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Interface models for faults in geophysics

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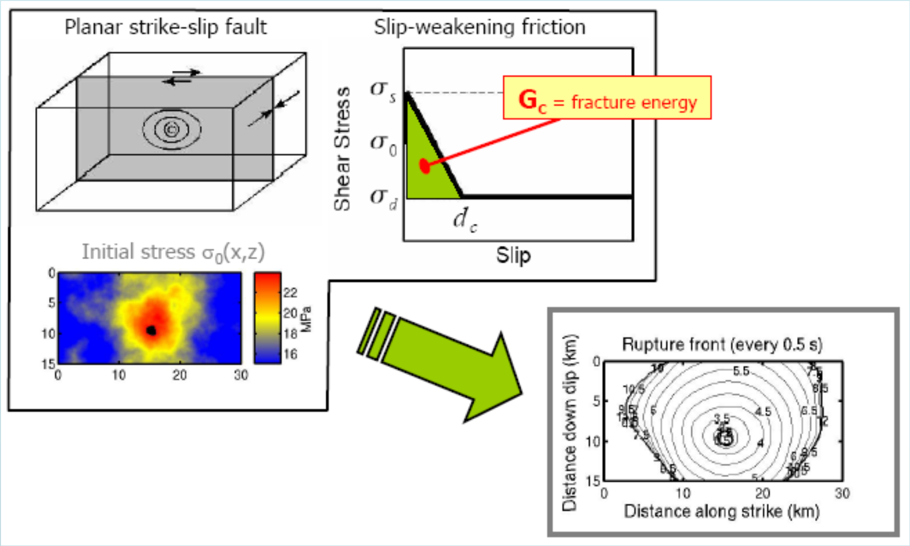
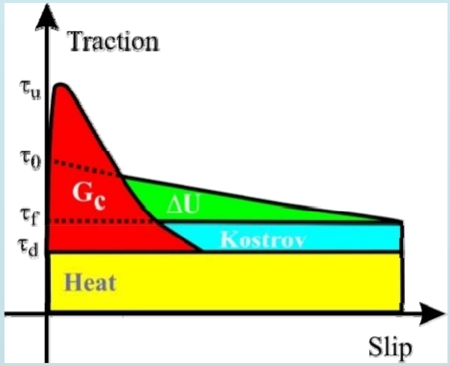
Outline

- 1 - Introduction
- 2 - Constitutive laws in geomechanics
- 3 - RCCM in the geomechanics context
- 4 - Numerical simulation : nucleation and wave generation
- 5 - Conclusion

1 - Introduction

The classical earthquake model: fault interface with effective friction





Rupture propagation model

*Problem : **scaling from laboratory to natural faults** (fracture energy, stress heterogeneities)*

- **Seismic rupture occurs within complex fault zones** (Chester et al, 1993)

F. Chester, J. Evans and R. Biegel, *J. Geoph. Res.*, **98** (B1), 771-786 (1993)

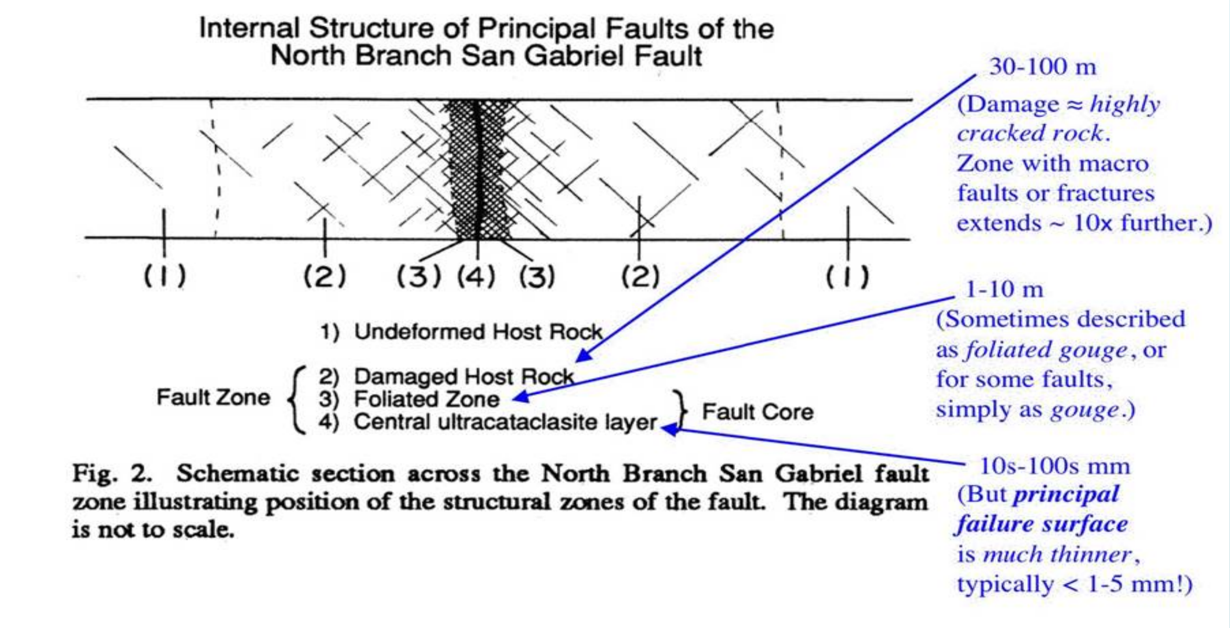
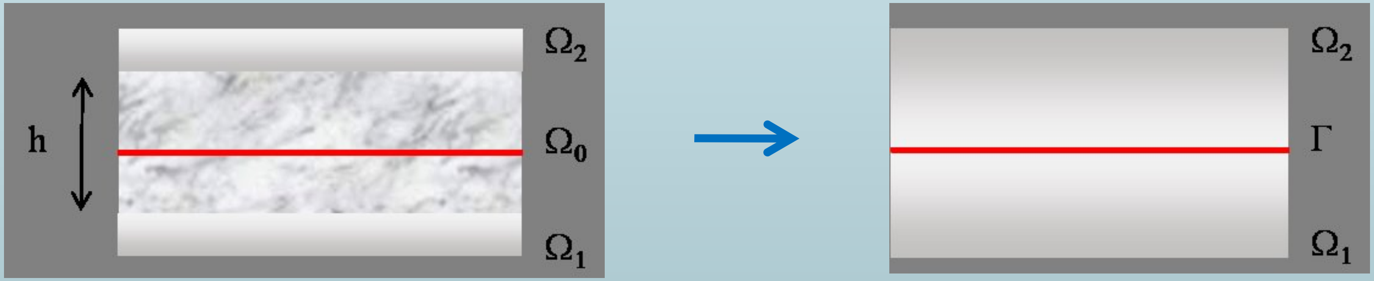


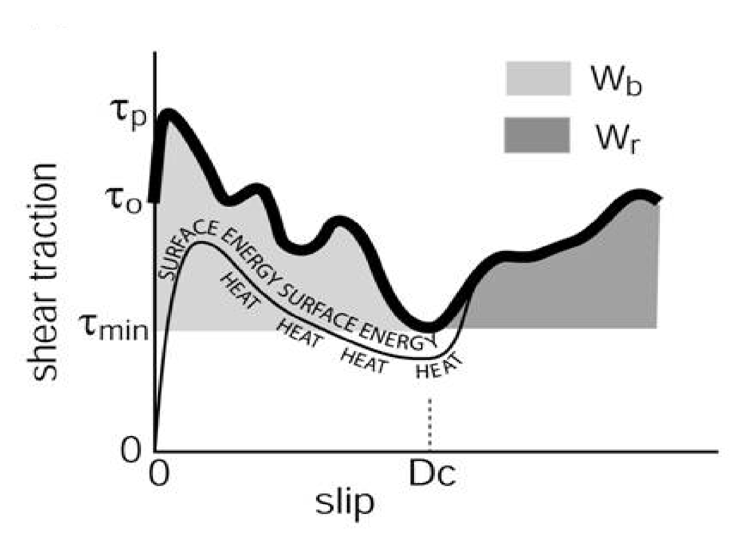
Fig. 2. Schematic section across the North Branch San Gabriel fault zone illustrating position of the structural zones of the fault. The diagram is not to scale.

