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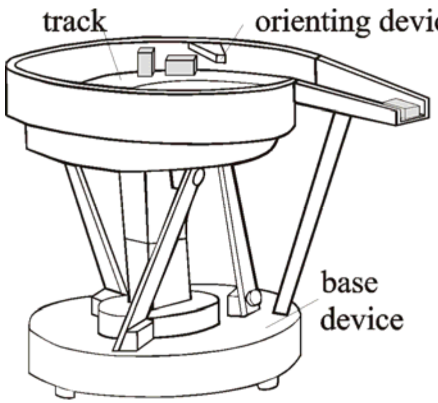
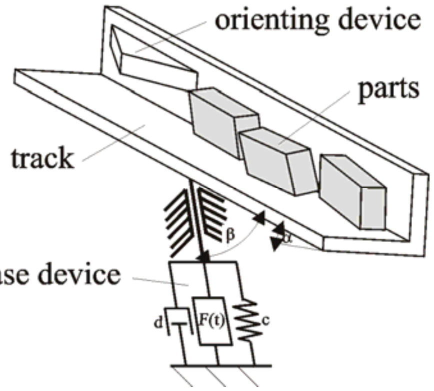
Reduction of a Vibration Conveyor Process

Friedrich Pfeiffer, Garching

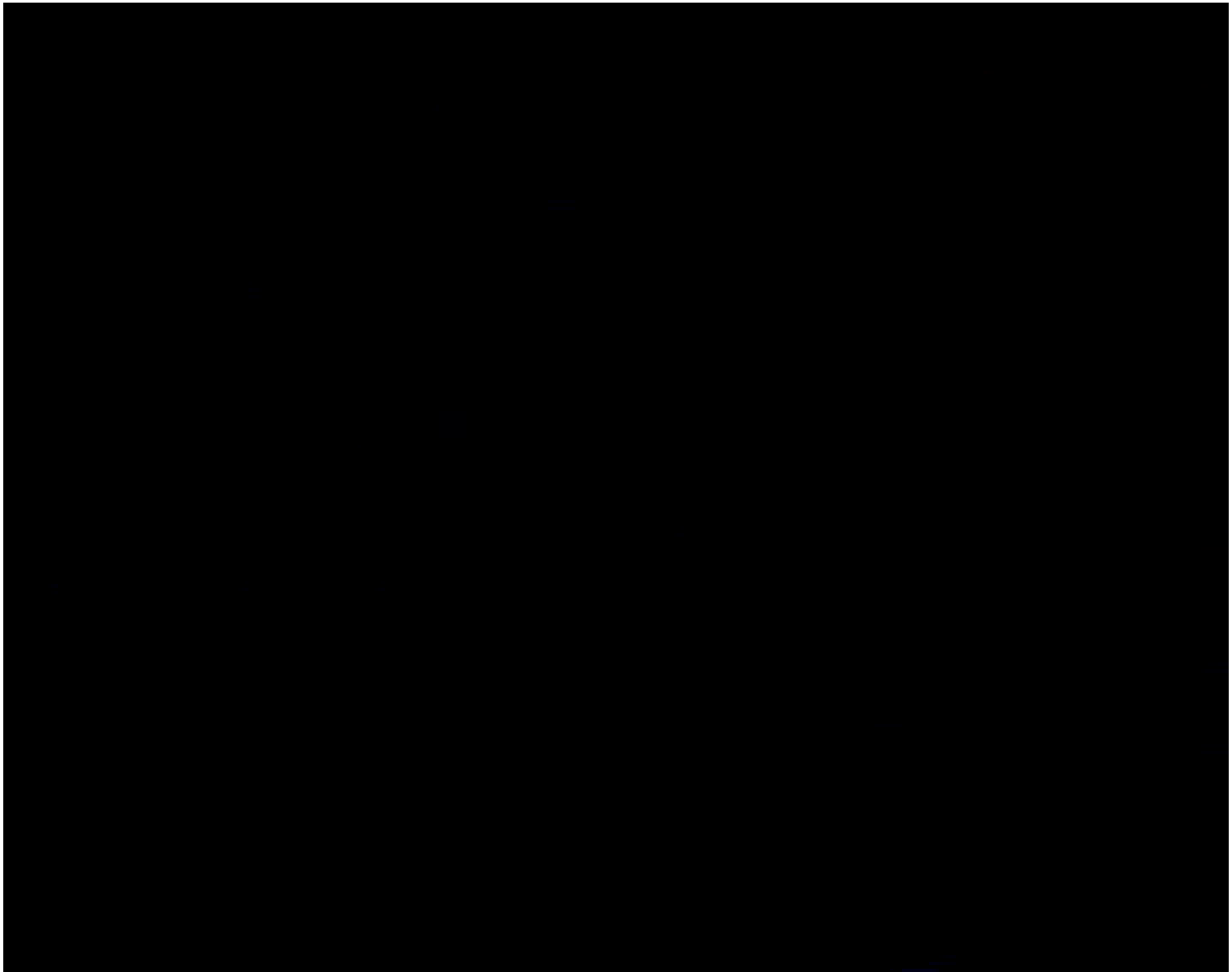


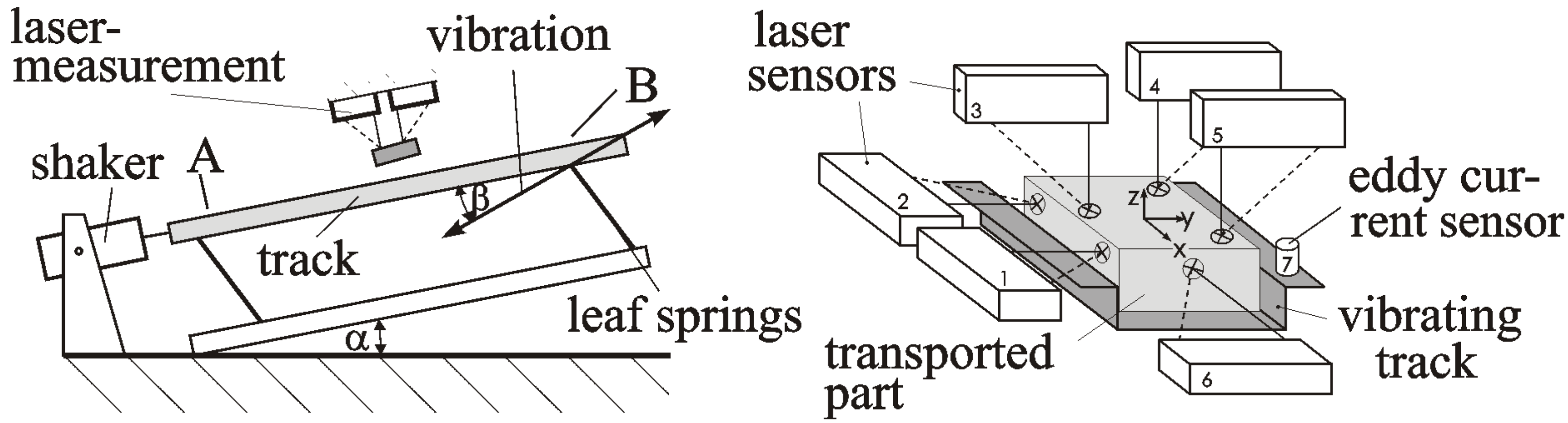


- Problem Definition
- General Solution
- Former Results
- Simplified Model
- Comparison Measurements
- Summary

Unilateral Problems in Structural Analysis. June 17-19, 2010, Palmanova

Typical Conveyor Operation

