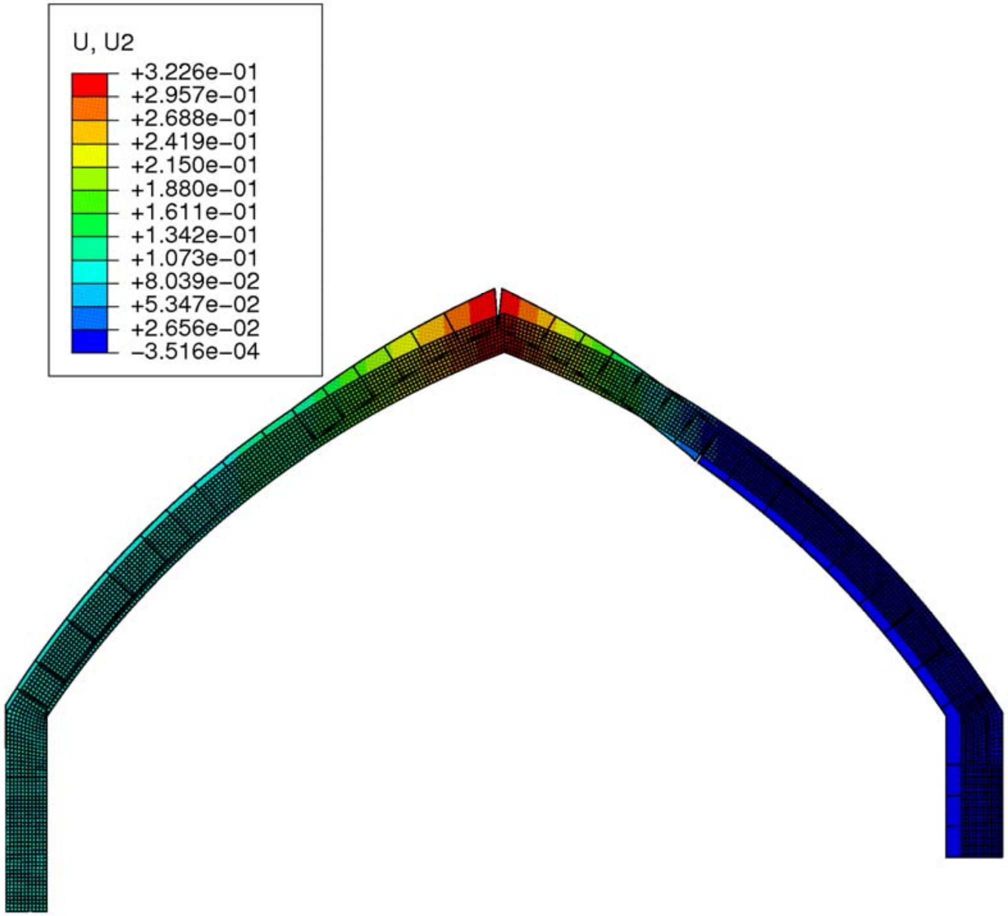
Estimation of initial geometry from existing situation and geometric concepts

Inverse analysis: case study



Verification of arising crack patterns (compatible to hypothesis)

Inverse analysis: case study



Inverse analysis: mpec formulation

Parametrized L.C.P.

$$\mathbf{y} = \mathbf{M}(\mathbf{w})\mathbf{x} + \mathbf{b}(\mathbf{w}), \mathbf{y} \geq \mathbf{0}, \mathbf{x} \geq \mathbf{0}$$

Optimal design problem

Find parameter vector w, such that

$$min_w \left\{ \frac{1}{2}(\mathbf{y} - \mathbf{y_0})^2 + \frac{\mathbf{1}}{\mathbf{2}}(\mathbf{x} - \mathbf{x_0})^2 \right\}$$

Where y, x are subjected to the above L.C.P.

And x_0, y_0 are the vectors of desired solutions.