- **Earthquake rupture investigated with wavelengths larger than the fault-zone thickness : homogeneization and energy dissipation principles**



Material surface hypothesis (*but a characteristic length will be introduced*)

➢ **Kinematic models show a complex traction vs slip dependence (Cocco et al., 2008)**

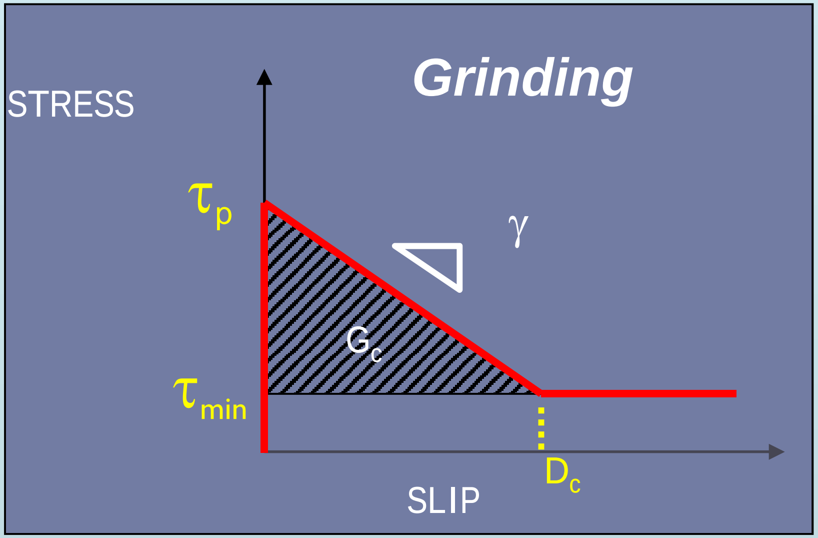


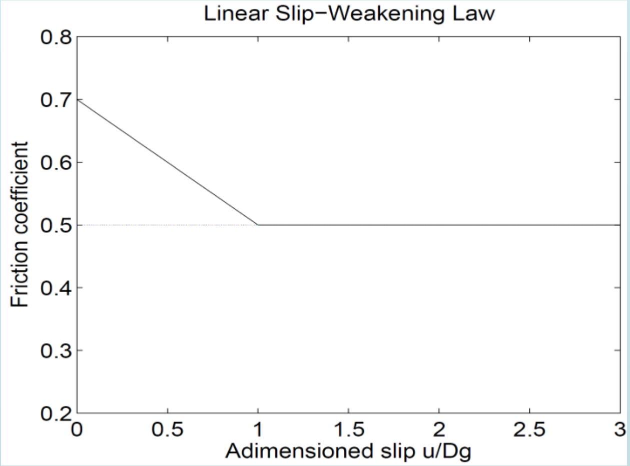
Not only friction…

- **Dc depends on the observation scale (*mm* for lab, *m* for earthquakes)**
- **How large is μ_d (τ_0 ,τ_p?), as compared to μ_s (τ_{min}) ?**
- **How to account for small and large earthquakes ?**

2 - Constitutive laws

2.1 - Slip-weakening friction **(Ida, 1972; Andrews, 1976)**





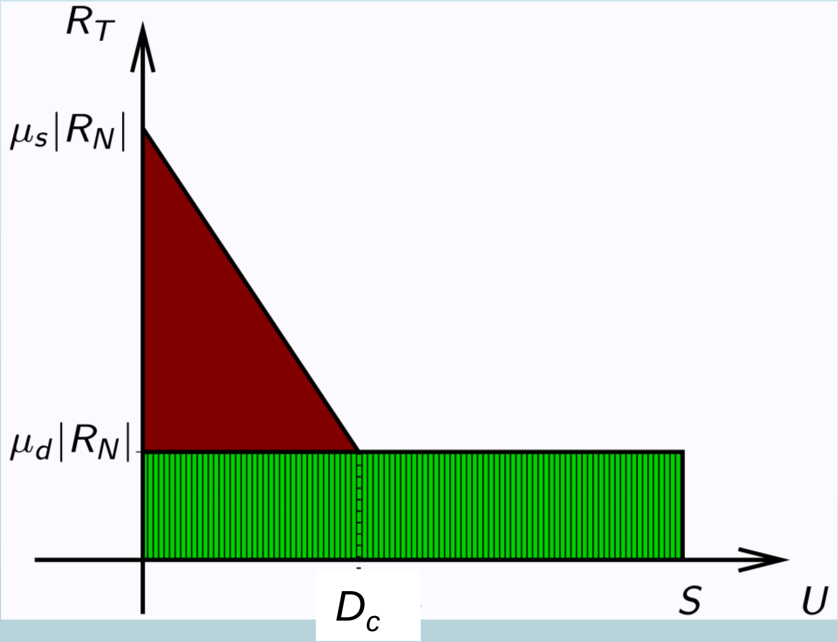
$$\mu = \begin{cases} \mu_s - (\mu_s - \mu_d)\frac{u}{D_g} \text{ if } u < D_g \\ \mu_d \text{ if } u \geq D_g \end{cases}$$

- **simple, fits the behavior of breaking interfaces (G_c)**
- **depending on normal stresses**
- **unloading not considered (monotone loading)**
- **rate and time independant**
- **also exponential decreasing laws (Cocco-Bizzari, 2002) (Campillo-Ionescu, 1997)**

Slip-weakening friction (2)

Dissipations :

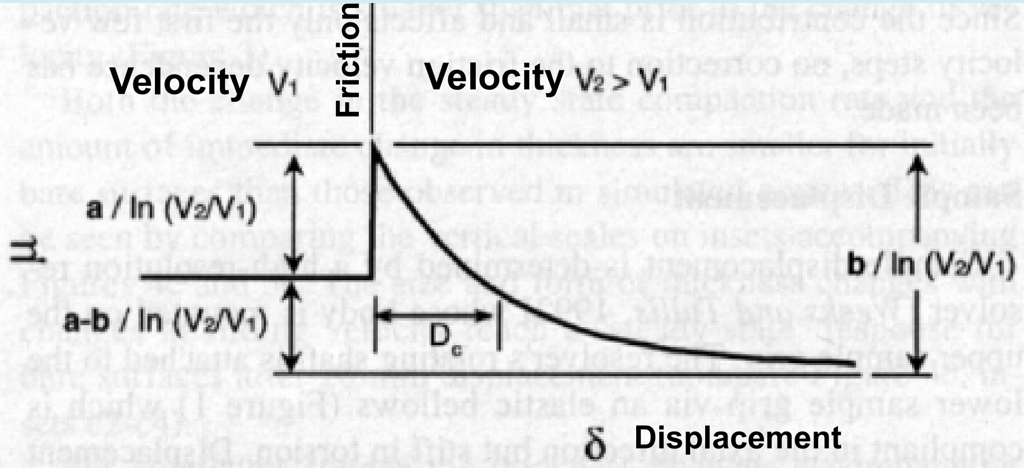
- similar to fracture energy (Gc) (irreversible grinding but ….)
- friction dissipation



References :

(Abercrombie & rice, 2005) (Ampuero& Vilotte, 2005) (Campillo et al, 2006) (Favreau et al, 1999) (Festa & Vilotte, 2006) (Guatteri & Spudich, 2000) (Ionescu & Paumier, 1996) ….