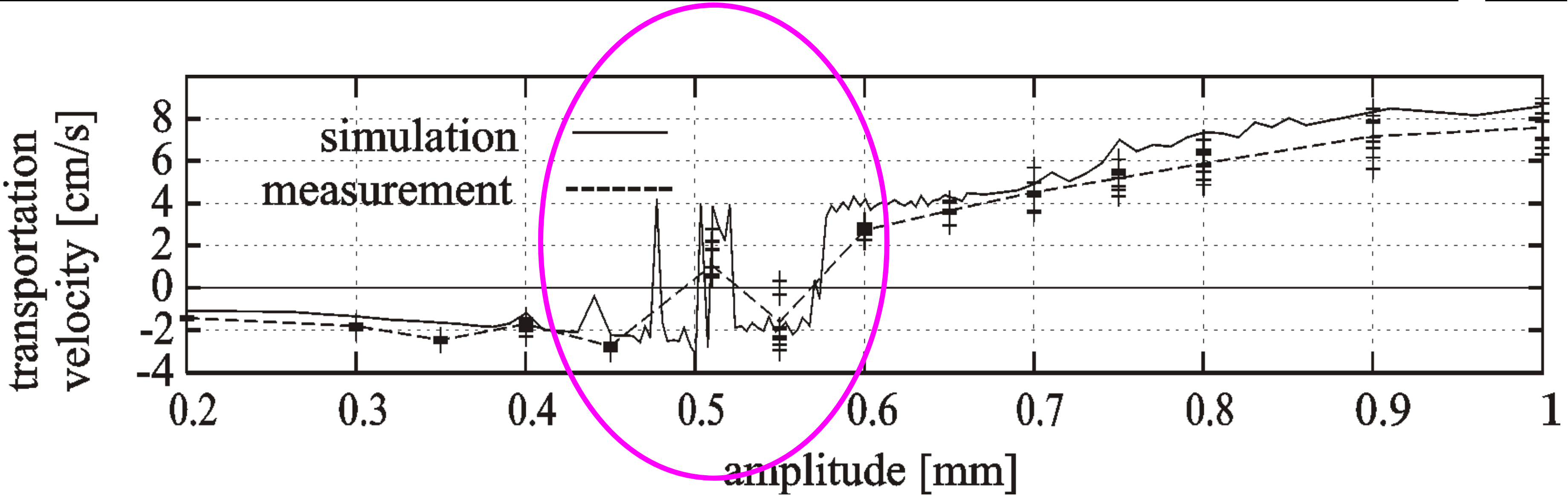




$$
\begin{aligned}
\boldsymbol{M}(\boldsymbol{q},t)d\boldsymbol{u} - \boldsymbol{h}(\boldsymbol{u},\boldsymbol{q},t)dt - \boldsymbol{W}_N(\boldsymbol{q},t)d\boldsymbol{\Lambda}_N - \boldsymbol{W}_T(\boldsymbol{q},t)d\boldsymbol{\Lambda}_T &= 0 \\
\boldsymbol{\lambda}_N - \mathrm{prox}_{C_N}(\boldsymbol{\lambda}_N - r\ddot{\boldsymbol{g}}_N) &= 0 \\
\boldsymbol{\lambda}_T - \mathrm{prox}_{C_T(\lambda_N)}(\boldsymbol{\lambda}_T - r\ddot{\boldsymbol{g}}_T) &= 0\,. \qquad (1)
\end{aligned}
$$

