Qualitative dynamics of a simple model with unilateral contact and Coulomb friction

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Motivation

- Occurrence of vibrations in sliding systems
- Control mechanisms submitted to dry friction

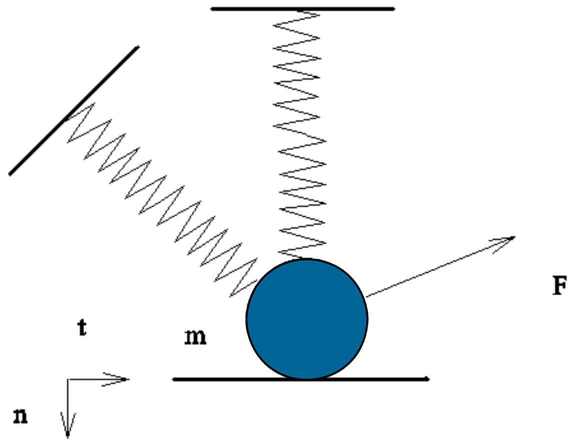
Tools

•Simple model but reveals interesting behaviour
•Dynamical approach

Klarbring's mass spring system

$$\begin{cases} m\ddot{u}_t + K_t u_t + W u_n = F_t + R_t \\ m\ddot{u}_n + W u_t + K_n u_n = F_n + R_n \end{cases} \qquad t \geq 0$$

$$\begin{cases} u_n \leq 0 \quad R_n \leq 0 \quad u_n R_n = 0 \\ \mu R_n \leq R_t \leq -\mu R_n \\ \left|R_t\right| = -\mu R_n \quad \Rightarrow \; \exists \lambda \geq 0 \;\; \dot{u}_t = -\lambda R_t \end{cases}$$



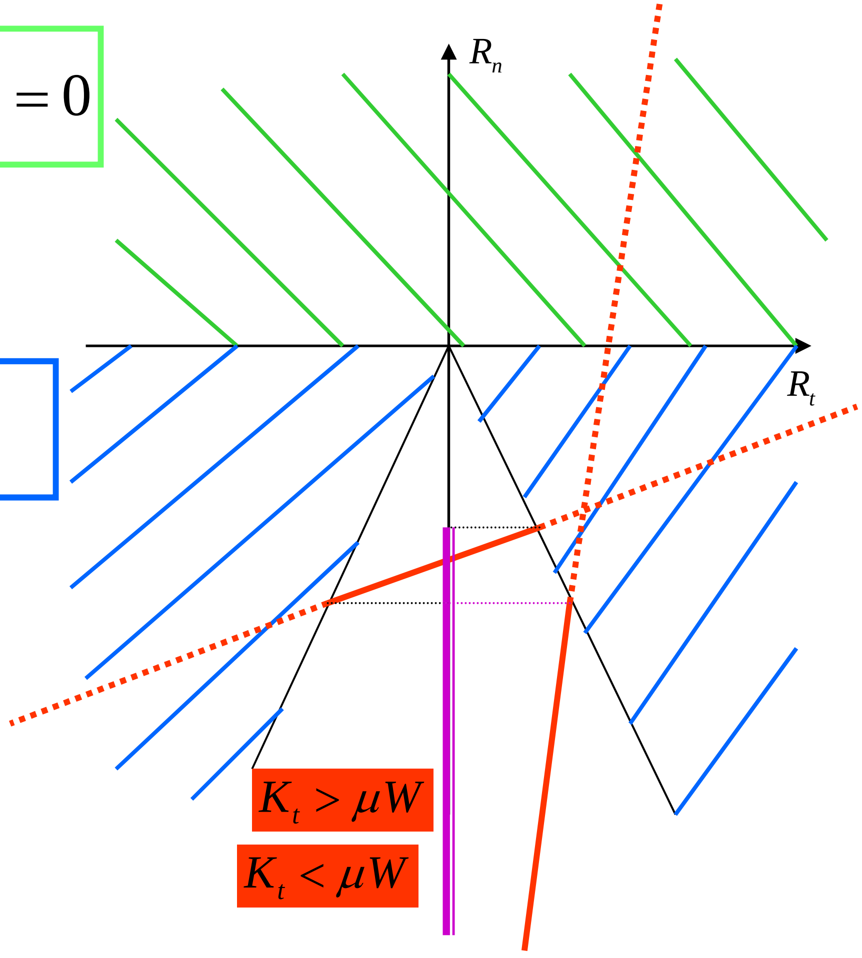
+ initial conditions   + impact law

Equilibrium solutions in strict contact

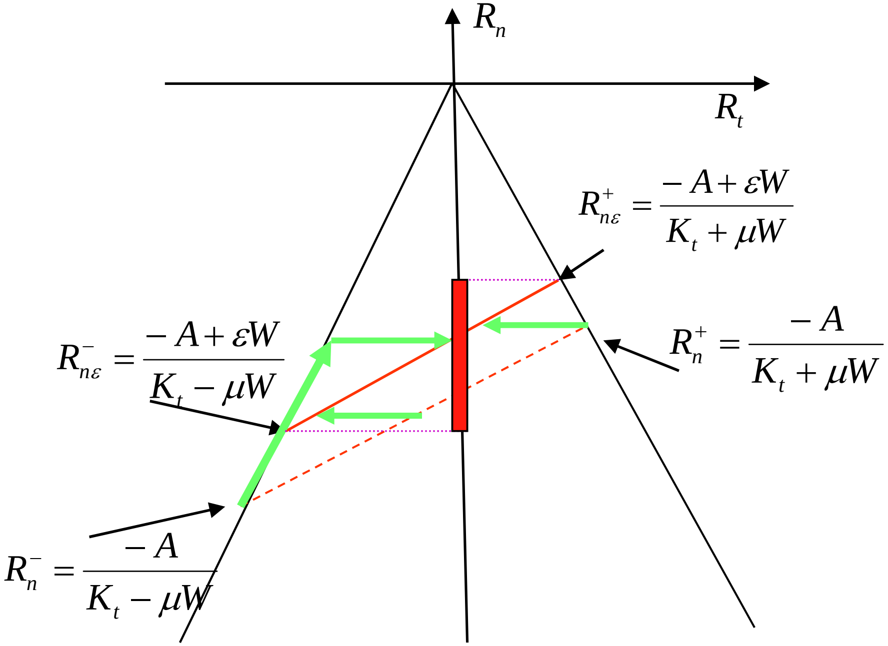
$$u_n \le 0 \quad R_n \le 0 \quad u_n R_n = 0$$

