Seventh Meeting- UNILATERAL PROBLEMS IN STRUCTURAL ANALYSIS Palmanova, June 16-19 2010

Dynamic delamination phenomena in composite structures

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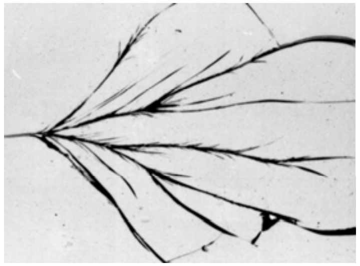
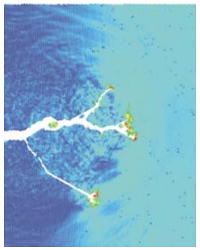
DYNAMIC FRACTURE MECHANICS

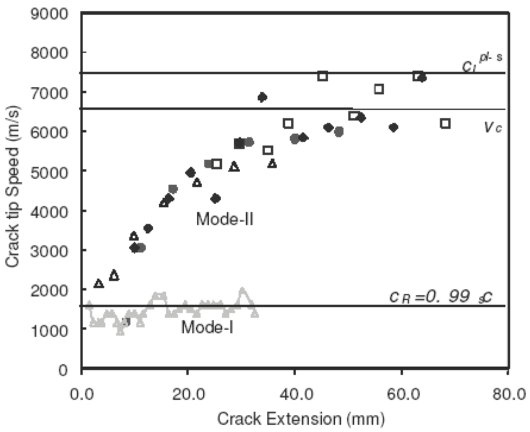
MONOLITIC MATERIALS

- Crack Branching phenomena
- Crack speeds are limited
- Unknown path of the crack



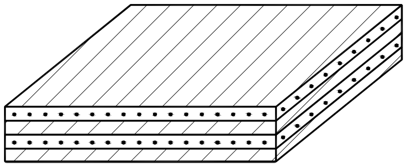


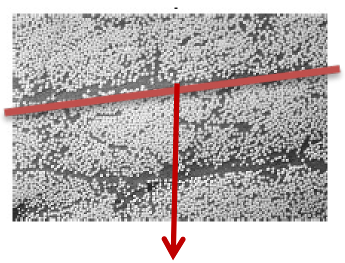
Ravi-Chandar and Knauss, *Int J Fract*, 1984



(Rosakis, A.J., "Intersonic shear cracks and fault ruptures propagation", Advances in Physics, 2002)

COMPOSITE STRUCTURES





Weak plane

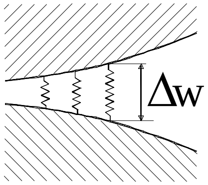
- High crack speed
- Crack constrained along the interfaces

DYNAMIC CRACK GROWTH MODELING

INDEX:

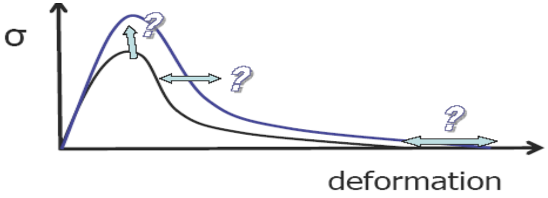
Cohesive modeling

- **Interface elements are introduced at the crack region**
- **Damaged constitutive relationship is required**



Fracture Mechanics approaches

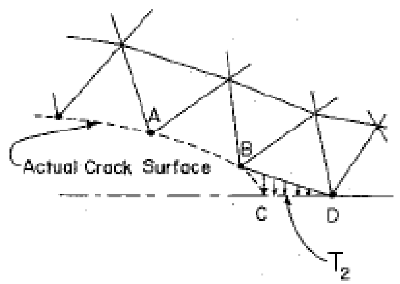
- **Static analyses:**
  (the time dependence is neglected "a priori")
- **Steady state crack growth approaches:**
  (A moving reference system is fixed at the tip, crack tip speed is constant)
- **Unsteady models :**
  Full Time dependence , inertial forces, loading rate,….