Inverse analysis: mpec formulation

Parametrized L.C.P.

$$\mathbf{y} = \mathbf{M}(\mathbf{w})\mathbf{x} + \mathbf{b}(\mathbf{w}), \mathbf{y} \geq \mathbf{0}, \mathbf{x} \geq \mathbf{0}$$

Extension for time- or parameter-dependent problems

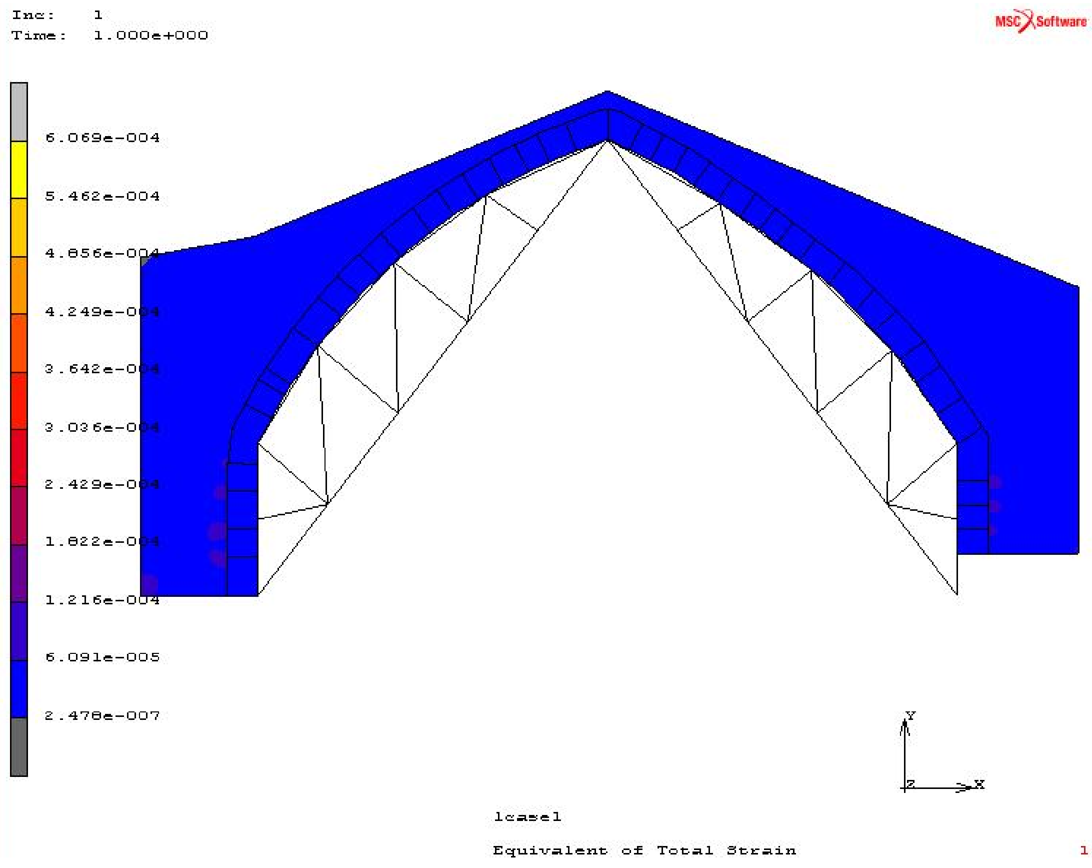
Find parameter vector w, such that

$$min_w \int_{t=0}^{t=T} \left\{ \frac{1}{2}(\mathbf{y}(\mathbf{t}) - \mathbf{y_0}(\mathbf{t}))^{\mathbf{2}} + \frac{\mathbf{1}}{\mathbf{2}}(\mathbf{x}(\mathbf{t}) - \mathbf{x_0}(\mathbf{t}))^{\mathbf{2}} \right\}$$

Where y, x are subjected to the above L.C.P.