$$\mu R_n \le R_t \le -\mu R_n$$

$$R_n = -\frac{K_t F_n - W F_t}{K_t} + \frac{W}{K_t} R_t$$

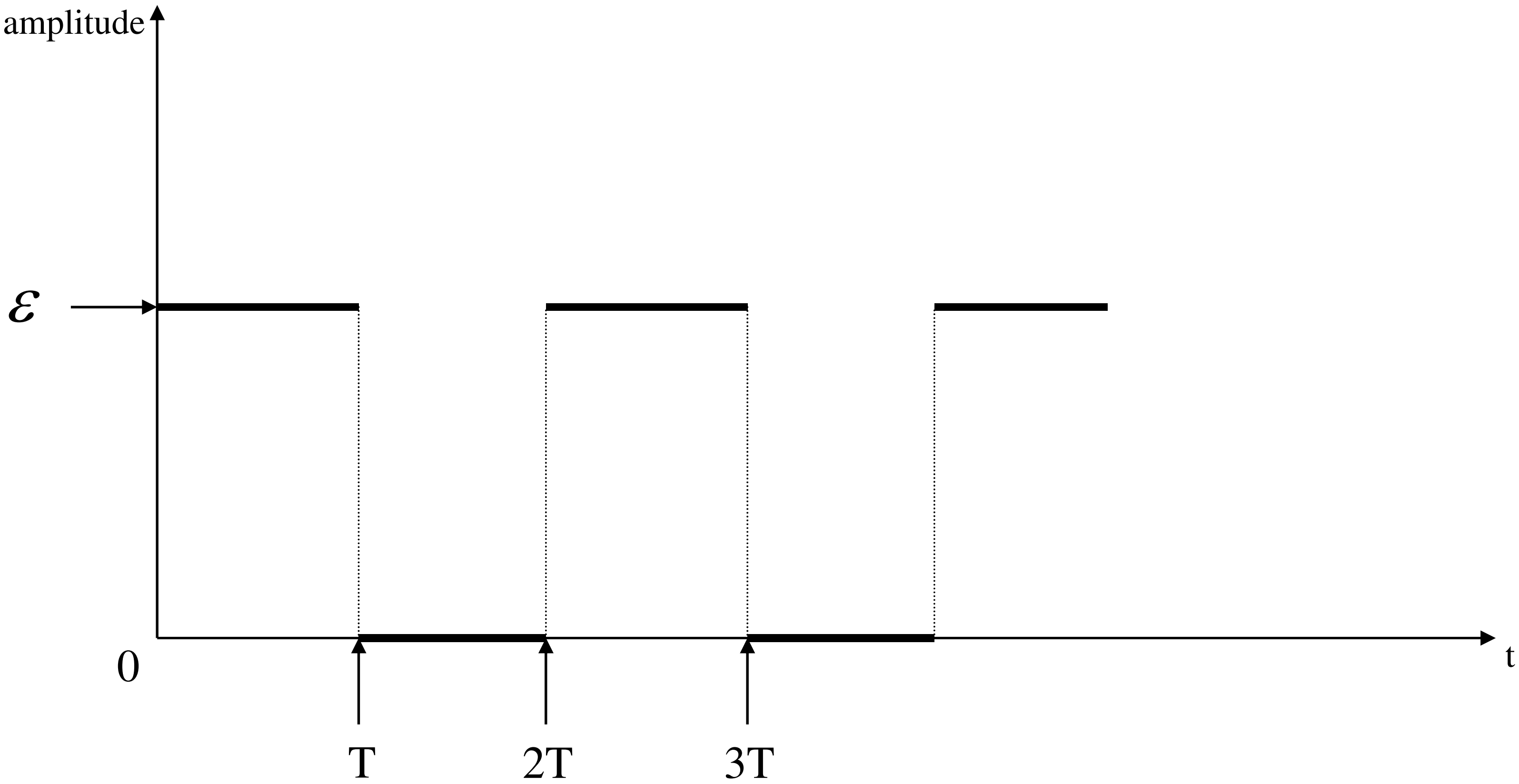


Adding a small constant to the tangential loading

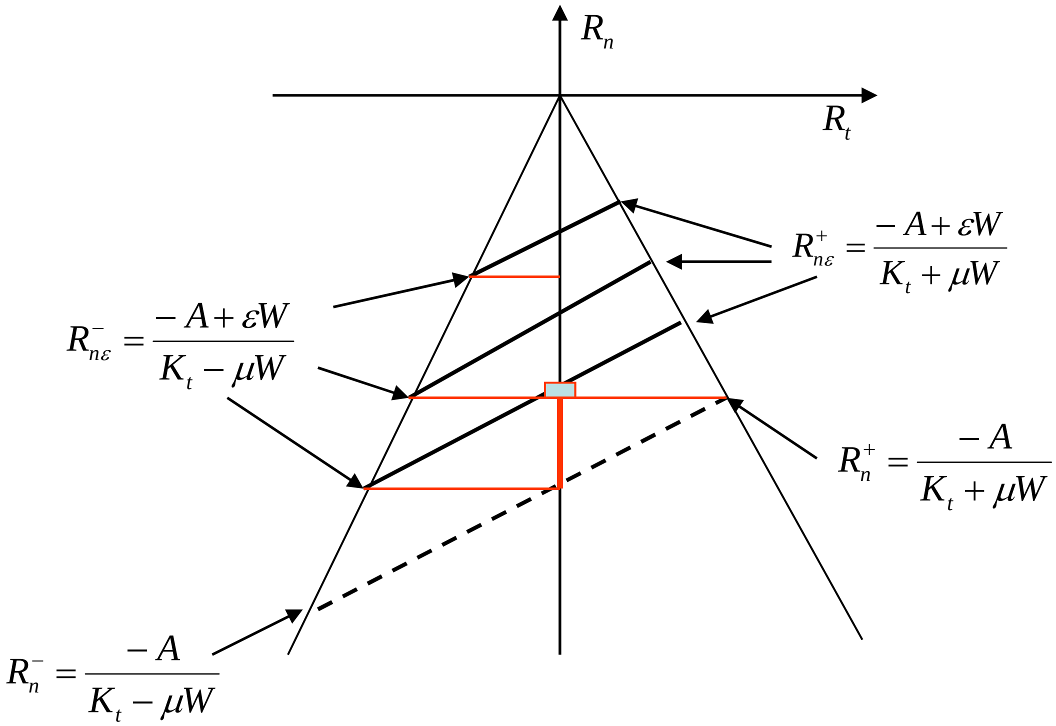


where $A = K_t F_n - W F_t$

Applying an oscillating tangential loading