2.2 - Rate-and-state friction (Ruina, 1983) (Rice-Ruina, 1983)(Dieterich, 1986)



- V_0 ref. velocity
- μ_0 ref friction coefficient
- a parameter **(velocity dependance)**
- b parameter **(delay effect)**
- D_c characteristic length **(very small)**

$$\tau = \sigma_n [\mu_0 + a \ln(V / V_0) + b \ln(V_0 \theta / D_c)]$$

$$\frac{d\Theta}{dt} = 1 - \frac{V\Theta}{D_c}$$

General form

$$\tau = \mu \, \sigma_n$$
$$\mu = \mu(V, \Theta)$$
$$d\,\Theta / dt = f(\Theta, V)$$

Θ state variable
(accomodation time)

Other laws with 2 state variables :
(Ruina, 1983) (Guet al, 1984) (Blanpied & Tullis, 1986)

Rate-and-state friction (2)

- **no unilateral conditions (monotone loadings) and normal stress is usually given**
- **rate dependant**
- **Θ state variable (accomodation time) :**

  when $V\ \Theta = D_c$, μ does not change anymore

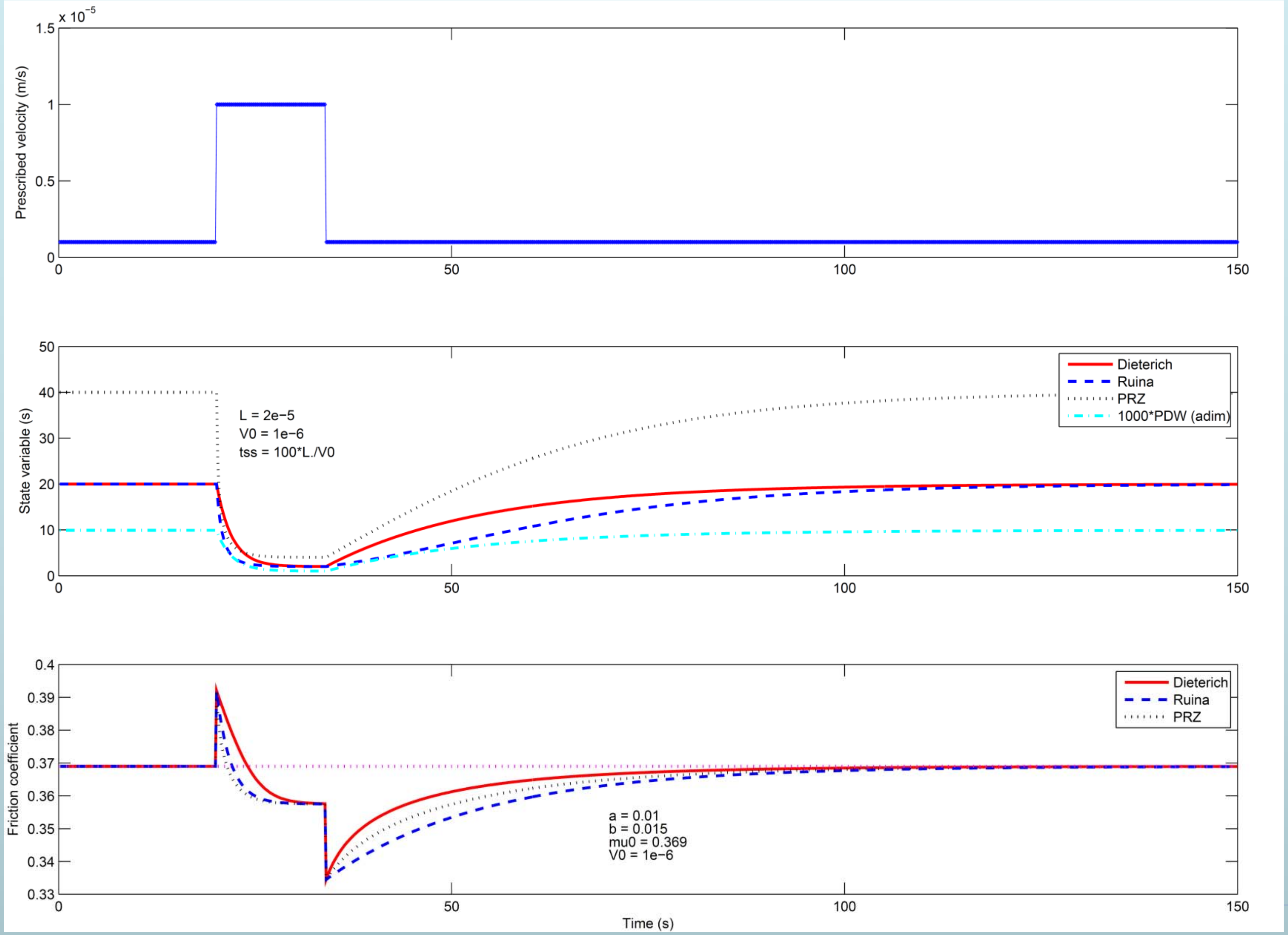
- **inferred from quasi-static laboratory experiments**

References :

(Rice & Ruina, 1983) (Rice & Tse, 1986) (Ranjith & Rice, 1999) (Campillo et al, 1996) (Favreau et al, 1999) (Ionescu & Paumier, 1993, 1994) (Gu et al., 1984) (Blanpied & Tullis, 1986) (Putelat, 2007) (Marone, 1998) ….

Rate-and-state friction (3)

Different forms : (Dieterich & Ruina) (Ruina) (Perrin-Rice-Zheng [PRZ]) (Putelat-Dawes-Willis [PDW])



Rate-and-state friction (4)

General comments

➢ difficult to identify the dissipations : fracture energy, …

➢ creep (and relaxation) : evolution of the friction coefficient (experiment of Dieterich-Kilgre)

➢ very useful for evolutive faults (condition $V \neq 0$) :

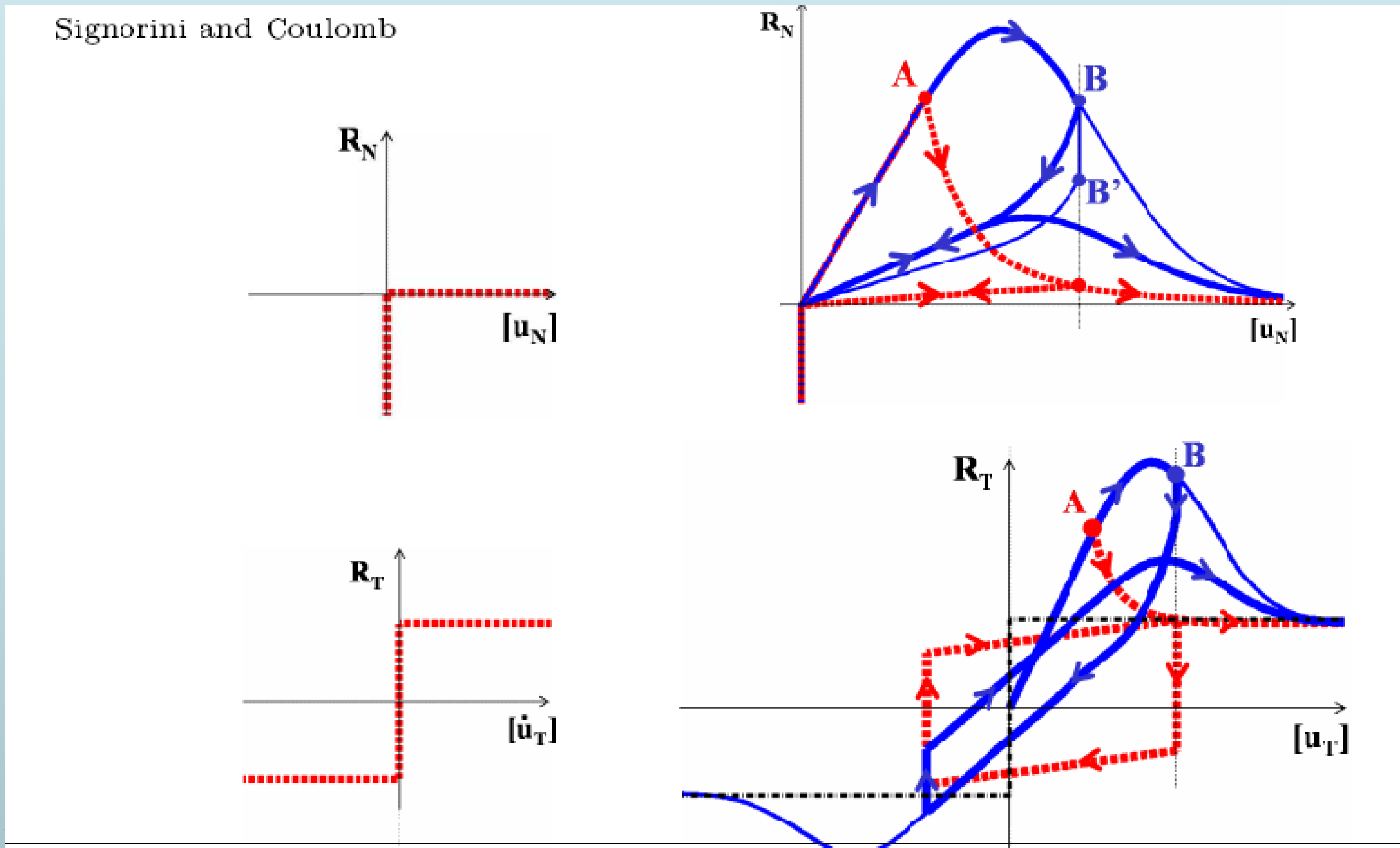
- effects of velocity jumps
- stability of steady sliding (Hopf bifurcation for a single mass-spring system)
- aftershocks and cycles
- quasi-static localisation, sismic rupture, bimaterial, …