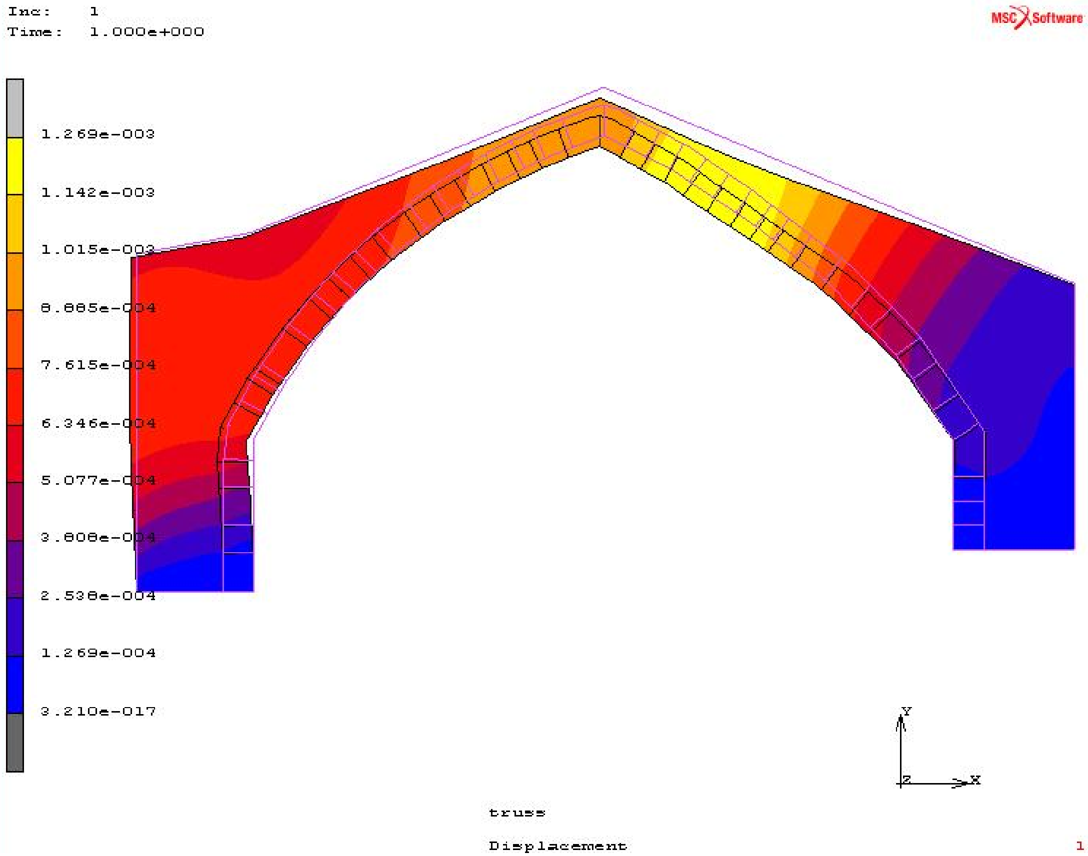
And x_0, y_0 are the vectors of desired solutions.