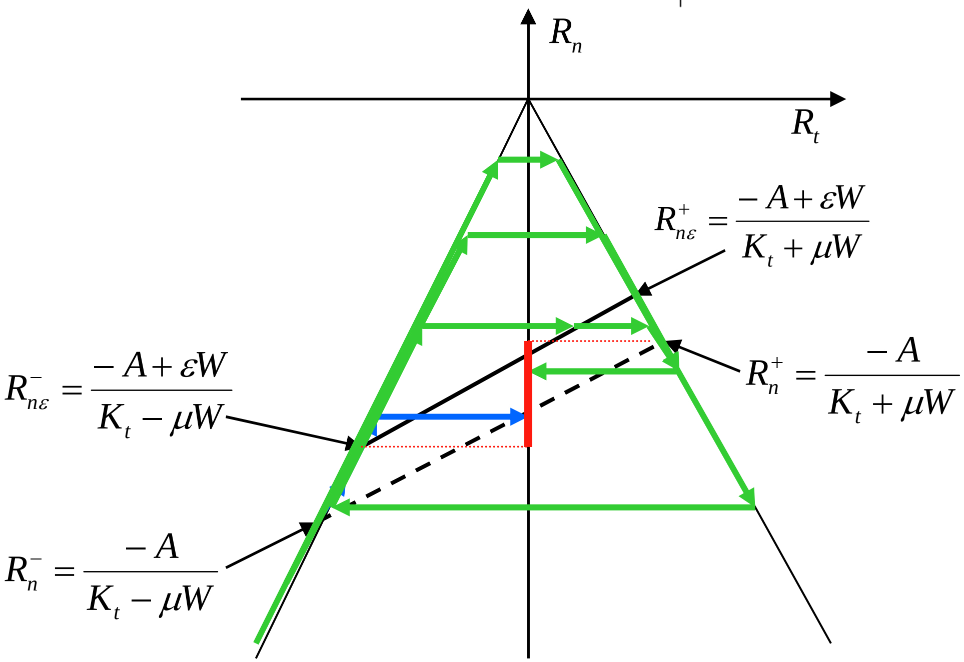


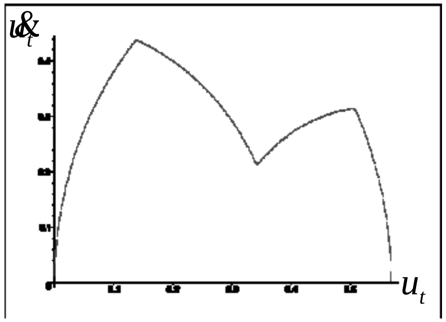
Effect of the amplitude of the loading

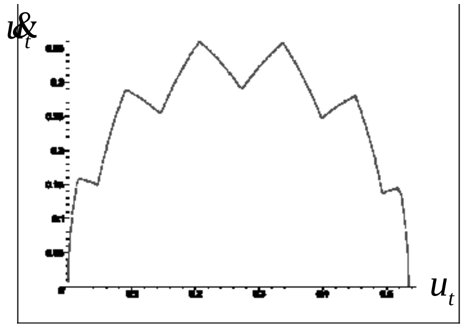


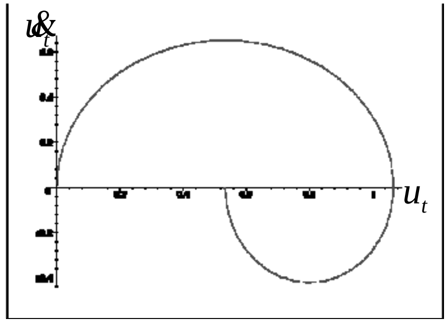
represented in the (R_t, R_n) plane

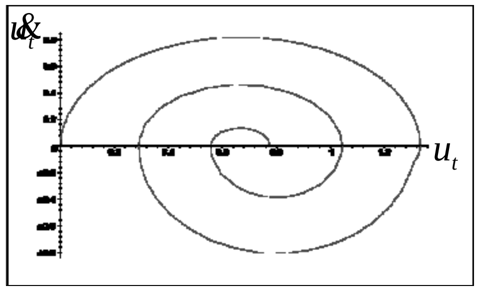
When $\varepsilon < \dfrac{2\mu A}{K_t + \mu W}$