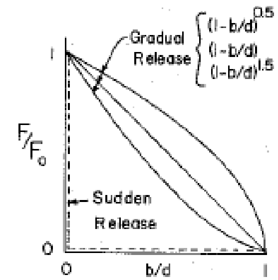


DYNAMIC CRACK GROWTH MODELING

Node release technique

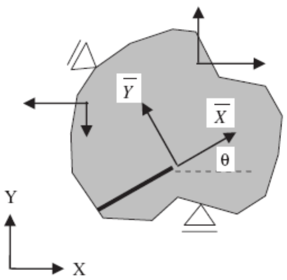
- Gradual release of the nodal forces behind the crack tip

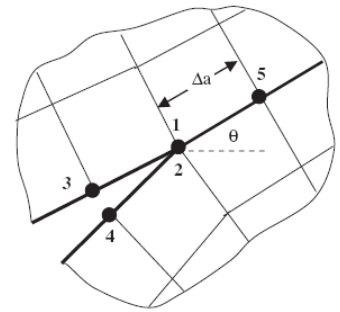




Virtual crack closure methods

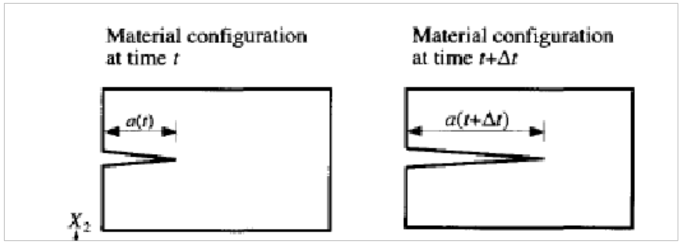
- The ERR is evaluated by the mutual work at the crack tip and behind the crack tip





Moving mesh methodology

- The nodes are moved to predict changes of the geometry produced by the crack motion



MOTIVATION OF THE WORK AND SUMMARY

AIM OF THE WORK

- **Propose a generalized modeling based on Fracture mechanics and moving mesh methodology to predict the dynamic behavior of composite laminated structures**

SUMMARY

- **Review the main equations of the ALE formulation in view of the Dynamic Fracture Mechanics approach**

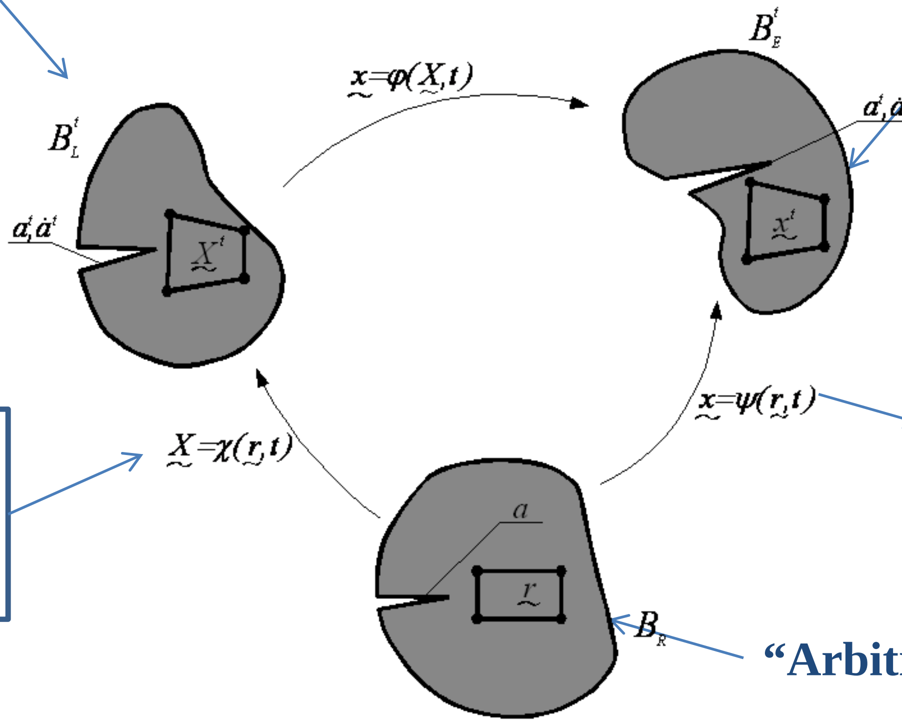
- **Evaluate the specialized expressions of the ERR by the use of the decomposition methodology of the J-integral and propose a proper mixed mode crack toughness criterion**