➢ strengthening ($a > b$) (stable sliding) and weakening ($a < b$) (conditionally unstable sliding)

2.2 – Friction and adhesion (damage)

RCCM (Raous-Cangémi-Cocou-Monerie, 1997 & 1999)

Coupling unilateral conditions + friction + adhesion (damage)
Viscosity (rate dependant)



RCCM (2)

u_N is the gap and R the force between two deformable bodies

β (state variable) is the intensity of adhesion (first introduced by (Frémond, 1988))

Unilateral contact and adhesion

$$-R_N^r + \beta^2 C_N\, u_N \geq 0\,, \quad u_N \geq 0\,, \quad \left(-R_N^r + \beta^2 C_N\, u_N\right)\, u_N = 0.$$

Adhesion-dependent Coulomb friction and adhesion

$$\begin{aligned}
&R_T^r = \beta^2 C_T\, u_T\,, \qquad R_N^r = R_N\,,\\
&\left\|R_T - R_T^r\right\| \leq \mu f(\beta) \left|R_N - \beta^2 C_N\, u_N\right|\,,\\
&\left\|R_T - R_T^r\right\| < \mu f(\beta) \left|R_N - \beta^2 C_N\, u_N\right| \Rightarrow \dot{u}_T = 0,\\
&\left\|R_T - R_T^r\right\| = \mu f(\beta) \left|R_N - \beta^2 C_N\, u_N\right| \Rightarrow \exists \lambda \geq 0\,,\ \dot{u}_T = \lambda(R_T - R_T^r).
\end{aligned}$$

Evolution of intensity of adhesion

$$\dot{\beta} = -\left[\frac{1}{b}\left(w - \beta\,(C_N\, u_N^2 + C_T\, \|u_T\|^2)\right)^-\right]^{1/p}.$$

Parameters of the model :

- μ friction coefficient
- C_N, C_T initial stiffness of the interface
- *w* *adhesion energy*
- b viscosity of the interface

p = 1

Usually f(β) = 1 - β

RCCM (3)

With no viscosity, instead of the differential equation, the evolution of β, which depends on the loading path, is given by :

$$\beta = \min_{\tau \leq t}\{ \beta(\tau), \min\{\beta(t), 1\}\}$$

with

$$\beta = \frac{w}{C_t \delta u_t^2} \quad \beta \in [0,1[$$

Dissipation powers

- ***damage (adhesion)***

$$D_d(\dot{\beta}) = \frac{\partial}{\partial \dot{\beta}} \Phi_d(\dot{\beta})\, \dot{\beta} = -\omega \dot{\beta}$$

- ***friction***

$$D_f(\beta, \sigma^-, \dot{v}) = (1-\beta)\, \mu\, \sigma^- \,|\dot{v}|$$

- ***viscosity***

$$D_v(\beta) = \dot{\beta} \frac{\partial}{\partial \dot{\beta}} \Phi_v(\dot{\beta}) = b \dot{\beta}^2$$

The energies

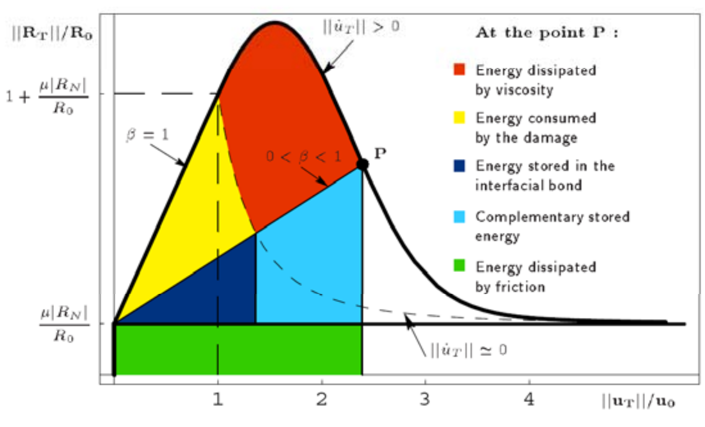


Figure 1: *Case* $f(\beta) = 1$

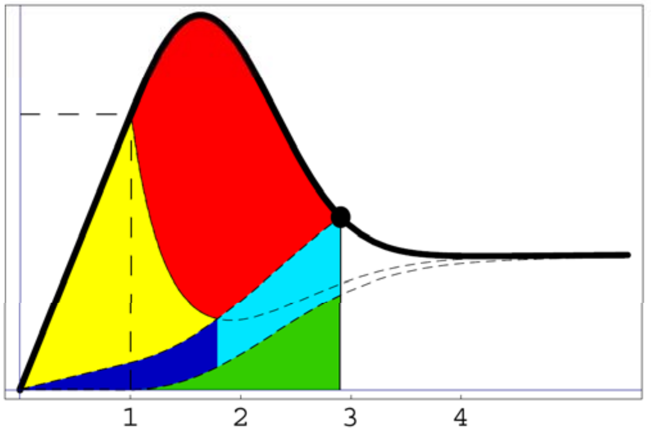


Figure 2: *Case* $f(\beta) = 1 - \beta$

The thermodynamics potentials (Del Piero - Raous, 2010)

Strain energy

$$\Psi(u, v, \beta) = \frac{1}{2}(C_N u^2 + C_T v^2)\, \beta^2$$

Damage dissipation potential

$$\Phi_d(\dot{\beta}) = -\omega\dot{\beta}$$

Viscosity dissipation potential

$$\Phi_v(\dot{\beta}) = \frac{1}{2} b\dot{\beta}^2$$

Friction dissipation potential

$$\Phi_f(u, \beta, \dot{v}, \dot{\beta}) = -(1-\beta)\, \mu\, (\sigma - C_N u \beta^2)\, |\dot{v}|\, \ln(|\dot{\beta}|)$$

RCCM (7)

Comments

- no scaled parameter (no given Dc) :

  adhesive energy *w*, initial stiffness *C*, friction μ, viscosity *b*

- any kind of loadings (unloading, cycle, …)

  Signorini cds = non penetration (think about opened faults at the surface)

- elastic behavior for small stresses

- damage process (variable β, intensity of adhesion (Frémond, 1987))

  It has to do with slip weakening but intrinsic (independant of normal stresses : considerations on asperities but also on physico-chemical forces and other …)

- viscosity ⇒ dissipation, possible relaxation and creep phenomenas

  (evolution of the damage and of the displacement along time when stresses remain constant) + viscous dissipation

- possibility of extension to recovered adhesion either when new compression is applied after an earthquake or because of time effects

  (thesis of Schryve on recoverable adhesion)