Inverse analysis: case study

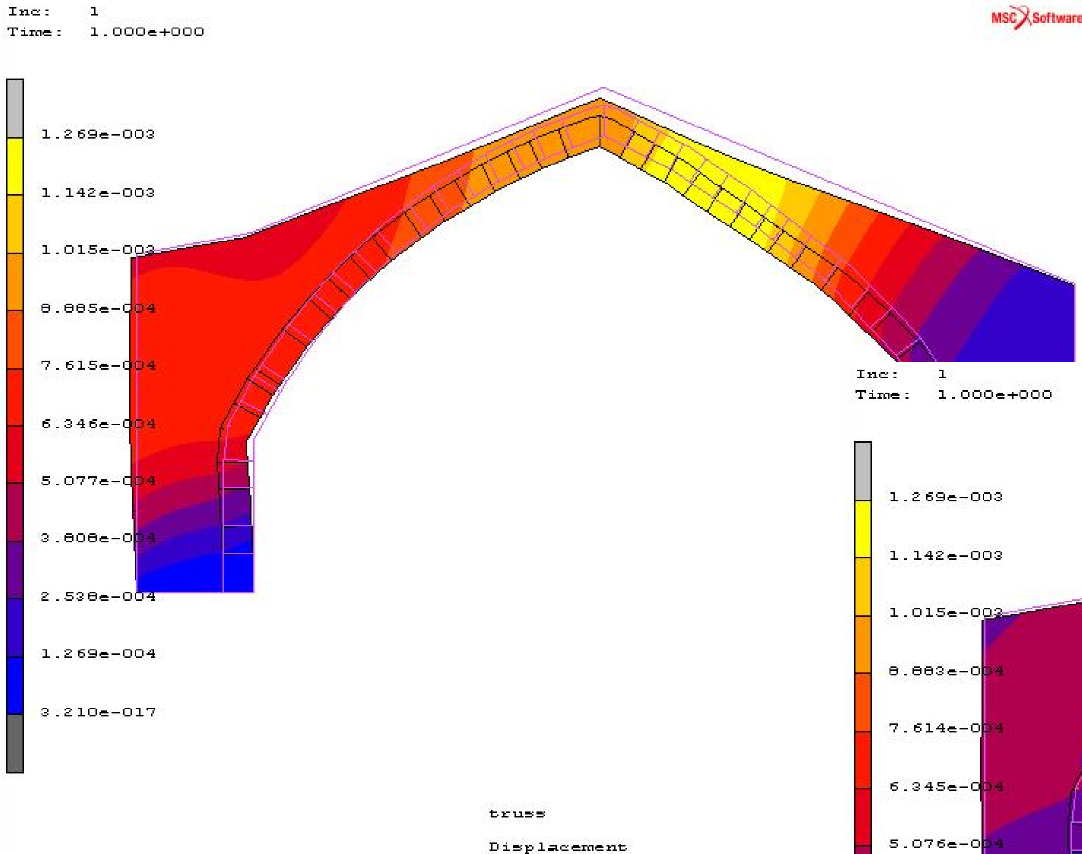


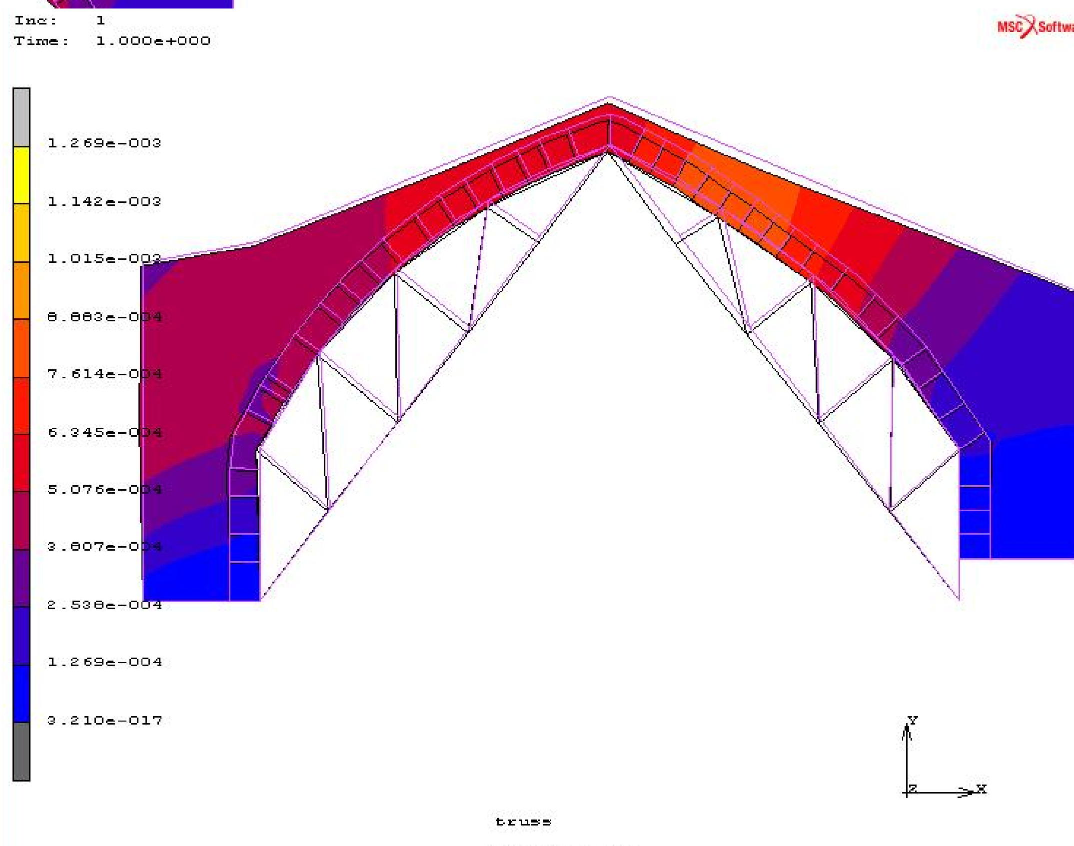
Proposal for supporting steel structure: compatibility of deformation such as to avoid enhancement of collapse

Inverse analysis: case study



Inverse analysis: case study





Effectiveness of

Supporting structure

Inverse analysis: conclusions

A theoretically interesting formulation (MPEC)

Has been used partially for the comparison of damage scenaria

A thorough investigation, with algorithm development will follow but,

Applicability to real cases is difficult, due to high uncertainties and lack of data.

