

7th International Meeting on UNILATERAL PROBLEMS IN STRUCTURAL ANALYSIS

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A model for the mechanical response with damage of collagenous biostructures

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MOTIVATION





MOTIVATION



Collagen organization reflects its key role in the mechanical strength and functionality of living tissue



Macro-, micro- and nano-scale characteristics of collagenous structures highly affect tissues mechanics

MULTISCALE MECHANICAL MODELING OF COLLAGENOUS TISSUES

At macroscale: mechanics of tissues affects organs functionality

At microscale: Cellular stress environment affects molecular pathways leading to tissue remodeling

At nanoscale: Injuries, diseases and healing are often related with molecular alterations

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 \Rightarrow Mechanics of soft tissues: macroscale \rightarrow nanoscale

⇒ A multiscale elasto-damaging model for collagenous fibrils:

> Nanoscale: Molecular model

Nanoscale: Cross-links model

Microscale: Fibril model

 \Rightarrow Conclusions and Perspective: Back to the macroscale



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Macroscale response is ruled by micro- and nano-scale mechanisms





TOE REGION:

removal of the fibers/fibrils microscopic crimps



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HEEL REGION:

kinks straightening due to entropic mechanisms





LINEAR REGION:

stretching of collagen triple-helices





YIELD AND FAILURE REGION:

Nanoscale mechanisms





YIELD AND FAILURE REGION:

Molecular slippage Molecular crack: due to cross-link damage: Г a m a g e Buehler (2008)

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Atomic Force Microscopy of an isolated molecule:





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Atomic Force Microscopy of an isolated molecule:



MECHANICS OF SOFT TISSUES: MICRO-/NANO- SCALE









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MECHANICS OF SOFT TISSUES: MICRO-/NANO- SCALE





NUMBER OF CROSS-LINKS

Reduced stiffness and ductile mechanisms



Increased strength and brittle-like behaviour



Fibrils elasto-damaging mechanical behaviour affects the overall tissue response *in corpore*.





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LOADING

Homogeneous traction



KINEMATICS

Two main deformation mechanisms:





• Molecular free-energy density and dissipative pseudopotential density:

 $\Psi_m(\varepsilon_m, \beta_m)$ Accounting for: Entropic elasticity Energetic elasticity

$$\Phi_m(\dot{arepsilon}_m,\dot{eta}_m)$$

Accounting for: **Brittle fracture**





<u>Cross-link state variables:</u> $\delta \qquad \beta_{cl} \in [0,1]$

- $\beta_{cl} \in [0,1] \quad \longleftarrow \quad \text{Damage parameter:} \\ \beta_{cl} = 0 \quad \rightarrow \text{cracked} \\ \beta_{cl} = 1 \quad \rightarrow \text{sound}$
- Cross-link free-energy and dissipative pseudopotential:

$$\mathcal{E}_{cl}(\delta,eta_{cl})$$

Assuming: Linear elastic behaviour

$$\mathcal{D}_{cl}(\dot{\delta},\dot{eta}_{cl})$$

Accounting for: **Ductile failure**









Fibril's Free-Energy density:



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MICRO-NANO KINEMATIC COMPATIBILITY



Kinematic assumption:

$$\Delta L_f = n_s (\Delta L_m + \delta) \quad \rightarrow \quad \varepsilon_f = \frac{\Delta L_f}{L_f} = \frac{1}{\mu_s} \left(\varepsilon_m + \frac{\delta}{L_m} \right)$$



CONSTITUTIVE LAWS AT NANOSCALE

Molecular stress:

$$\sigma_m = \frac{\partial \Psi_m}{\partial \varepsilon_m} + \frac{\partial \Phi_m}{\partial \dot{\varepsilon}_m}$$

Nanostress due to molecular damage:

$$b_m = \frac{\partial \Psi_m}{\partial \beta_m} + \frac{\partial \Phi_m}{\partial \dot{\beta}_m}$$

Cross-link reactive force:

$$R = \frac{\partial \mathcal{E}_{cl}}{\partial \delta} + \frac{\partial \mathcal{D}_{cl}}{\partial \dot{\delta}}$$

Nanoforce due to cross-link damage:

$$B_{cl} = \frac{\partial \mathcal{E}_{cl}}{\partial \beta_{cl}} + \frac{\partial \mathcal{D}_{cl}}{\partial \dot{\beta}_{cl}}$$