3 – RCCM in the geomechanics context

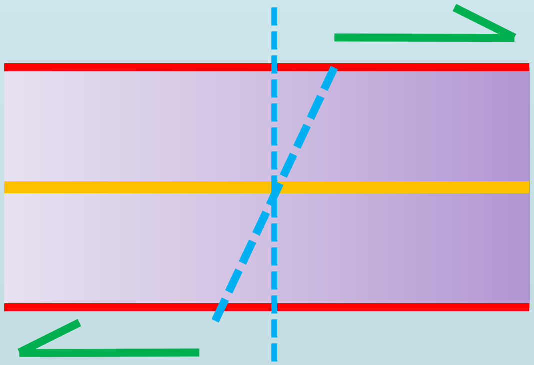
The interface stiffness C_T : a zero order homogeneization

h is the thickness of the interface

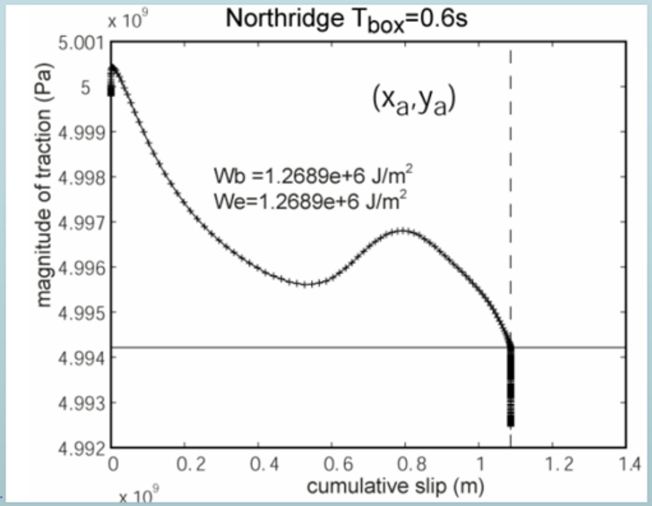
$$W_{el} = \frac{1}{2}\int_S \sigma_{xz}\varepsilon_{xz}dV = \frac{1}{2}A\int_0^h \frac{1}{G}\sigma_{xz}^2 dV = \frac{1}{2}\langle G\rangle \frac{\left(u^+ - u^-\right)^2}{h}A$$

$$\frac{1}{\langle G\rangle} = \int_0^h \frac{1}{G(z)}dz$$

$$C_t = \frac{\langle G\rangle}{h}$$



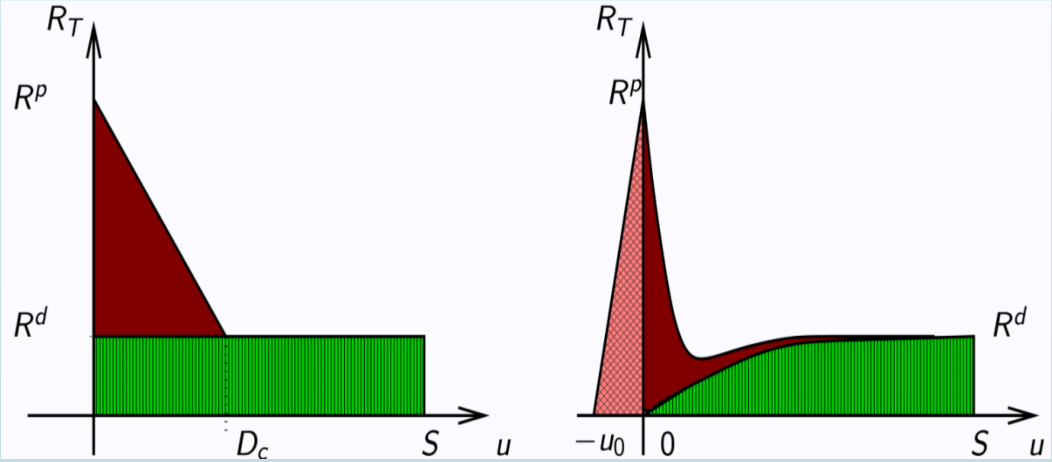
G(z) shear modulus in the interface



from Piatanesi et al. (2005)

γ	W_b (10^6 J/m^2)	τ_u (10^7Pa)	$h^{1km/s}$ (m)
0.7	1.548	1.3	139

The energies and the bounds (what are τ_p, τ_{min} and D_c for RCCM ?)



	SW	RCCM
Dissipation : damage or fracture	G_c	$w/2$
Friction dissipation		
Maximal stress (threshold) τ_p or τ_u	$\mu_s \sigma_n$	$\sqrt{w\, C_t}$
Asymptotic stress	$\mu_d \sigma_n$	$\mu \sigma_n$

Note : scale effect

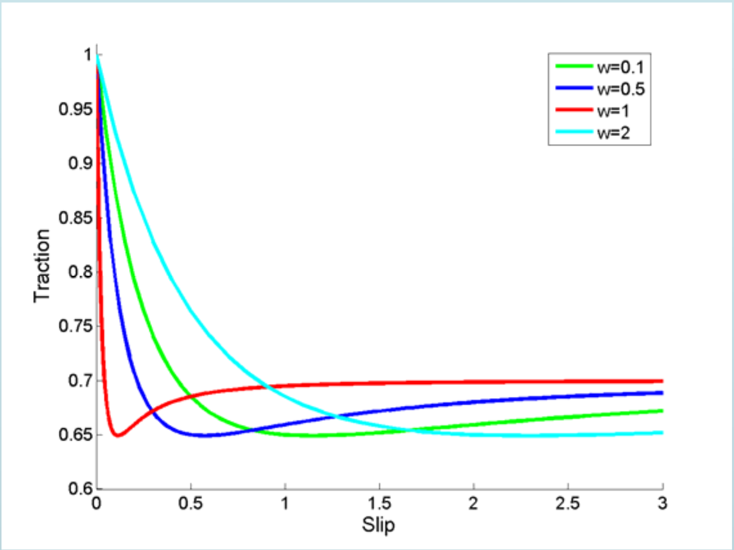
$$\tau_u = \sqrt{wC_t} = \sqrt{\frac{w\langle G\rangle}{h}}$$

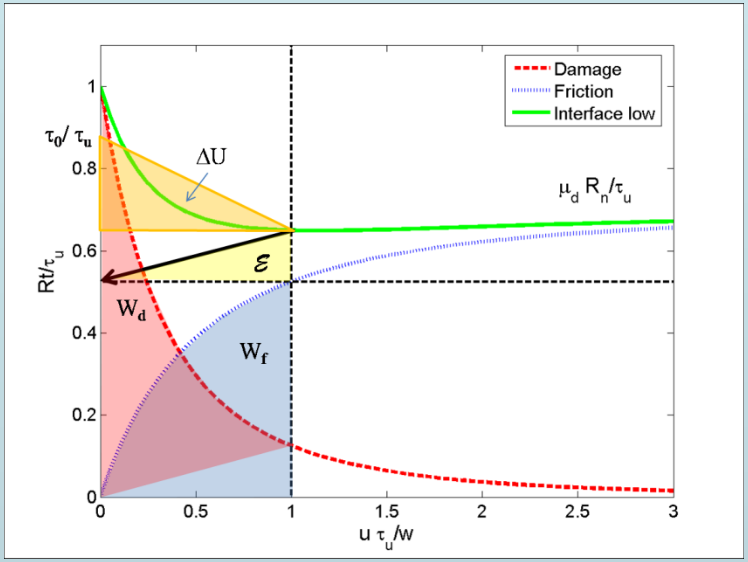
For a fixed ***w***, the smaller the layer thickness, the steeper the traction increases before breaking.
The fault strength can be written as a function of the interface stiffness and the damage surface energy.