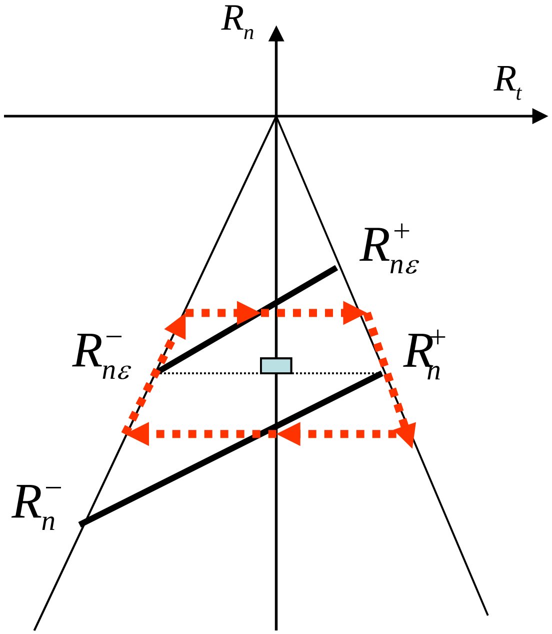






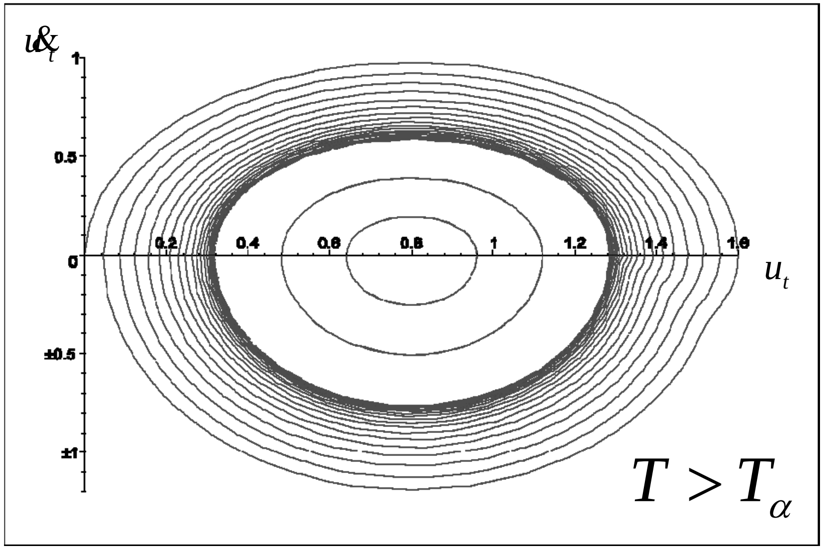


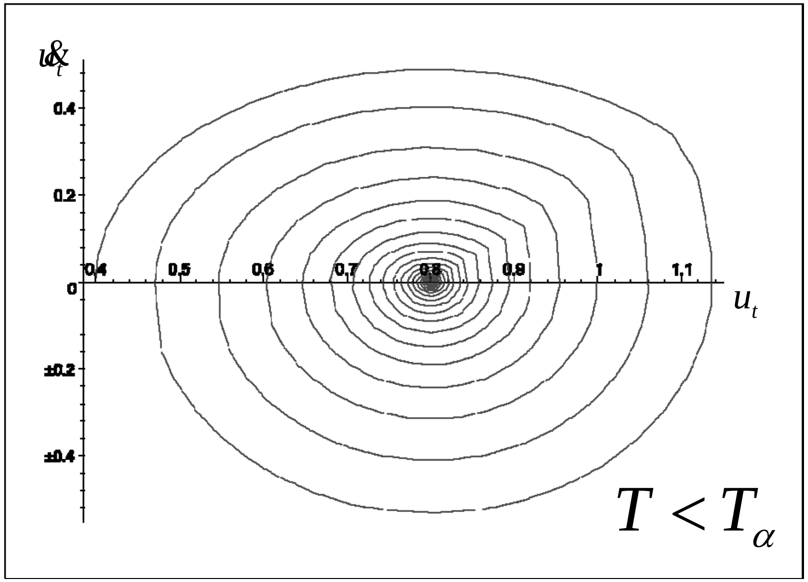
When $\varepsilon = \dfrac{2\mu A}{K_t + \mu W}$



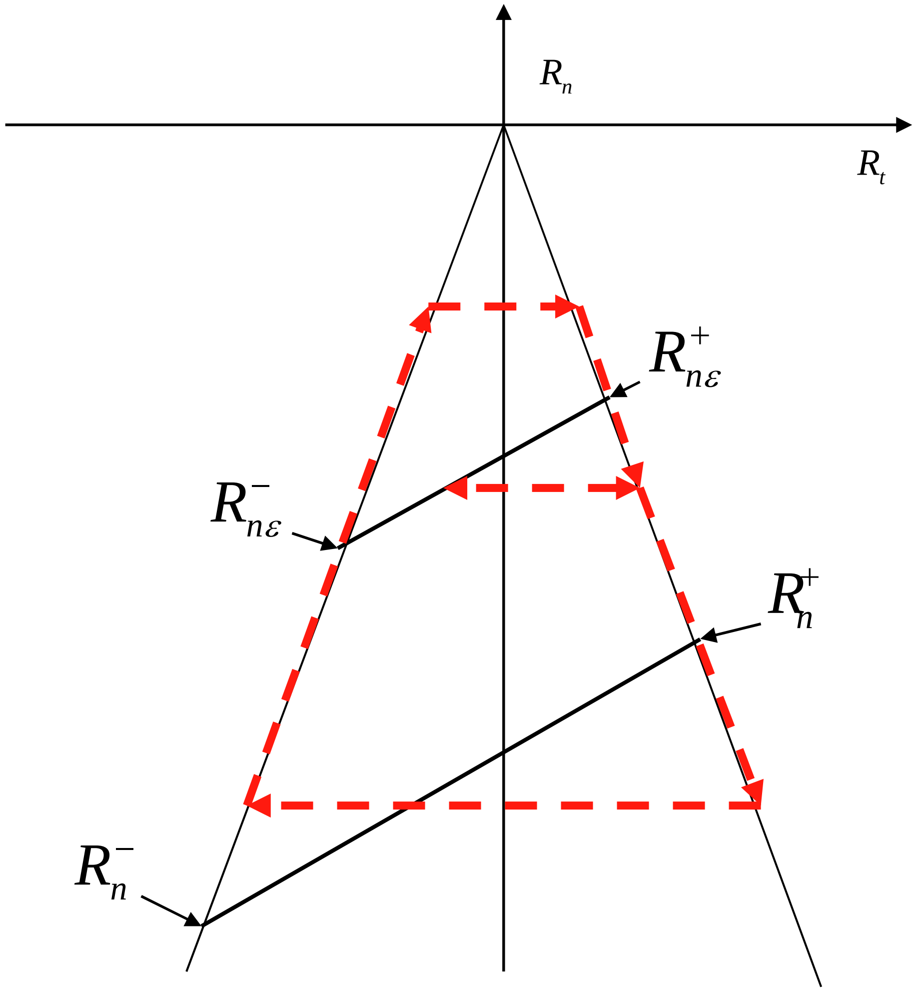
$$T_\alpha = \pi\sqrt{\frac{m}{K_t - \mu W}}$$