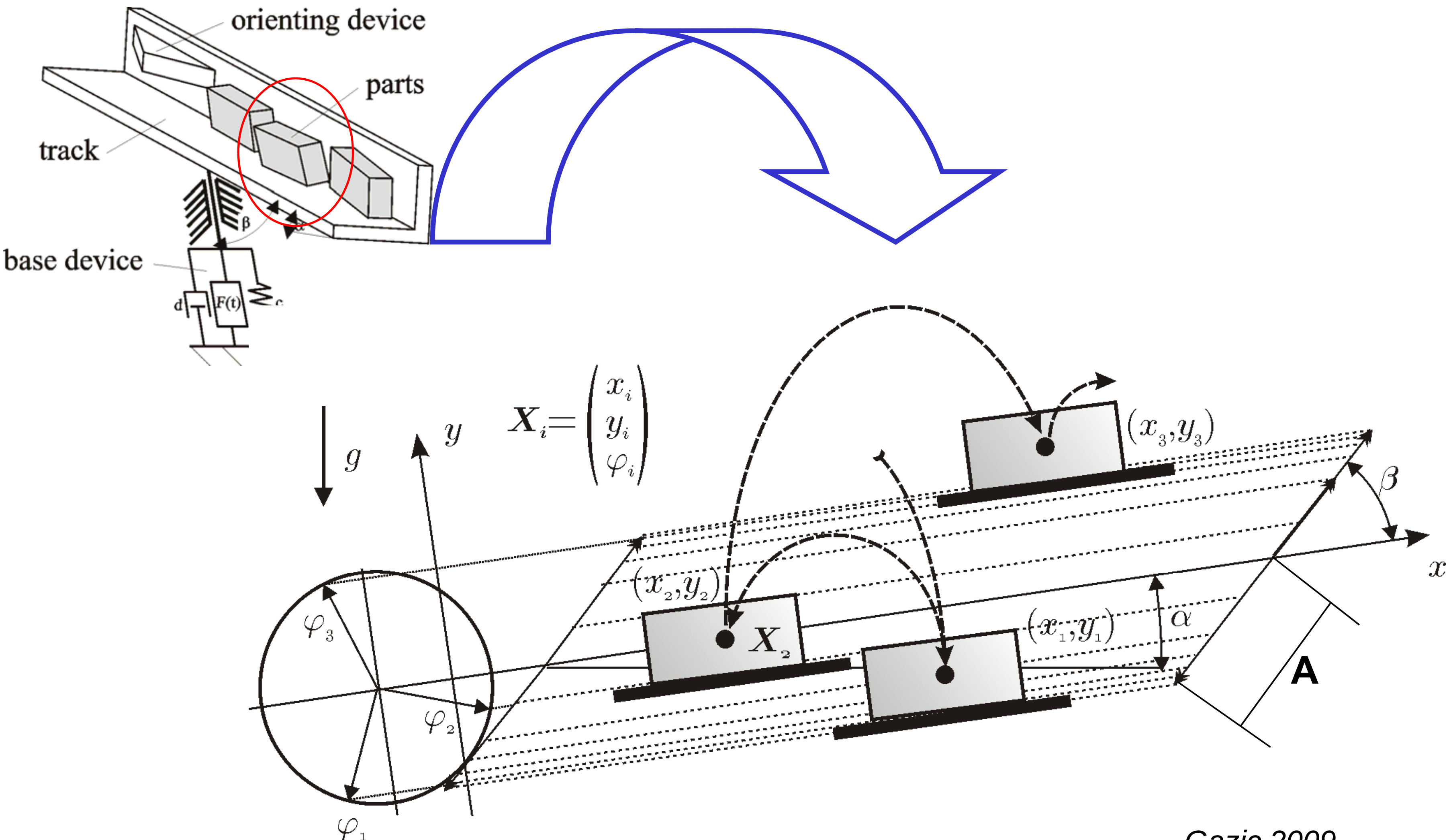
Wolfsteiner 1999 with Lemke-algorithm

Former Results

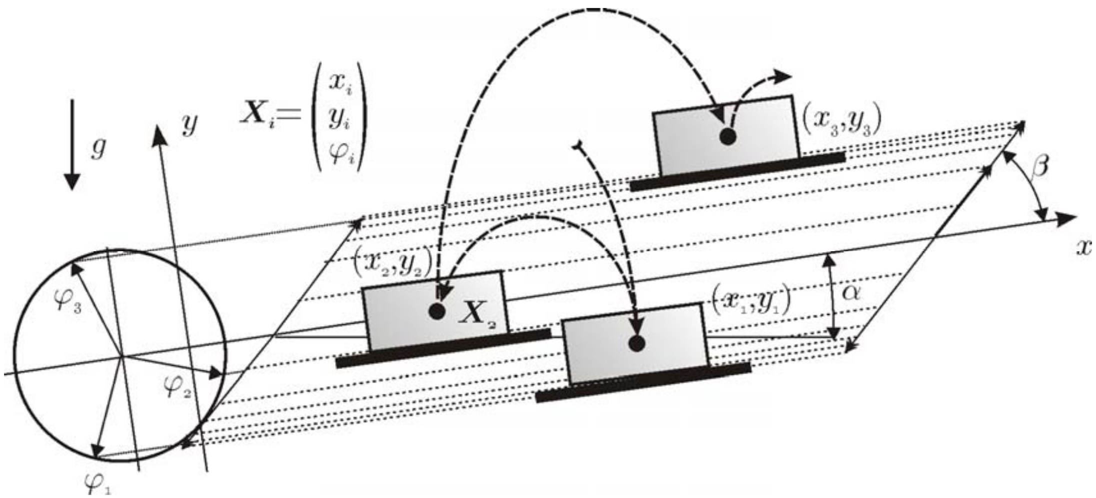


- **Not much difference between spatial and planar theoretical models (with the exception of the orienting devices)**
- **Even large heaps of parts in the conveyor base decompose by the conveyor process into individually moving parts**
- **Therefore an explanation of the above effects also possible by considering a single part dynamics with planar theory**

Single Part - Point Mass - Model



Gazic 2009



Feeder Motion:

$$\begin{pmatrix} \bar{x} \\ \bar{y} \end{pmatrix}_F = \begin{pmatrix} +cos\beta sin(\Omega t) \\ +sin\beta sin(\Omega t) \end{pmatrix},$$

$$\begin{pmatrix} \dot{\bar{x}} \\ \dot{\bar{y}} \end{pmatrix}_F = \begin{pmatrix} +cos\beta cos(\Omega t) \\ +sin\beta cos(\Omega t) \end{pmatrix},$$