- **Develop the finite element implementation. Propose validation by means of comparisons with experimental data and a parametric study to analyze dynamic crack behavior (i.e. crack arrest phenomena, allowable tip speeds, cracks interaction)**

BASICS OF MOVING MESH STRATEGY: ARBITRARY-LAGRANGIAN EULERIAN FORMULATION

“Lagrangian Approach”

“Eulerian Approach”



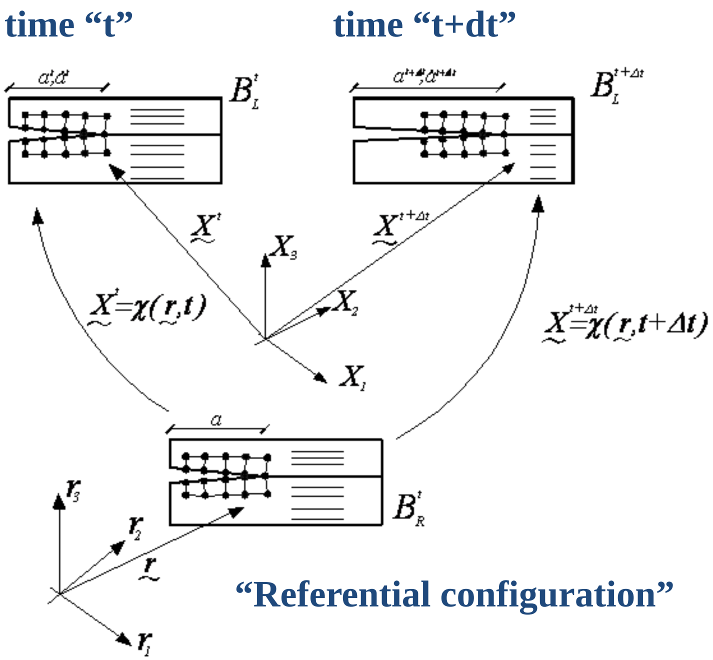
$$\psi : B_L \rightarrow B_E$$
$$x = \psi(r, t)$$

$$\chi : B_R \rightarrow B_L$$
$$X = \chi(r, t)$$

“Arbitrary Lagrangian Eulerian”

The Reference configuration is fixed and independent of any placement of the material body

BASICS OF MOVING MESH STRATEGY: ARBITRARY-LAGRANGIAN EULERIAN FORMULATION



"Referential configuration"

Physical quantities:

$$\tilde{v} = \frac{d}{dt}\varphi(\tilde{X}, t)\Big|_{\tilde{X}}, \qquad \tilde{X}' = \frac{d}{dt}\chi(\tilde{r}, t)\Big|_{\tilde{r}}$$

"Material" "Referential"

$$\dot{f} = f' - \tilde{X}'\frac{d}{d\tilde{X}} f(\tilde{X}, t)$$

Time derivative rule

Physical fields in ALE formulation

$$\ddot{\tilde{u}} = \tilde{u}'' - 2\nabla_x \tilde{u}' \cdot \tilde{X}' - \nabla_x \tilde{u}\tilde{X}'' + \nabla_x(\nabla_x \tilde{u})\tilde{X}'\tilde{X}' + \nabla_x \tilde{u}\nabla_x \tilde{X}'\tilde{X}'$$

"Material accel."