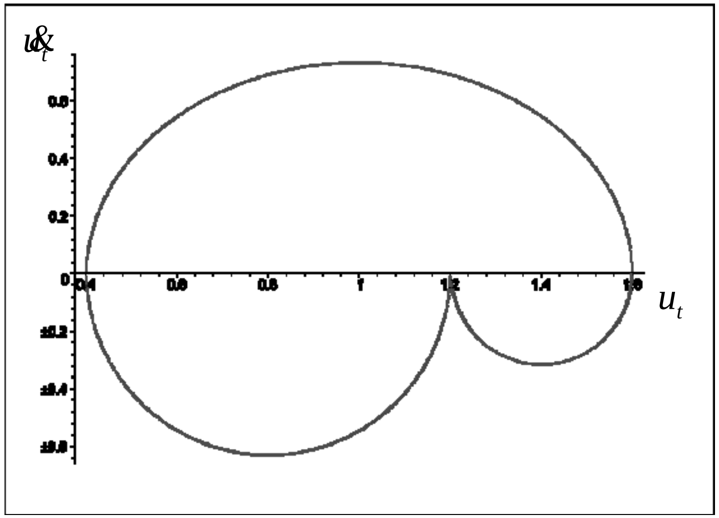
$$T_\beta = \pi\sqrt{\frac{m}{K_t + \mu W}}$$





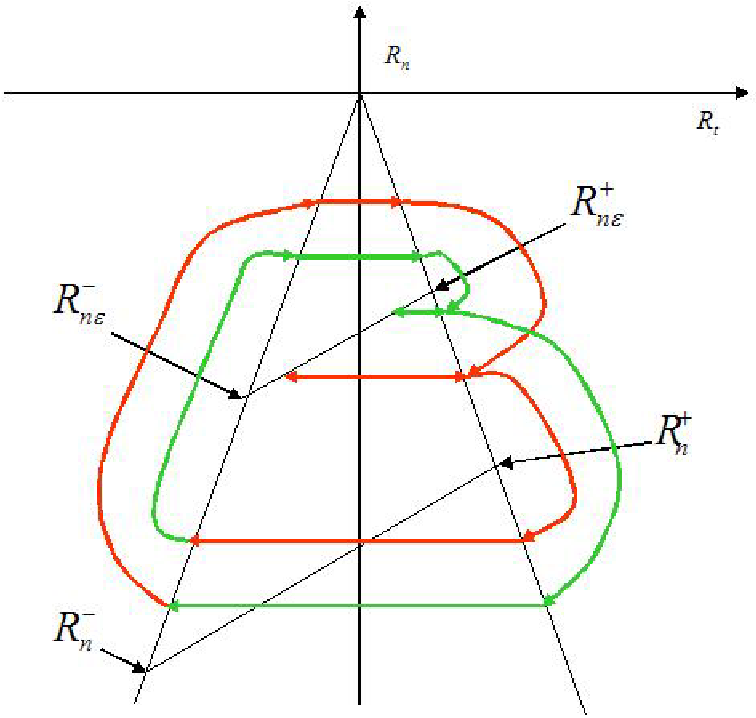
Do periodic solutions exist when $\varepsilon > \dfrac{2\mu A}{K_t + \mu W}$ and $T > T_\alpha + T_\beta$?

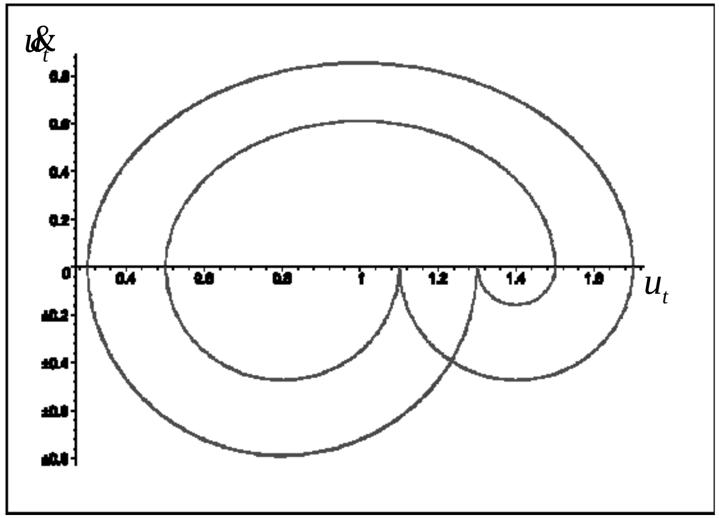


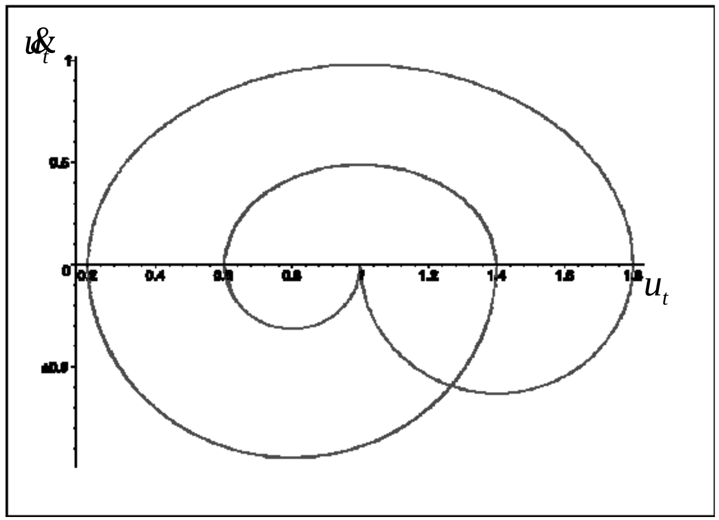


How about double period solutions ?