Relevant publications

M. Campo, G.A. Drosopoulos, G.E. Stavroulakis: Unilateral contact and damage analysis in masonry arches. Vietnam Journal of Mechanics, 31(3-4) (2009) 185-190

G.A. Drosopoulos, G.E. Stavroulakis, C.V. Massalas: Frictional contact analysis of masonry bridges, FRP reinforcement and estimation of collapse. In: Computer Analysis and Design of Masonry Structures, Ed. J.W. Bull, CIVIL-COMP Ltd – SAXE-COBURG Publications, U.K. (invited chapter, in press).

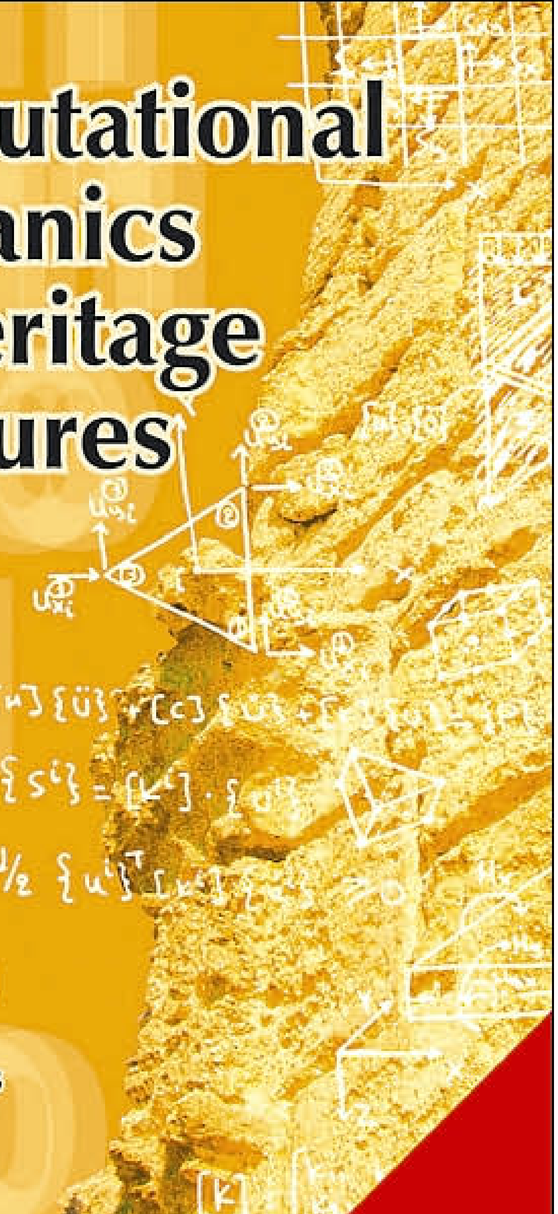
M.E. Stavroulaki, G. Bartoli, M. Betti, G.E. Stavroulakis: Strengthening of masonry using metal reinforcement. A parametric numerical investigation. Prohitech2009 Protection of Historical Buildings, Conference Proceedings, Volume 2, pp. 1187, Ed. F.M. Mazzolani, CRC Press, 2009.

M. Betti, G.A. Drosopoulos, G.E. Stavroulakis: Two non-linear finite element models developed for the assessment of failure of masonry arches. Comptes Rendus Mecanique, Special Issue on Duality, inverse problems and nonlinear problems in solid mechanics, dedicated to Prof. H. Bui, Vol 336/1-2 pp 42-53, 2008.

G.A. Drosopoulos, G.E. Stavroulakis, C.V. Massalas: Influence of the geometry and the abutments movement on the collapse of stone arch bridges. Construction and Building Materials, 22(3), 200-210, 2008.

B. Leftheris, A. Sapounaki, Maria E. Stavroulaki, George E. Stavroulakis: Computational mechancs for heritage structures. Series: High Performance Structures and Materials, Vol. 9, WIT PRESS, Date: 2006, Pages: 288, Southampton, Boston, UK

Computational Mechanics for Heritage Structures



B.P. Leftheris
M.E. Stavroulaki
A.C. Sapounaki
G.E. Stavroulakis

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Thank you for your attention !!!

Georgios E. Stavroulakis

Co-authors:

Georgios Drosopoulos

Maria Stavroulaki