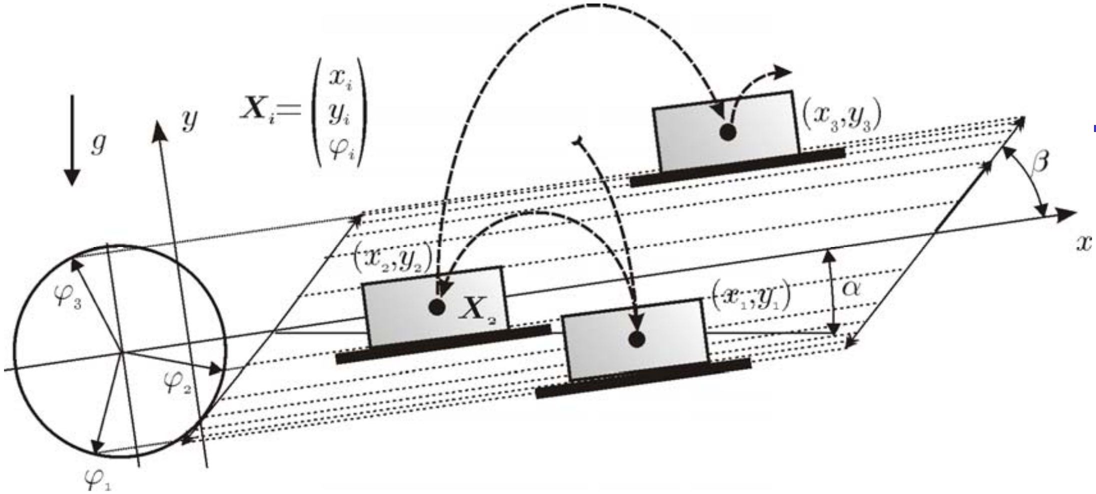
$$\begin{pmatrix} \ddot{\bar{x}} \\ \ddot{\bar{y}} \end{pmatrix}_F = \begin{pmatrix} -cos\beta sin(\Omega t) \\ -sin\beta sin(\Omega t) \end{pmatrix}.$$

Relative Motion:

$$\begin{pmatrix} \bar{g}_N \\ \bar{g}_T \end{pmatrix} = \begin{pmatrix} \bar{y}_P - \bar{y}_F \\ \bar{x}_P - \bar{x}_F \end{pmatrix}, \qquad \begin{pmatrix} \dot{\bar{g}}_N \\ \dot{\bar{g}}_T \end{pmatrix} = \begin{pmatrix} \dot{\bar{y}}_P - \dot{\bar{y}}_F \\ \dot{\bar{x}}_P - \dot{\bar{x}}_F \end{pmatrix}, \qquad \begin{pmatrix} \ddot{\bar{g}}_N \\ \ddot{\bar{g}}_T \end{pmatrix} = \begin{pmatrix} \ddot{\bar{y}}_P - \ddot{\bar{y}}_F \\ \ddot{\bar{x}}_P - \ddot{\bar{x}}_F \end{pmatrix}.$$

Dimensionless Magnitudes:

$$(\bar{x}, \bar{y}) = \frac{(x, y)}{A}, \qquad (\dot{\bar{x}}, \dot{\bar{y}}) = \frac{(\dot{x}, \dot{y})}{A\Omega}, \qquad (\ddot{\bar{x}}, \ddot{\bar{y}}) = \frac{(\ddot{x}, \ddot{y})}{A\Omega^2}, \qquad K = (\frac{g}{A\Omega^2}).$$



ballistics of the mass

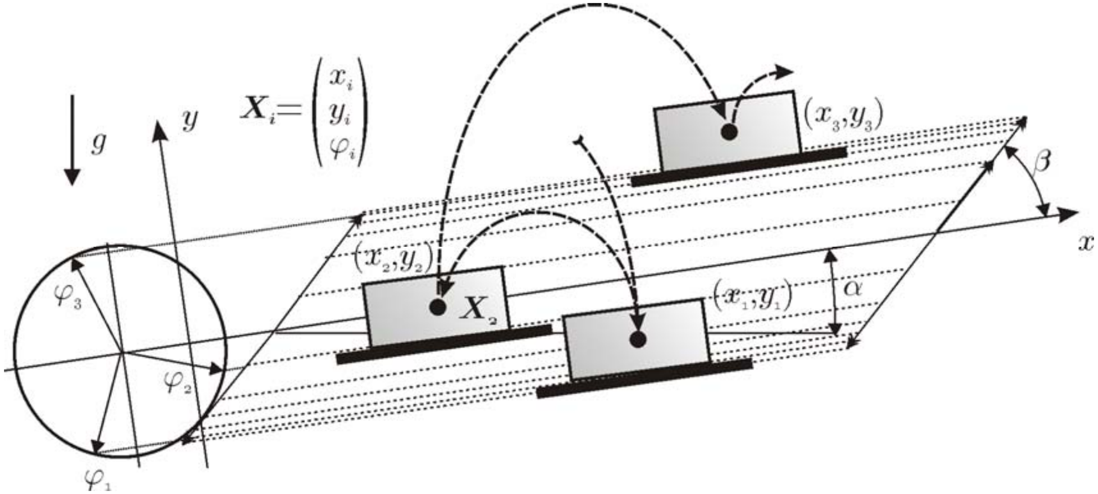
$$t = \frac{\varphi}{\Omega}, \qquad \Delta\varphi = \Omega t - \varphi_0$$