$$\nabla_{\tilde{x}} \tilde{u} = \nabla_{\tilde{r}} \tilde{u}\, \tilde{J}^{-1}$$

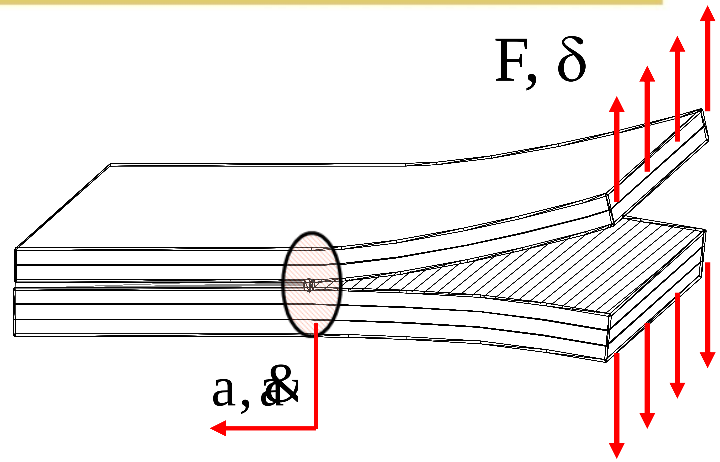
"Grad. transform."

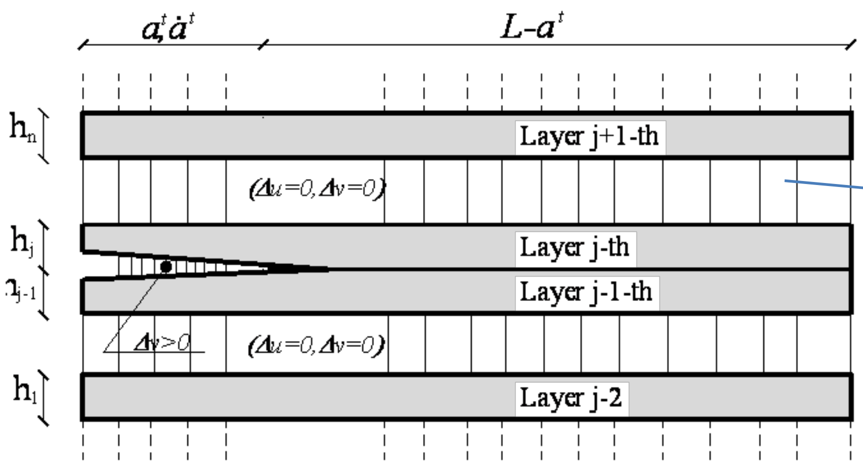
$$\det \tilde{J} \neq 0$$

"one-to-one relationship"

DESCRIPTION OF THE DELAMINATION MODEL

- **Multi-layer Modeling**
- **2D Kinematic formulation**
- **The laminate is divided into *n* mathematical layer representing the staking sequence**





- **Compatibility equations LMM:**

$$\Delta u_i = u_{i+1} - u_i = 0, \quad \Delta v_i = v_{i+1} - v_i = 0,$$

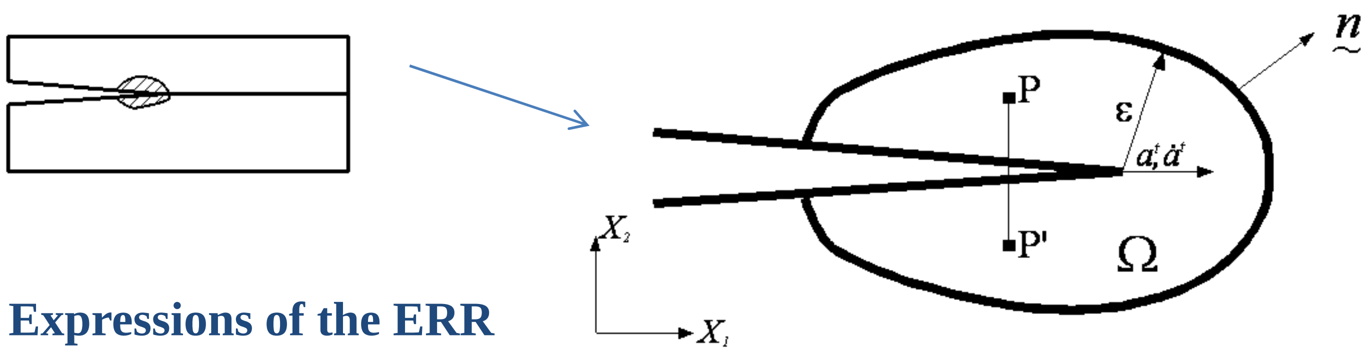
“undelaminated interfaces”

$$\Delta v_i = v_{i+1} - v_i \geq 0,$$

“delaminated interfaces”