CONSTITUTIVE LAWS AT MICROSCALE

Fibril stress:

$$\sigma_f = \frac{\partial \Psi_f}{\partial \varepsilon_f} + \frac{\partial \Phi_f}{\partial \dot{\varepsilon}_f}$$

Cross-links total reactive force at fibrillar level:

$$R_f = \left(\frac{\partial \Psi_f}{\partial \delta} + \frac{\partial \Phi_f}{\partial \dot{\delta}}\right) \Omega_f$$

EQUILIBRIUM AT NANOSCALE

By the Principle of Virtual Powers:
$$\mathcal{P}_{int} = \mathcal{P}_{ext}$$

 $\dot{\varepsilon}_m = \frac{dv}{dz}$
 $\mathcal{P}_{int} = N_m \left(\int_{\Omega_m} \sigma_m \dot{\varepsilon}_m \, d\Omega + \int_{\Omega_m} b_m \dot{\beta}_m \, d\Omega \right) + \lambda N_m \left(R \dot{\delta} + B_{cl} \dot{\beta}_{cl} \right)$
 $\mathcal{P}_{ext} = N_m \mathcal{F} \left[v(L_m) - v(0) \right] + \lambda N_m \mathcal{F}_{cl} \dot{\delta}$

where σ_m , b_m , R, and B_{cl} are static quantities at nanoscale, dual to kinematic variables

Equilibrium equations:
(Nanoscale)
$$\frac{d\sigma_m}{dz} = 0 \quad \text{for } z \in [0, L_m] \\
\mathcal{F} = \sigma_m A_m \\
\mathcal{F}_{cl} = R \\
b_m = 0 \\
B_{cl} = 0$$
where:
$$\frac{\mathcal{F}}{\mathcal{F}_{cl}} \xrightarrow{\mathcal{F}}{\mathcal{F}_{cl}} \xrightarrow{\mathcal{F}}{\mathcal{F}_{cl}}$$

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EQUILIBRIUM AT MICROSCALE

Nanoscale state variables:

 $(\varepsilon_m, \delta, \beta_m, \beta_{cl})$



Microscale state variables:

$$(\varepsilon_f, \, \delta, \, \beta_m, \, \beta_{cl})$$

 $\dot{\varepsilon}_f = \frac{dV}{dz}$

By the Principle of Virtual Powers at the microscale:

$$\mathcal{P}_{int} = \int_{\Omega_f} \sigma_f \dot{\varepsilon}_f \, d\Omega + N_m \int_{\Omega_m} b_m \dot{\beta}_m \, d\Omega + R_f \dot{\delta} + \lambda N_m B_{cl} \dot{\beta}_{cl} \qquad \mathcal{P}_{ext} = F \left[V(L_f) - V(0) \right]$$

where σ_f and R_f are static quantities at microscale, dual to ε_f and δ .

Equilibrium equations:
(Microscale)
$$\begin{aligned}
\frac{d\sigma_f}{dz} &= 0 \quad \text{for } z \in [0, L_f] \\
F &= \sigma_f A_f \quad \text{where:} \quad \overleftarrow{F} \quad \overrightarrow{F} \quad \overrightarrow{F} \\
R_f &= 0 \\
B_{cl} &= 0
\end{aligned}$$

A BRIDGE FROM NANOSCALE TO MICROSCALE

By homogenization and compatibility:

$$\Psi_{f}(\varepsilon_{m},\delta,\beta_{m},\beta_{cl}) = \frac{\Psi_{m}(\varepsilon_{m},\beta_{m})}{\mu} + \frac{\lambda}{\mu\Omega_{m}} \mathcal{E}_{cl}(\delta,\beta_{cl}) \qquad \Longrightarrow \qquad \Psi_{f}(\varepsilon_{f},\delta,\beta_{m},\beta_{cl})$$

$$\Phi_{f}(\dot{\varepsilon}_{m},\dot{\delta},\dot{\beta}_{m},\dot{\beta}_{cl}) = \frac{\Phi_{m}(\dot{\varepsilon}_{m},\dot{\beta}_{m})}{\mu} + \frac{\lambda}{\mu\Omega_{m}} \mathcal{D}_{cl}(\dot{\delta},\dot{\beta}_{cl}) \qquad \Longrightarrow \qquad \Phi_{f}(\dot{\varepsilon}_{f},\dot{\delta},\dot{\beta}_{m},\dot{\beta}_{cl})$$

$$\sigma_m = \sigma_f \,\mu_p \qquad \qquad R_f = \left(\frac{\partial \Psi_f}{\partial \delta} + \frac{\partial \Phi_f}{\partial \dot{\delta}}\right)\Omega_f = \lambda N_m \left(R - \frac{\mathcal{F}}{\lambda}\right)$$