$$K = \left(\frac{g}{A\Omega^2}\right)$$

$$\begin{pmatrix} \ddot{\bar{x}} \\ \ddot{\bar{y}} \end{pmatrix}_P = \begin{pmatrix} -K sin\alpha \\ -K cos\alpha \end{pmatrix},$$

$$\begin{pmatrix} \dot{\bar{x}} \\ \dot{\bar{y}} \end{pmatrix}_P = \begin{pmatrix} \dot{\bar{x}}_0 \\ \dot{\bar{y}}_0 \end{pmatrix} + \begin{pmatrix} -K sin\alpha \\ -K cos\alpha \end{pmatrix} \Delta\varphi$$

$$\begin{pmatrix} \bar{x} \\ \bar{y} \end{pmatrix}_P = \begin{pmatrix} \bar{x}_0 \\ \bar{y}_0 \end{pmatrix} + \begin{pmatrix} \dot{\bar{x}}_0 \\ \dot{\bar{y}}_0 \end{pmatrix} \Delta\varphi + \begin{pmatrix} -K sin\alpha \\ -K cos\alpha \end{pmatrix} \frac{\Delta\varphi^2}{2}.$$



Contact Phase

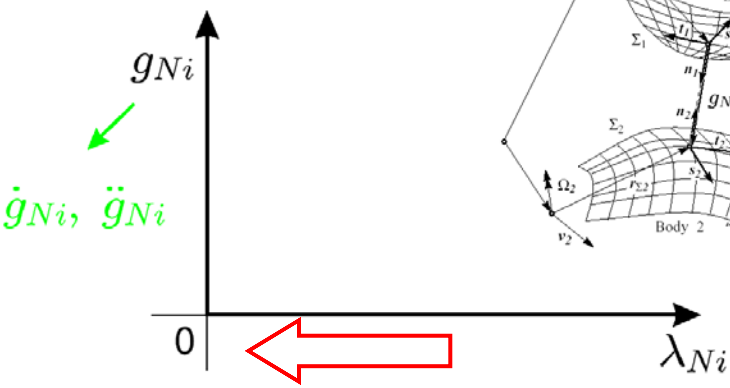
$$\bar{g}_N = \bar{y}_P - \bar{y}_F = [\bar{y}_0 + \dot{\bar{y}}_0 \cdot (\Delta\varphi) - K cos\alpha \cdot (\frac{\Delta\varphi^2}{2})] - [sin\beta \sin(\varphi_0 + \Delta\varphi)] = 0,$$

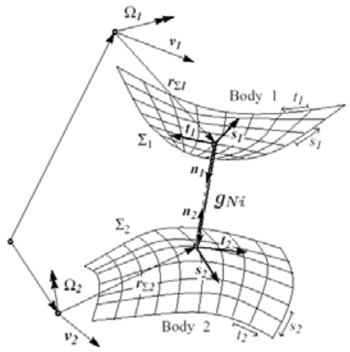
$$\Longrightarrow \boxed{\Delta\varphi}$$

Closing Condition

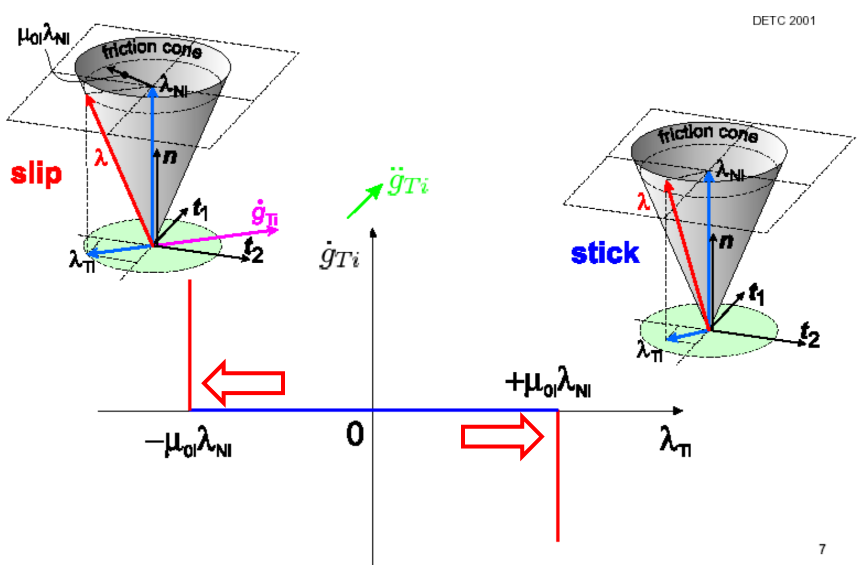
$$\dot{\bar{g}}_N = \dot{\bar{y}}_P - \dot{\bar{y}}_F = [\dot{\bar{y}}_0 - K cos\alpha \cdot (\Delta\varphi)] - [sin\beta \cos(\varphi_0 + \Delta\varphi)] < 0$$