ERR RATE EVALUATION : J-INTEGRAL APPROACH

Revision of the J-integral Dec. procedure (Rigby & Aliabady, 1998, Greco & Lonetti, 2009))



Expressions of the ERR

$$J = \lim_{\varepsilon \to 0} \oint_{\Omega} \left[(W+K) n_1 - \tilde{t} \frac{\partial \tilde{u}}{\partial \tilde{X}_1} \right] ds$$

$$J = \oint_{\partial\Omega} \left[(W+K) n_1 - \tilde{t} \frac{\partial \tilde{u}}{\partial \tilde{X}_1} \right] ds + \int_{\Omega} \left[\rho \left(\ddot{\tilde{u}} - \tilde{f} \right) \nabla \tilde{u} - \rho \dot{\tilde{u}} \nabla \dot{\tilde{u}} \right] dA$$

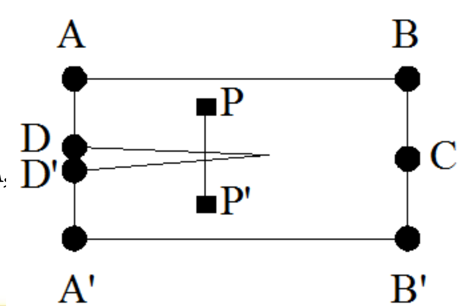
"Path independent"

(Nishioka,T, 2001)\

Decomposition of the ERR into symmetric and antisymmetric fields

$$J_I = G_I = \oint_{\partial\Omega} \left[\left(W^S + K^S \right) n_1 - \sigma_{ij}^S n_j \frac{\partial u^S}{\partial x} \right] ds + \int_{\Omega} \left[\rho \left(\ddot{\tilde{u}}^S - \tilde{f}^S \right) \nabla \tilde{u}^S - \rho \dot{\tilde{u}}^S \nabla \dot{\tilde{u}}^S \right] dA,$$

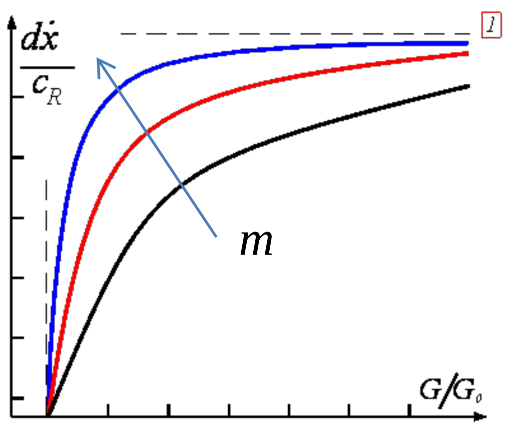
$$J_{II} = G_{II} = \oint_{\partial\Omega} \left[\left(W^{AS} + K^{AS} \right) n_1 - \sigma_{ij}^{AS} n_j \frac{\partial u^{AS}}{\partial x} \right] ds + \int_{\Omega} \left[\rho \left(\ddot{\tilde{u}}^{AS} - \tilde{f}^{AS} \right) \nabla \tilde{u}^{AS} - \rho \dot{\tilde{u}}^{AS} \nabla \dot{\tilde{u}}^{AS} \right] dA,$$



(Lonetti et al., 2010)

DYNAMIC CRACK PROPAGATION ANALYSIS: GROWTH CRITERION

Crack growth criterion



$$G_D = \frac{G_0}{1 - \left(\frac{c_t}{V_R}\right)^m}$$

Material parameter (m)

"Critical value of the ERR"