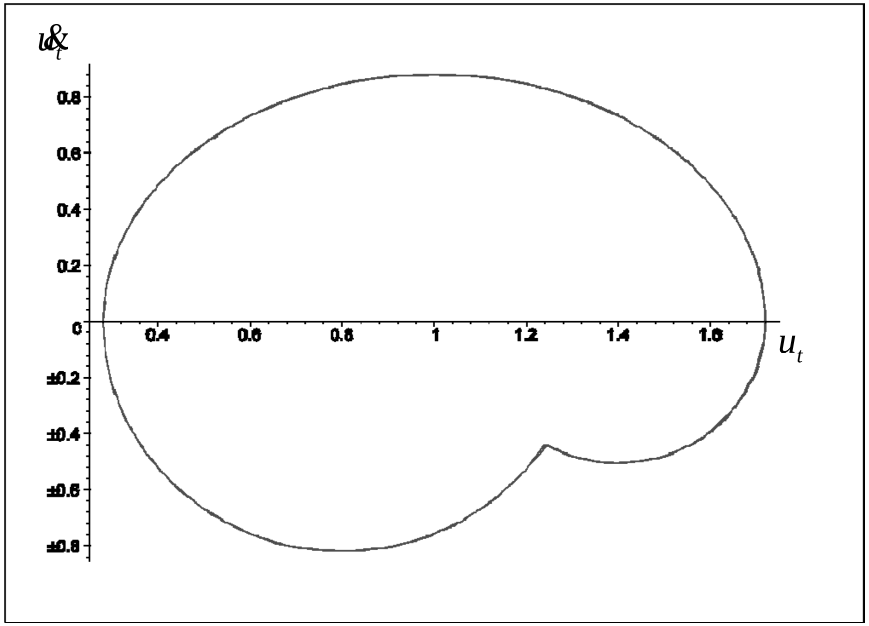
$$\varepsilon > \frac{2\mu A}{K_t + \mu W} \quad \text{and} \quad T > T_\alpha + T_\beta$$



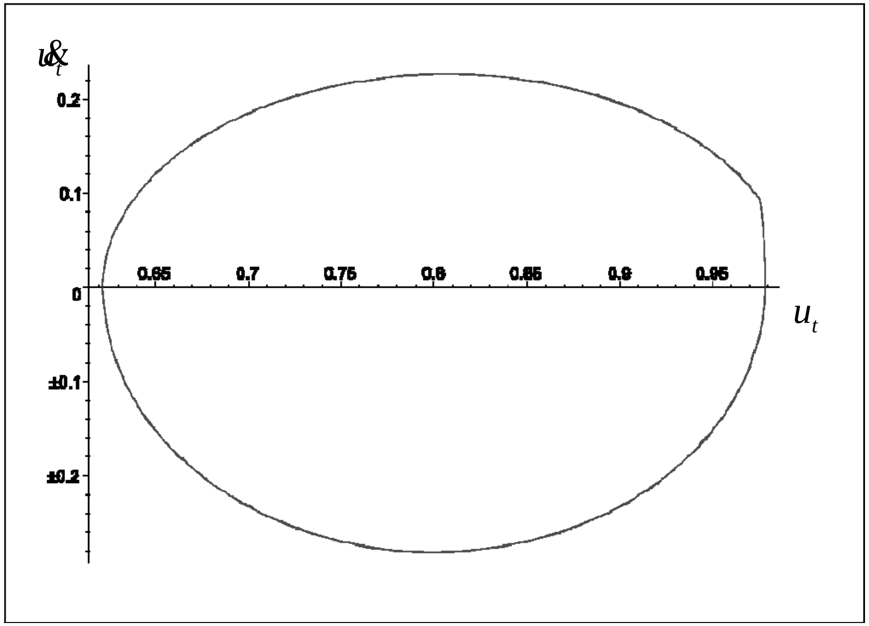




Are there still periodic solutions when $T < T_\alpha + T_\beta$?