D_c is estimated from the minimum of the traction curve

$D_c = \delta U_{min}$

$$R_t = \left(\frac{w}{\tau_u \delta U + w}\right)^3 \tau_u + \left[1 - \left(\frac{w}{\tau_u \delta U + w}\right)^2\right] \mu_d R_n$$





Threshold

$$\tau_u = \sqrt{w C_t}$$

A characteristic slip length

$$\delta U_{\min} = \frac{w}{\tau_u}\left(\frac{3}{2\gamma} - 1\right); \quad \gamma = \frac{\mu_d}{\mu_s}$$

4 – Numerical simulations

4.1 - Nucleation

Extension of (Uenishi-Rice, 2003) for LSW to RCCM

Universal nucleation length for slip-weakening rupture instability under nonuniform fault loading

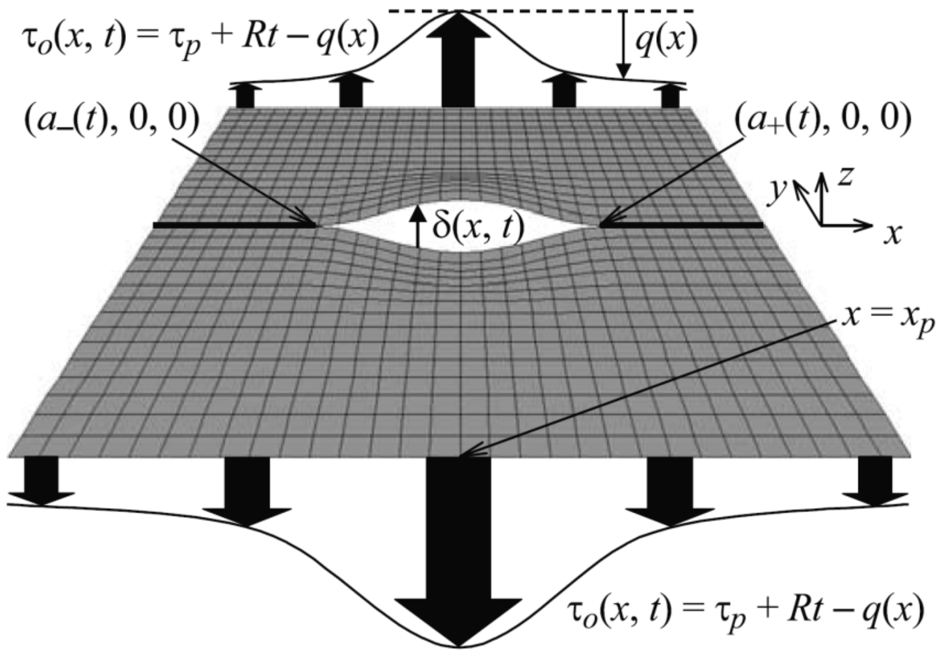
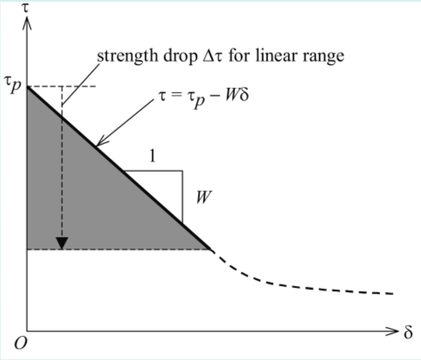


Figure 1. A displacement field associated with antiplane shear (mode III) rupture in an infinite, homogeneous, linear elastic space. The loading shear stress $\tau_o(x, t)$ is locally peaked in space and increases gradually with time, at rate R. Similarly, we can define the problem for in-plane shear (mode II), or for tensile (mode I) rupture.

(Uenishi-Rice, 2003) for LSW



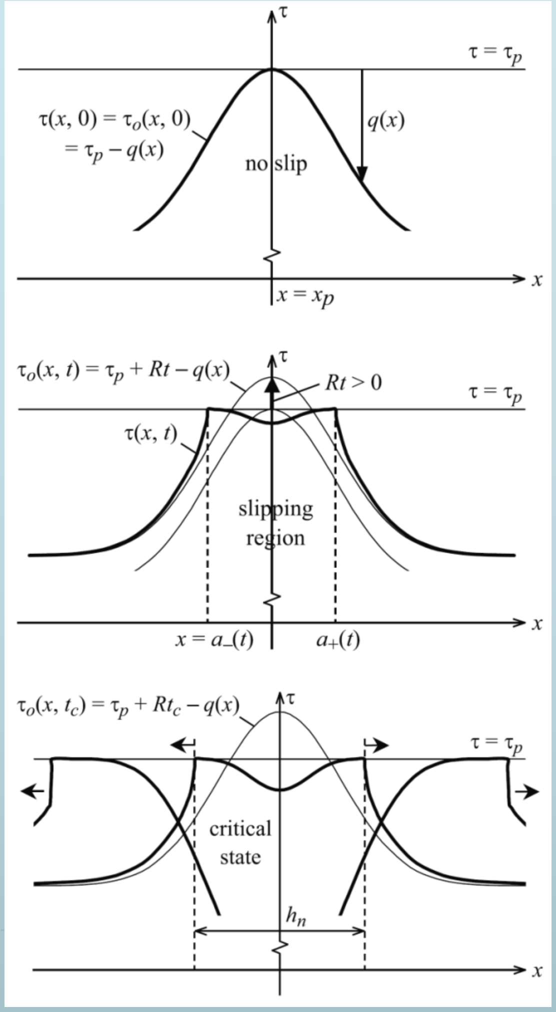
- increasing peaked loading stress (Gaussian)

- semi-analytic solution of the quasi-static problem

$$\tau(x,t) = \tau_0(x,t) - \frac{\mu^*}{2\pi} \int_{-a(t)}^{a(t)} \frac{dU(\xi,t)/d\xi}{x-\xi} d\xi$$

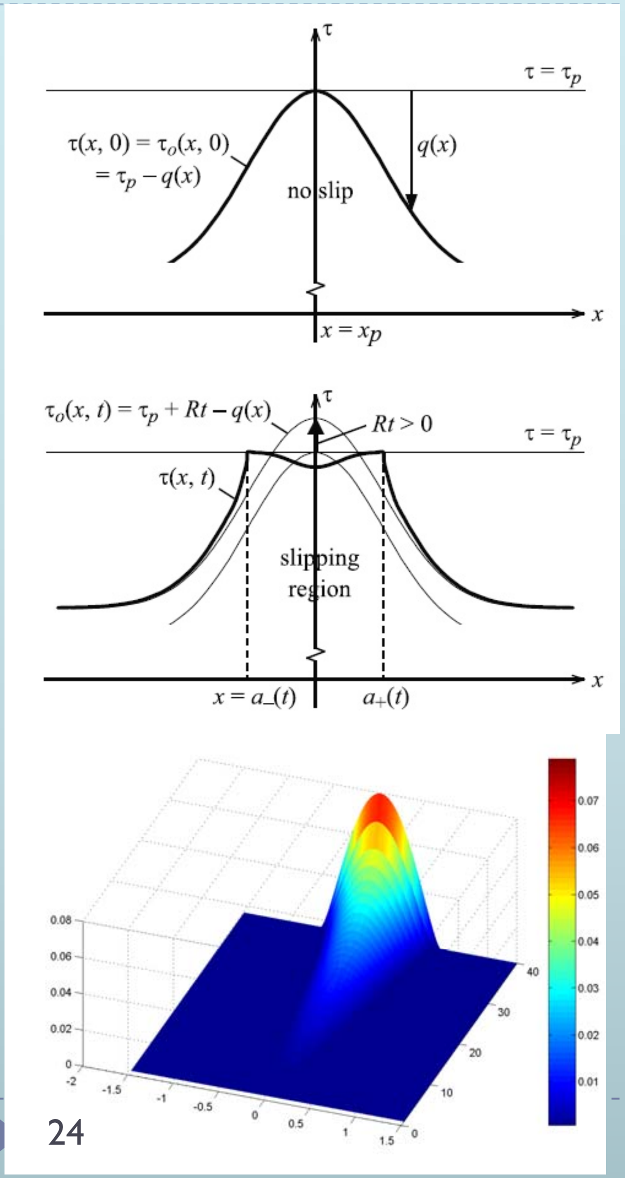
- the critical nucleation length h_n is given by the condition on the stresses and the displacement to be finite at the end of the slipping region

- when using a Chebyshev polynomial representation of the solution, h_n is given by the solution of an eigenvalue problem



With LSW, the nucleation length is shown to be independant of the shape of the loading stress distribution (any earthquake)

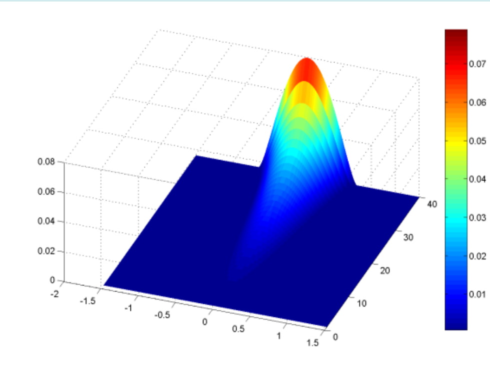
Extension of the Uenishi & Rice (2003) analysis to RCCM