By equilibrium:

$$\mathcal{F} = \sigma_m A_m$$

$$\mathcal{F}_{cl} = R$$

$$F = \sigma_f A_f$$

$$\mathcal{F}_{cl} = \frac{\mathcal{F}}{\lambda} \rightarrow \sigma_m = \frac{\lambda}{A_m} R \rightarrow \frac{\partial \Psi_m}{\partial \varepsilon_m} + \frac{\partial \Phi_m}{\partial \dot{\varepsilon}_m} = \frac{\lambda}{A_m} \frac{\partial \mathcal{E}_{cl}}{\partial \delta} + \frac{\partial \mathcal{D}_{cl}}{\partial \dot{\delta}}$$

$$R_f = 0$$





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A multiscale elasto-damaging model for collagenous fibrils:

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Free-energy density:

 $\mathbf{I}(x) = \begin{cases} 0 & \text{if } x \in [0, 1] \\ +\infty & \text{elsewhere} \end{cases}$

$$\Psi_m(\varepsilon_m,\beta_m) = \frac{\beta_m}{\uparrow} \Psi_m^{el}(\varepsilon_m) + (1-\beta_m)w_m + \mathbf{I}(\beta_m)$$

Threshold of damage activation

Dissipative pseudopotential density:

$$\Phi_{m}(\dot{\varepsilon}_{m},\dot{\beta}_{m}) = c_{m}\frac{\dot{\beta}_{m}^{2}}{2} + I_{-}(\dot{\beta}_{m}) \xleftarrow{} Irreversibility condition for damage evolution$$

It will give linear dependence for the evolution of damage

$$I_{-}(x) = \begin{cases} 0 & \text{if } x \in \mathbb{R}^{-} \\ +\infty & \text{elsewhere} \end{cases}$$



Free-energy density:

 $\mathbf{I}(x) = \begin{cases} 0 & \text{if } x \in [0, 1] \\ +\infty & \text{elsewhere} \end{cases}$

$$\Psi_m(\varepsilon_m,\beta_m) = \beta_m \Psi_m^{el}(\varepsilon_m) + (1-\beta_m)w_m + \mathbf{I}(\beta_m)$$

$$\Psi_m^{el}(\varepsilon_m) = \mathbf{e}(\varepsilon_m) - T\mathbf{s}(\varepsilon_m)$$



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$$\Psi_m^{el}(\varepsilon_m) = \mathbf{e}(\varepsilon_m) - T\mathbf{s}(\varepsilon_m) : \sigma_m(\varepsilon_m^*) = \left. \frac{\partial s}{\partial \varepsilon_m} \right|_{\varepsilon_m^*} = \left. \frac{\partial e}{\partial \varepsilon_m} \right|_{\varepsilon_m^*}$$

Elastic modulus in entropic elasticity:

$$s(\varepsilon_m) : E^s(\varepsilon_m) = \frac{\partial^2 s}{\partial \varepsilon_m^2} = r_a \left\{ \frac{r_\ell}{2 \left[1 - r_\ell(\varepsilon_m + 1) \right]^3} + r_\ell \right\} \implies \begin{array}{l} \text{Recovery of the classical} \\ \text{formulation for entropic elasticity} \\ \text{(Worm-Like Chain model)} \\ r_a = \frac{k_B T}{A_m L_p} \qquad r_\ell = \frac{L_m}{L_c} \end{array}$$

Elastic modulus in energetic elasticity

$$e(\varepsilon_m) : E^e(\varepsilon_m) = \frac{\partial^2 e}{\partial \varepsilon_m^2} = \frac{\widehat{E}}{1 + e^{-m(\varepsilon_m - \varepsilon_t)}}$$