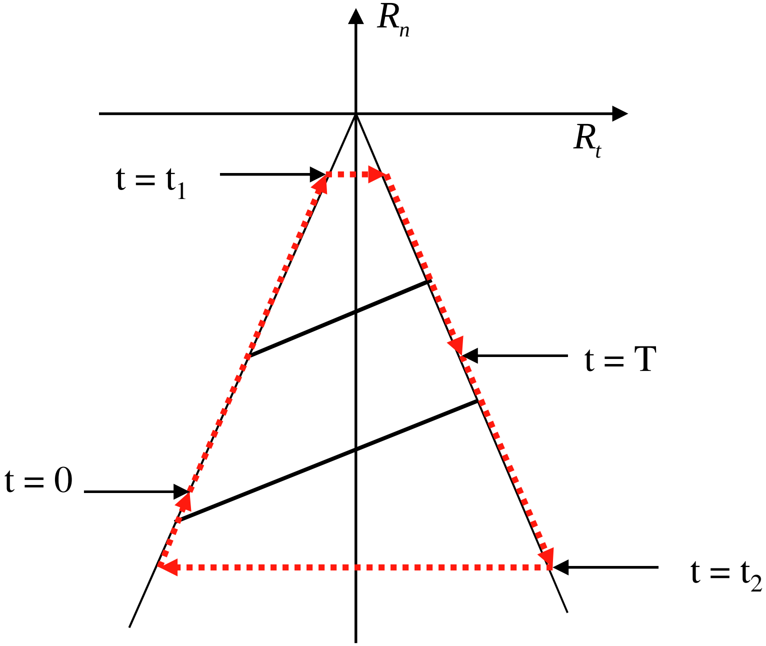


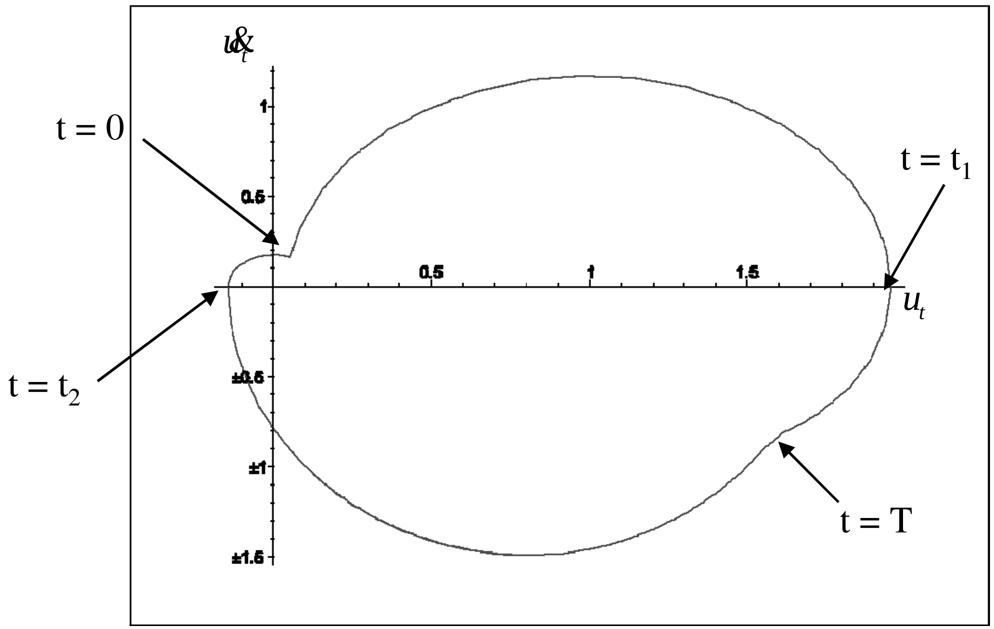
$T_\beta < T < T_\alpha$



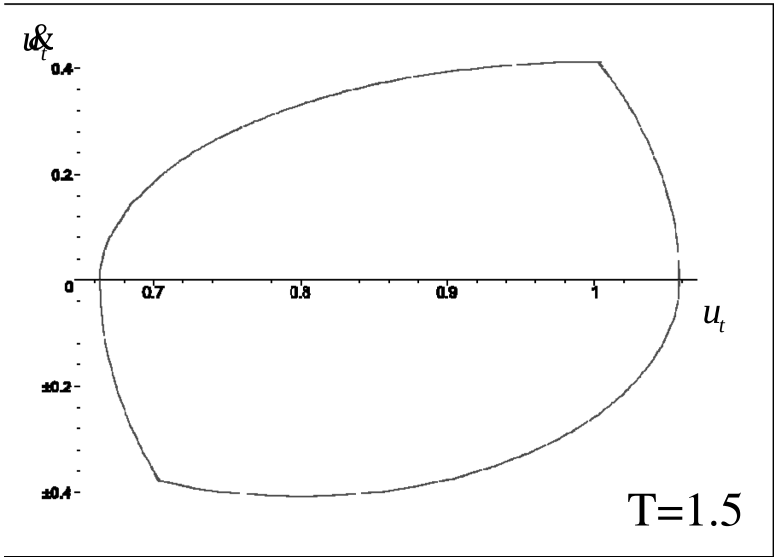
$$\varepsilon > \frac{2\mu A}{K_t + \mu W}$$