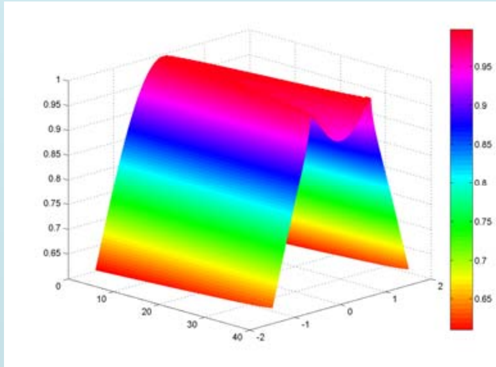


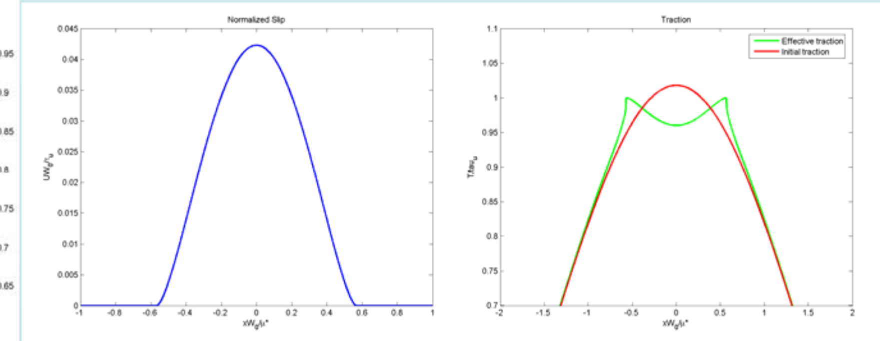
- ***Gaussian perturbation***
- **Recursive linearization of the incremental solution**
- ***Development of the solution onto Chebyshev polynomials***
- ***Finiteness condition***
- **Newton-Raphson method for convergence**

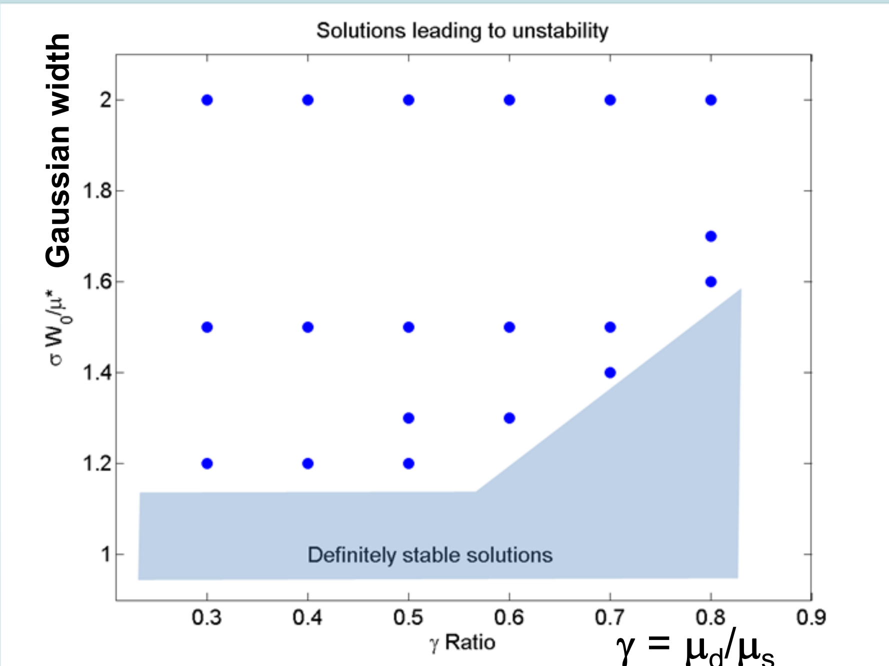
With RCCM the critical length and the stability depend on the Gaussian width, i.e. the loading stress condition has a consequence on the earthquake trigger.

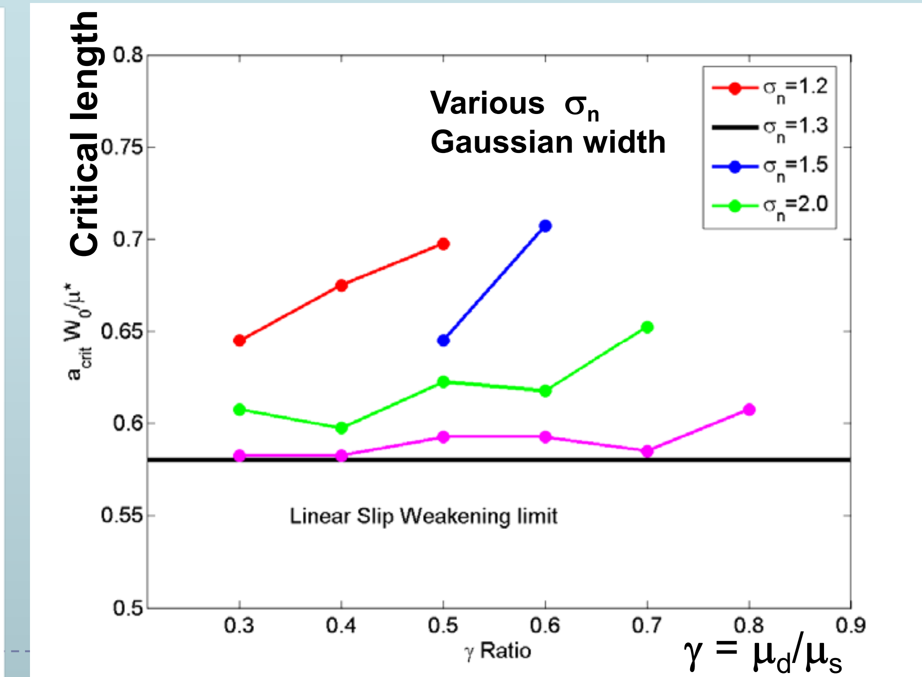
Segregation of small and strong earthquakes (and stable quasi-static solutions exist)