Compliant to results in Bueheler (2009) for non-linear energetic elasticity

Requirements:
$$\begin{cases} E^{s}(\varepsilon_{m}^{*}) = E^{e}(\varepsilon_{m}^{*}) \\ \frac{\partial E^{s}}{\partial \varepsilon_{m}}\Big|_{\varepsilon_{m}^{*}} = \frac{\partial E^{e}}{\partial \varepsilon_{m}}\Big|_{\varepsilon_{m}^{*}} & \underbrace{\text{Uniqueness of }}(\varepsilon_{m}^{*}, \varepsilon_{t}) \end{cases}$$

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MODEL VALIDATION: ENERGETIC ELASTICITY

$$E^{e}(\varepsilon_{m}) = \frac{\partial^{2} e}{\partial \varepsilon_{m}^{2}} = \frac{\widehat{E}}{1 + e^{-m(\varepsilon_{m} - \varepsilon_{t})}}$$

Benchmarks on an isolated straight molecule (i.e, $\varepsilon_m =$ molecular <u>material</u> strain)



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MODEL VALIDATION: ENTROPIC TO ENERGETIC TRANSITION

Equivalent elastic modulus of a collagen molecule:







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MODEL VALIDATION: MOLECULAR BREAKAGE



Collagen strength and brittle-like mechanism are accurately reproduced $(w_m \approx 1.8 \text{ J}, c_m = 10^{-3})$







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A MODEL FOR COLLAGENOUS FIBRILS: CROSS-LINKS MODEL



Free-energy:

the

 $\mathbf{I}(x) = \begin{cases} 0 & \text{if } x \in [0, 1] \\ +\infty & \text{elsewhere} \end{cases}$

$$\mathcal{E}_{cl} = \beta_{cl} \frac{\rho k}{2} \delta^2 + (1 - \beta_{cl}) w_{cl} + \mathbf{I}(\beta_{cl})$$

Threshold of damage activation

 ρ : parameter expressing covalent bonds density within a cross-link k: stiffness of a covalent bond

Dissipative pseudopotential:

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$$\mathbf{I}_{-}(x) = \begin{cases} 0 & \text{if } x \in \mathbb{R}^{-} \\ +\infty & \text{elsewhere} \end{cases}$$





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A MODEL FOR COLLAGENOUS FIBRILS: THE FULL MODEL



EQUILIBRIUM EQUATIONS:



$$G(\dot{\delta}) = \begin{cases} 1 & \text{if } \dot{\delta} \neq 0\\ p : |p| \le 1 & \text{if } \dot{\delta} = 0 \end{cases}$$

Damage evolution laws:

$$b_{m} = 0 \qquad \Longrightarrow \qquad c_{m}\dot{\beta}_{m} + \partial I(\beta_{m}) + \partial I_{-}(\dot{\beta}_{m}) \ni \left(w_{m} - \Psi_{m}^{el}(\varepsilon_{m}, T)\right)$$
$$B_{cl} = 0 \qquad \Longrightarrow \qquad c_{cl}\dot{\beta}_{cl} + \partial I(\beta_{cl}) + \partial I_{-}(\dot{\beta}_{cl}) \ni \left(w_{cl} - \frac{\rho k}{2}\delta^{2}\right)$$

A MODEL FOR COLLAGENOUS FIBRILS: THE FULL MODEL



MODEL VALIDATION:



 $\lambda \rho = 1 \leftrightarrow \beta = 15 (\rightarrow 1 \text{ covalent cross-link/molecule})$



A MODEL FOR COLLAGENOUS FIBRILS: THE FULL MODEL



MODEL RESULTS: $\lambda \rho = 1$



Nano-scale deformation mechanisms are effectively taken into account at the micro-scale 40 Molecular/Cross-links strain Fibril δ/L 35 0,3 'n Elastic modulus (GPa) Molecule 30 25 m 0,2 20 15 0,1 10 0,2 0.3 0,5 0,0 0,1 0,4 0.1 0,0 0.2 0.3 Fibril strain Fibril strain

Model takes into account the hierarchical and organized structure of collagen fibrils



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CONCLUSIONS AND **PERSPECTIVE**

BACK TO THE MACROSCALE



CONCLUSIONS AND PERSPECTIVE

BACK TO THE MACROSCALE

Lamellar unit degradation (-50%) is a key factor in the development of aneurismal dilatation (Zatina,1984 – Wilson,1999)



F. Maceri, M. Marino, G. Vairo (2010) *A unified multiscale mechanical model for soft collagenous tissues with regular fiber arrangement*, J. Biomechanics **43**:355-363



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Thank You





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