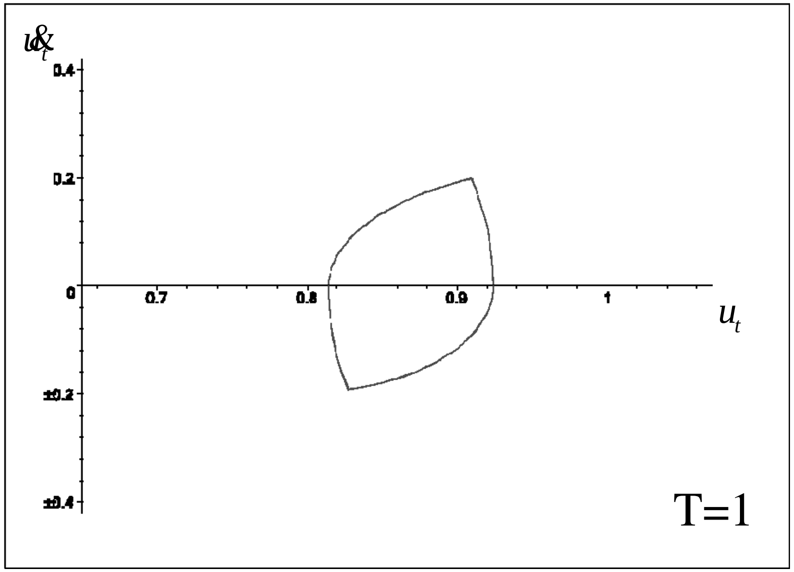
And when a phase difference appears

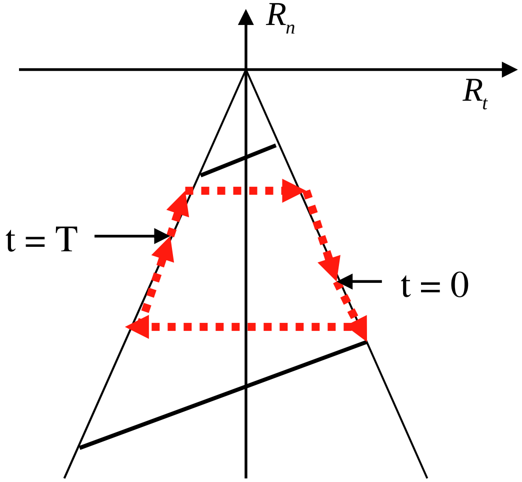


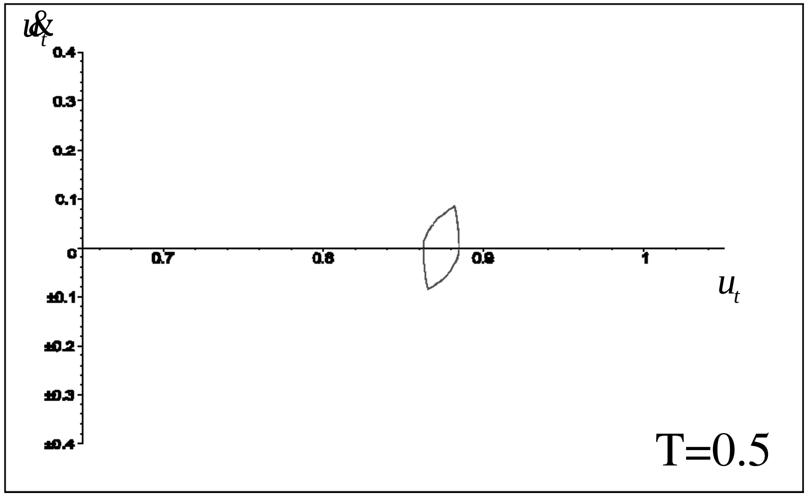


And when T is even smaller ?



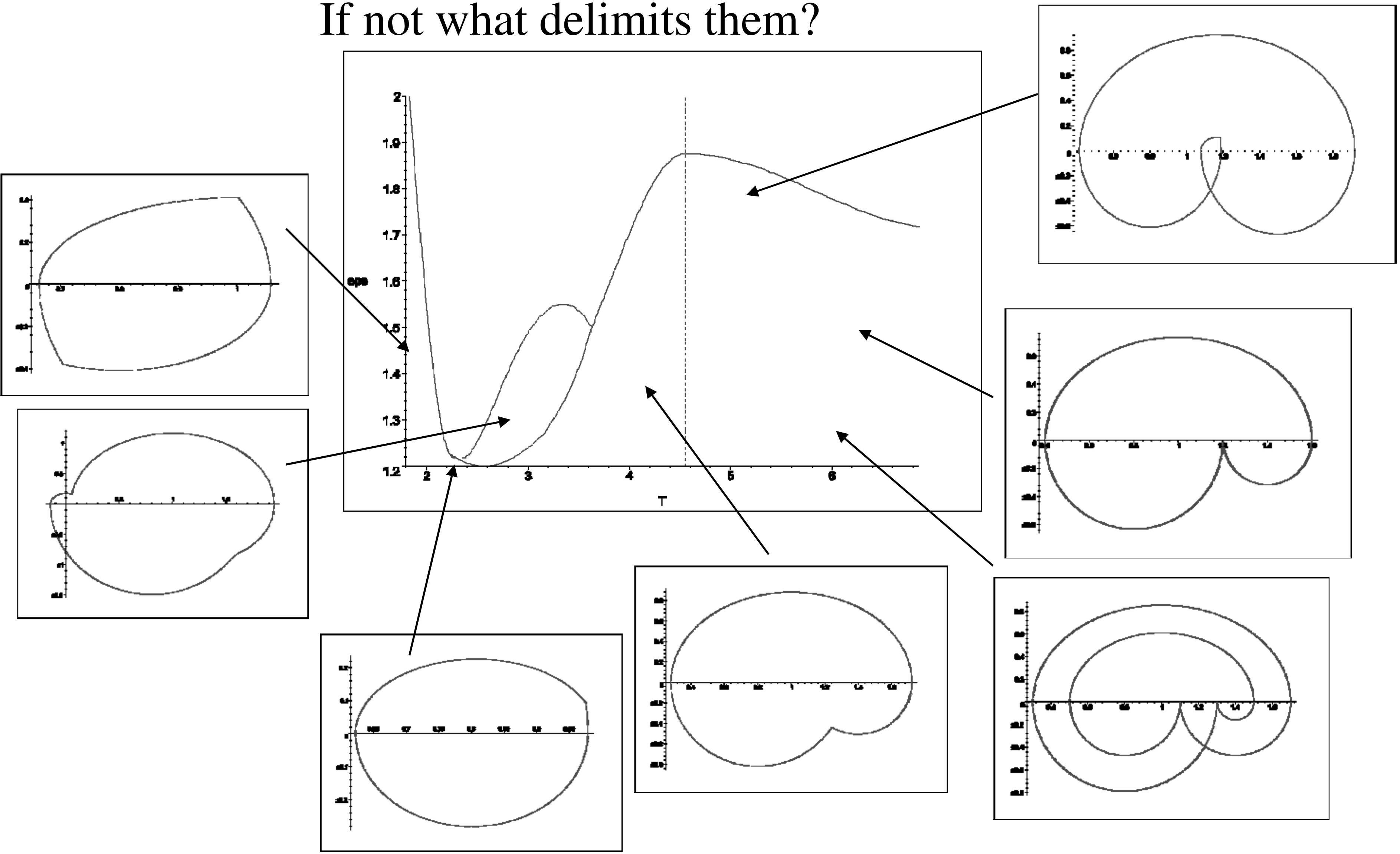






Do these periodic solutions exist for all values of ε ?

If not what delimits them?



To summarize