Start Ballistic Flight = End of Contact





for non-impulsive contacts only



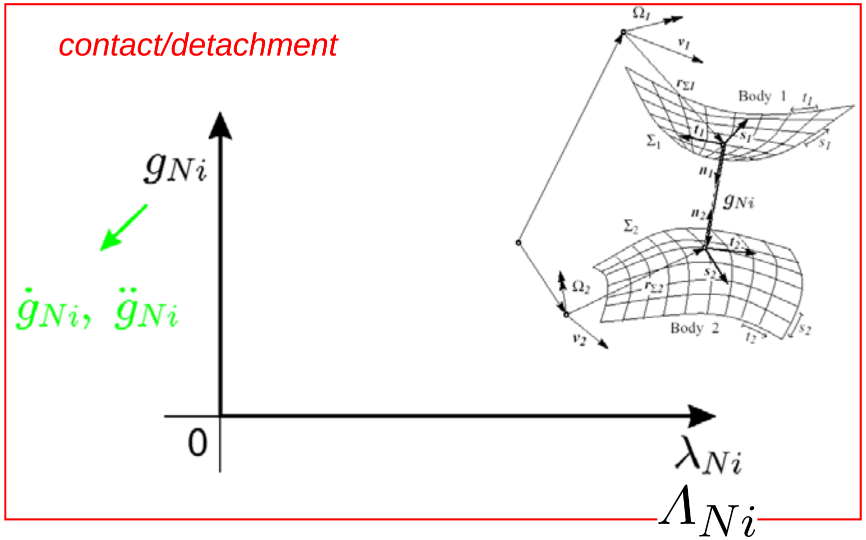
Impact with Friction

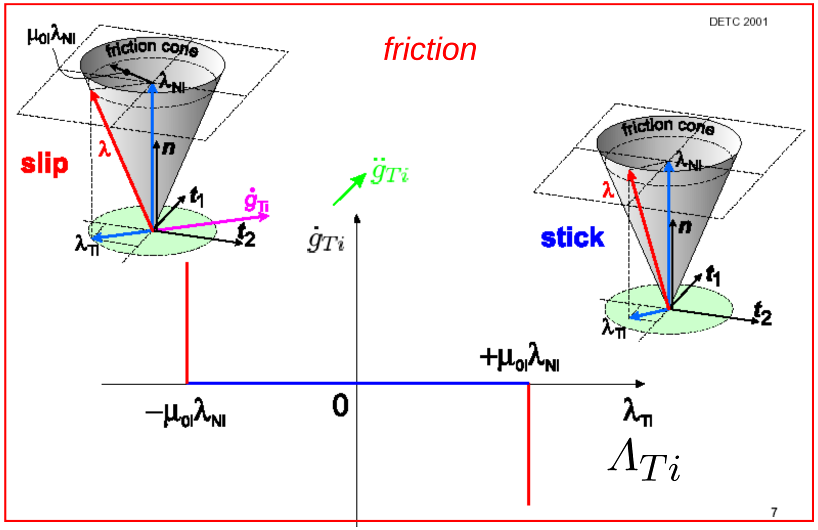
$$m(\dot{x}_C - \dot{x}_A) + \Lambda_{TC} + \mu sign(\dot{g}_{TC})\Lambda_{NC} = 0$$
$$m(\dot{y}_C - \dot{y}_A) + \Lambda_{NC} = 0$$

compression

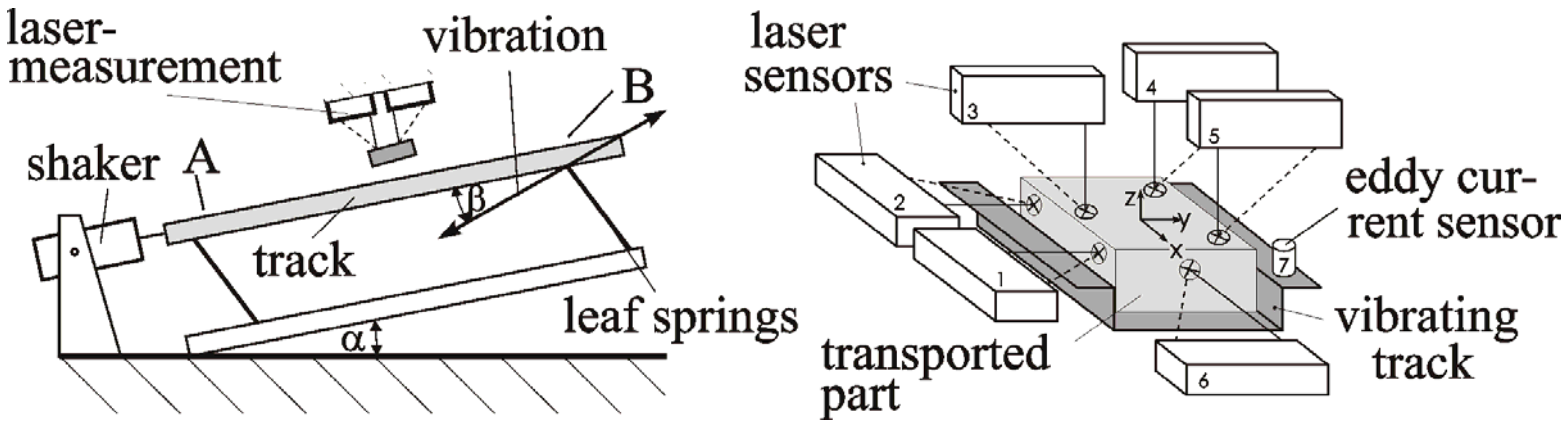
$$m(\dot{x}_E - \dot{x}_C) + \Lambda_{TE} + \mu sign(\dot{g}_{TE})\Lambda_{NE} = 0$$
$$m(\dot{y}_E - \dot{y}_C) + \Lambda_{NE} = 0$$