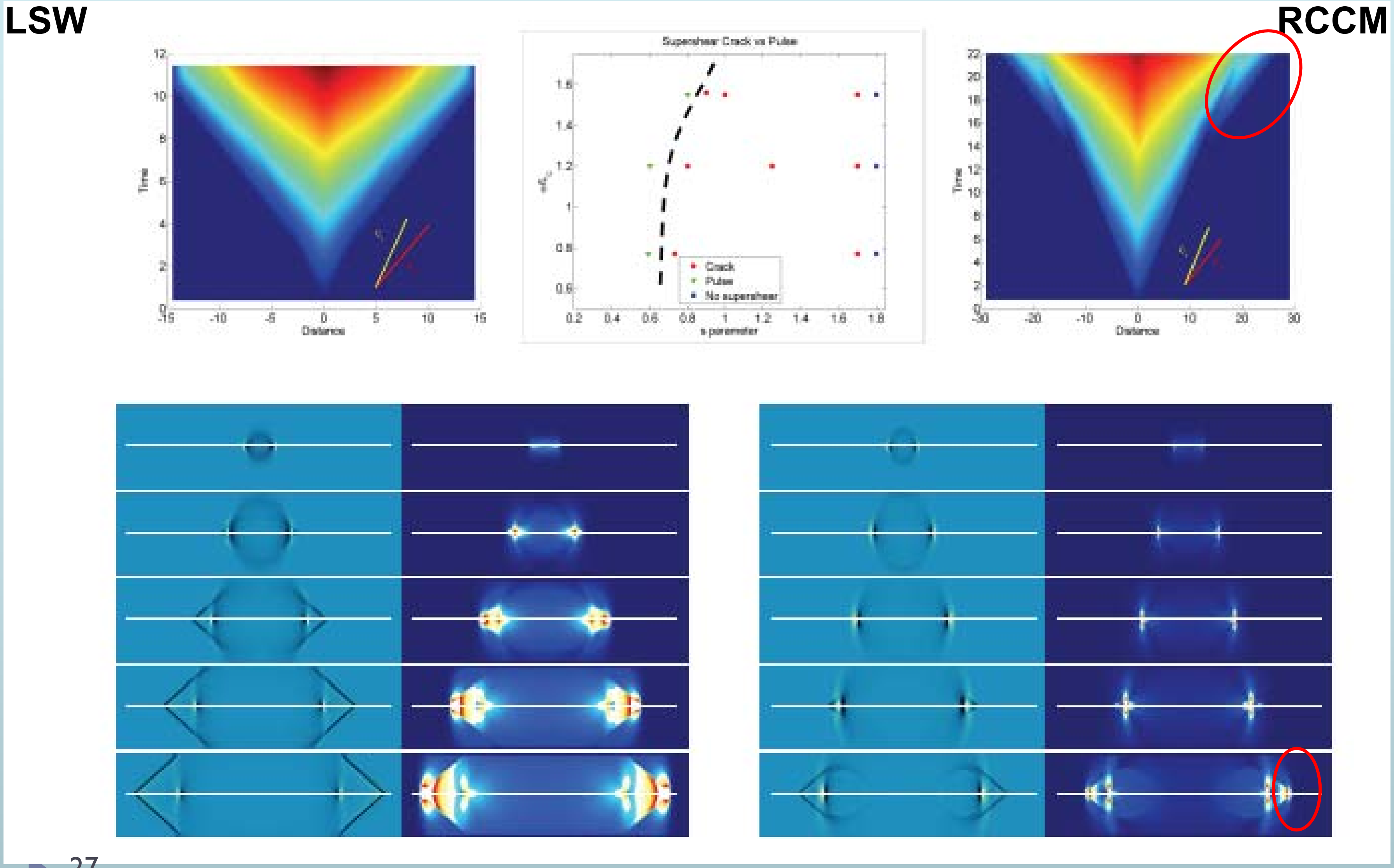




4.2 – Supershear

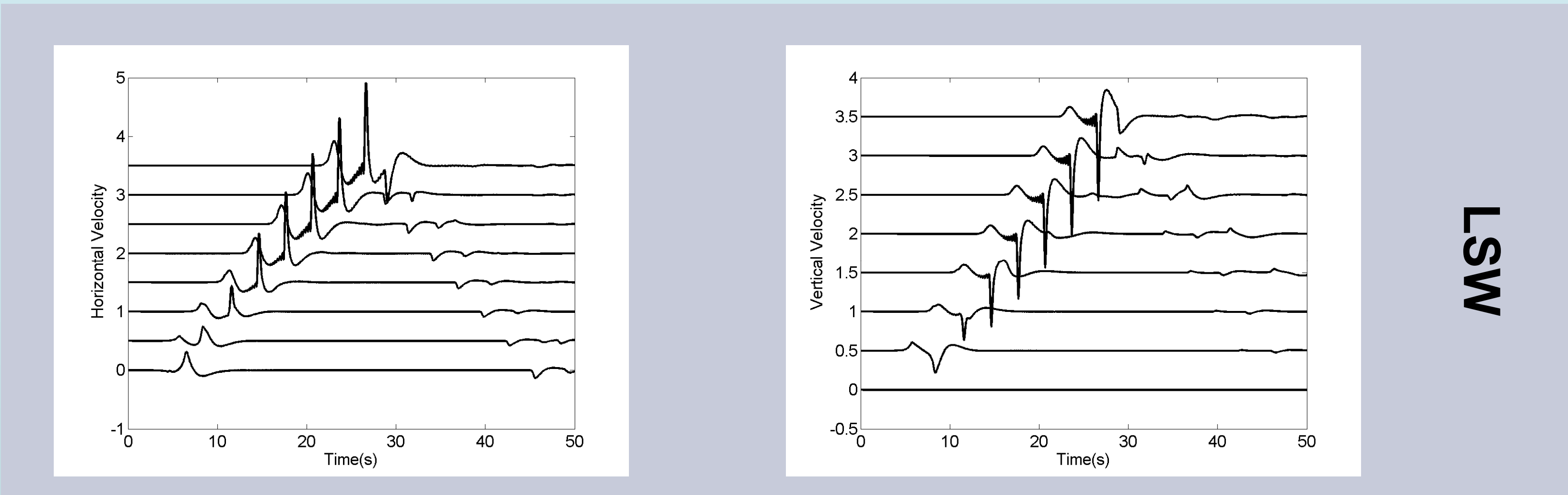
- **initial condition : solution at the critical length $a_c = h_m$**
- **perturbation of the solution (1%)**
- **computation of the elasto-dynamics solution (wave propagation) using a spectral element code**

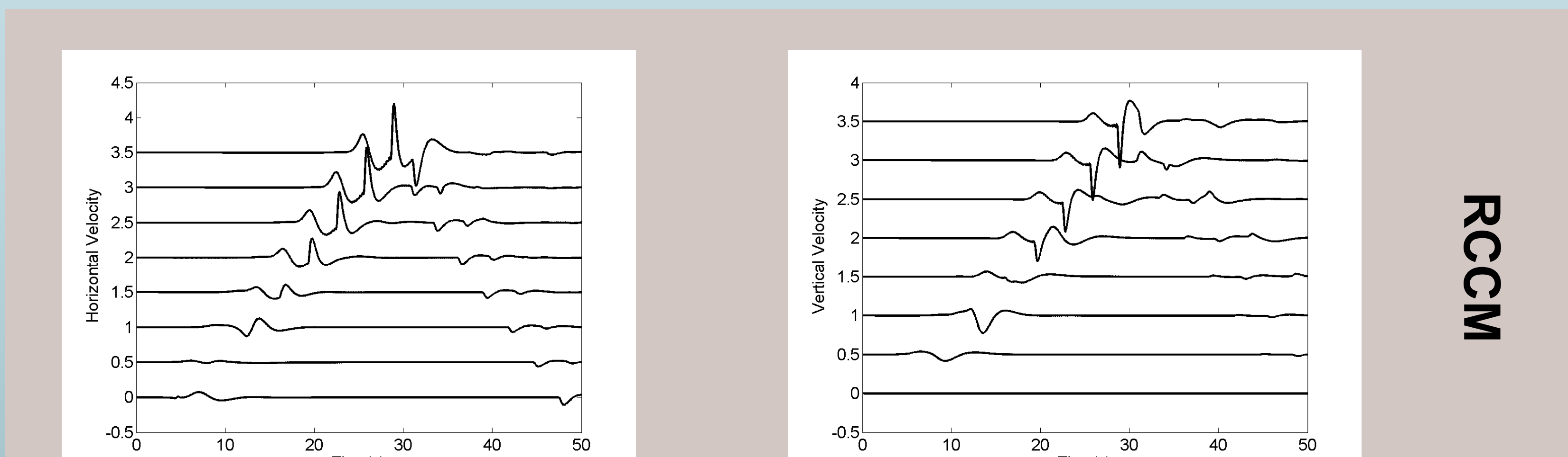
With RCCM : pulse in front of the wave, supershear occurs later, collapse slower (related to the extra dissipation) : … *and supershear is rare for earthquake ….*



4.3 – Strong motion

With RCCM strong motion is reduced (2 to 10 times) (HF filter)





5 - Conclusions

RCCM model

- ***Scale-independent interface law coupling friction and damage (parameters w,μ,C)***
- ***Concept of damage (SW and RS are only based on friction i.e. depending on normal stresses)***
- ***Dissipation : friction, damage and viscosity***
- ***General loadings (unloading, cycle, …) unilateral conditions + law takes into account the unloadings***
- ***Time effects (loading velocity, relaxation, creep, eventually recoverable adhesion)***

Numerical simulations (nucleation, supershear, strong motion)

- ***Nucleation size depends on the slope of the law at the beginning (small earthquakes may have steeper slopes) and on the shape of the loading stress (segregation between small and strong earthquakes)***
- ***RCCM interfaces may creep or rupture (stable quasi-static solutions exist!)***
- ***Strong motion is reduced as compared to LSW with the same energy balance***
- ***Supershear is delayed (μ_d is allowed to be 0.25 without having supershear all time).***

  The probability to observe supershear is smaller than for LSW models and supershear is rare for the earthquakes.