expansion

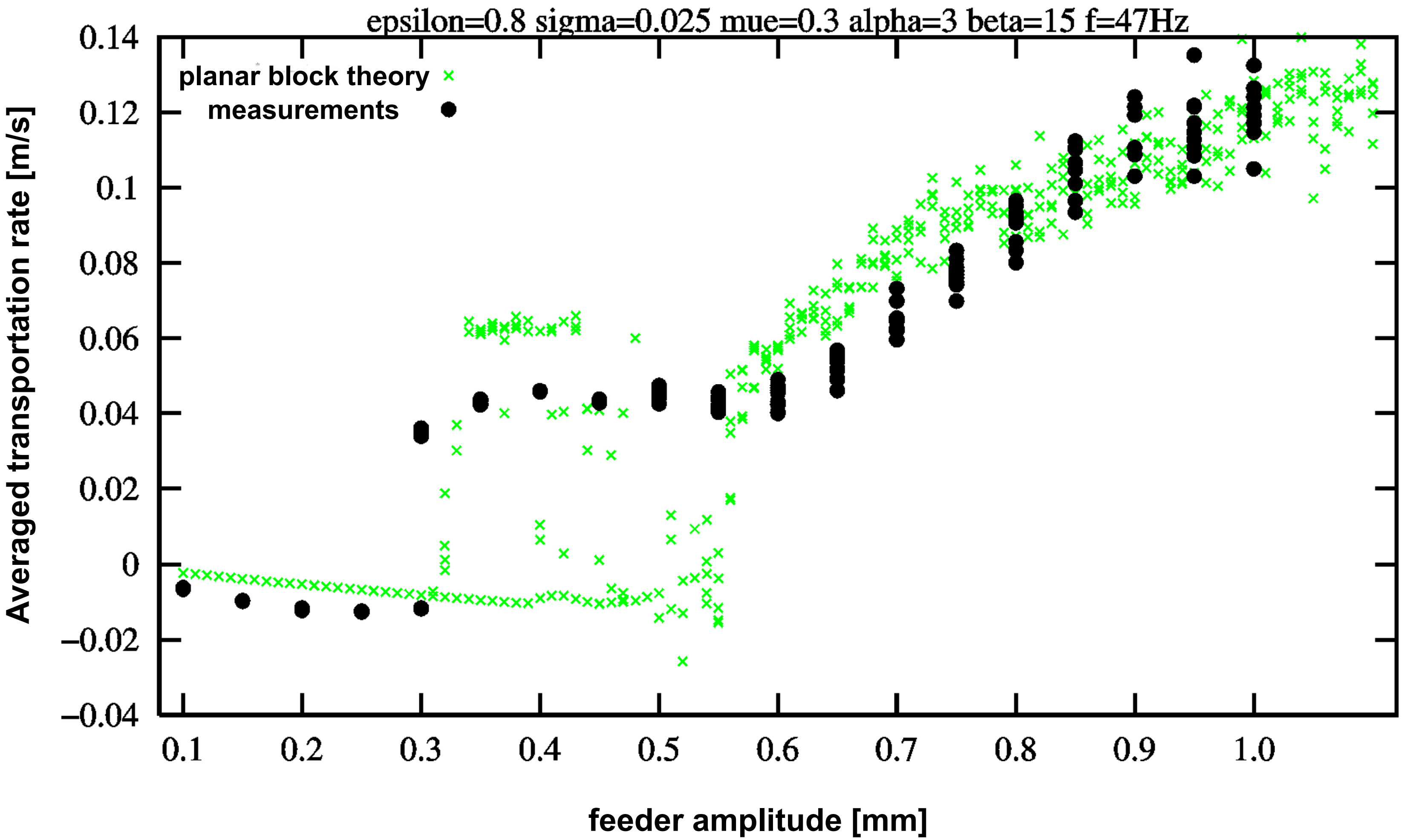




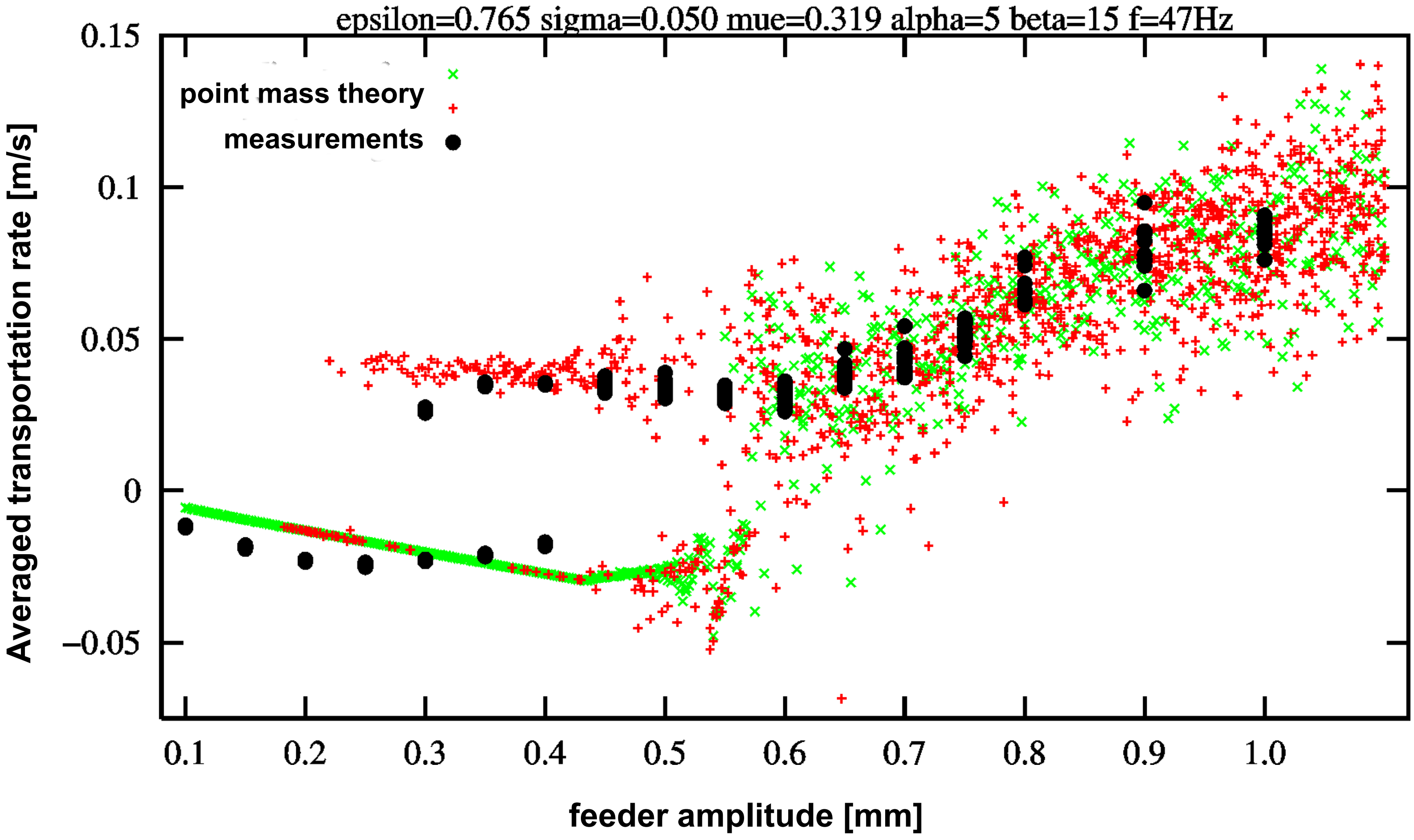
Test – Set – Up

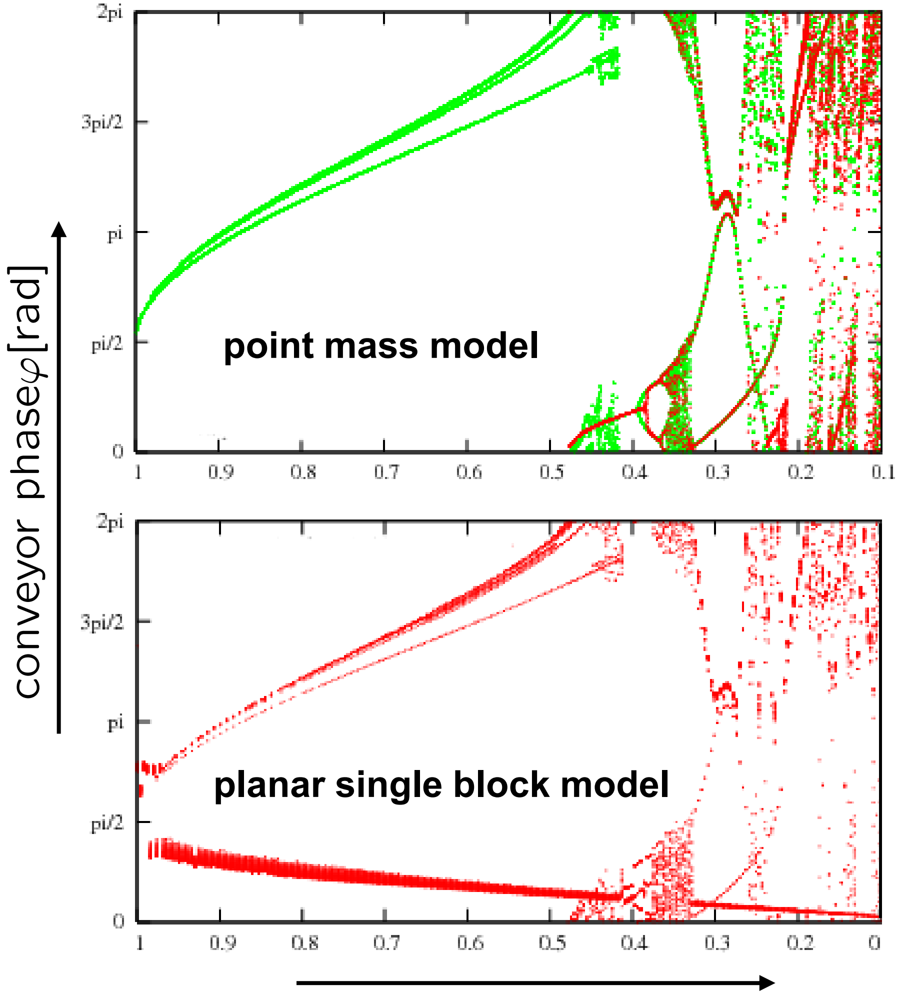

laser-
measurement
vibration
B
shaker
A
β
track
leaf springs
α
laser
sensors
3
4
5
2
z
y
x
1
7
6
eddy cur-
rent sensor
vibrating
track
transported
part

Planar Block Theory and Measurements



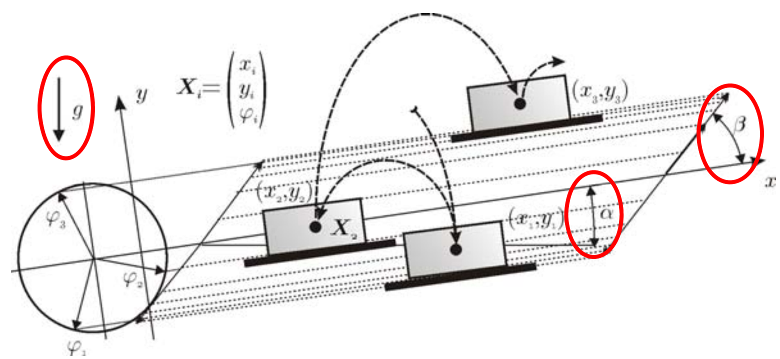
Point Mass Theory and Measurements





conveyor constant $K_g = |\frac{g \cdot cos(\alpha)}{A\Omega^2 \cdot cos(\beta)}|$

Poincaré Maps for a Conveyor Process with $\varepsilon = 0.2$



Summary

- **vibration conveyor with an arbitrary number of blocks**

  -- spatial theory – planar theory – planar single block – point mass

- **no difference of models with the exception of orienting devices (bock clusters decompose into single blocks)**

- **possibility of a break-down of the transportation rate by nonlinear dynamical effects like bifurcation (design restrictions)**

- **Theory and measurements compare well**