(Freund, 1990; Ravi-Chandar, 2004)

$$c \to V_R \qquad G_D\left(c_t\right) = \infty$$

"Rayleigh wave speed"

$$c \to 0 \qquad G_D\left(c_t\right) = G_0\left(0\right)$$

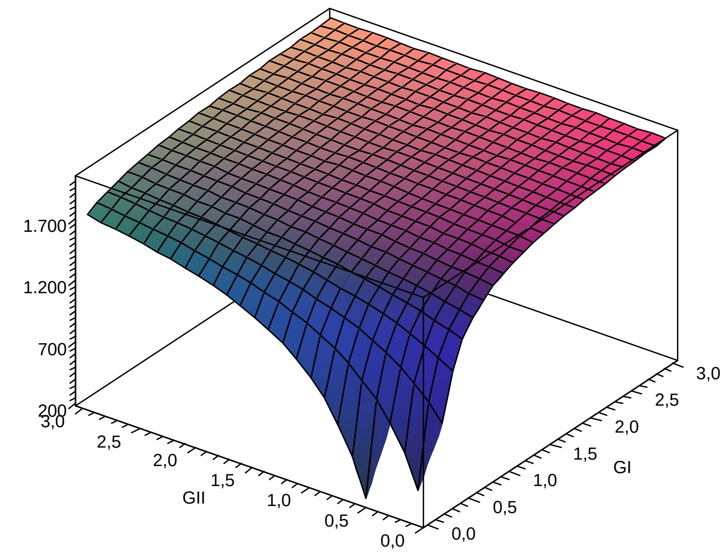
"initiation value"

1) Mixed mode crack growth criterion

$$g_f = \frac{G_I}{G_{ID}\left(c_t\right)} + \frac{G_{II}}{G_{IID}\left(c_t\right)} - 1 \le 0$$

Material parameter (m)

$$G_{ID}\left(c_t\right) = \frac{G_{0I}}{1 - \left(\frac{c_t}{V_R}\right)^m}, \qquad G_{IID}\left(c_t\right) = \frac{G_{0II}}{1 - \left(\frac{c_t}{V_R}\right)^m}$$



DESCRIPTION OF THE DELAMINATION MODEL IN THE REFERENTIAL CONFIGURATION

Governing Equations: “Principle of d’Alembert”

$$\sum_{i=1}^{n}\int_{V_i}\sigma\,\delta\nabla u\,dV+\sum_{i=1}^{n}\int_{V_i}\rho\,\ddot{u}\,\delta u\,dV=\sum_{i=1}^{n}\int_{\Omega_i}t\,\delta u\,dA+\sum_{i=1}^{n}\int_{V_i}f\,\delta u\,dV$$

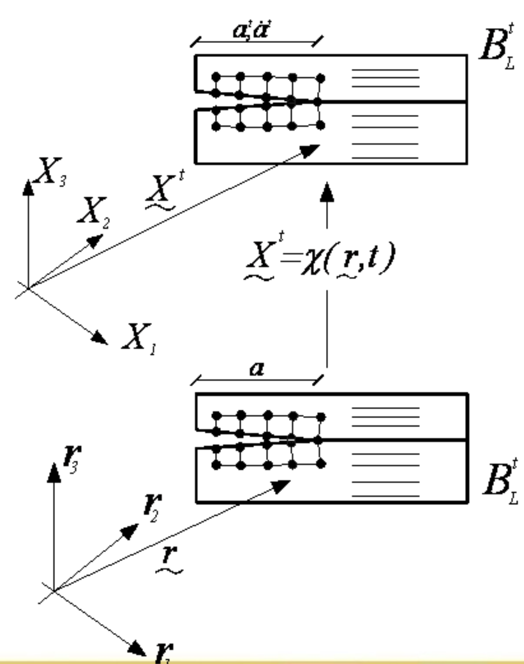
Internal work | **External work**

$$\sum_{i=1}^{n}\int_{V_i}\sigma\,\delta\nabla u\,dV=\sum_{i=1}^{n}\int_{V_{ri}}C\left(\nabla_r u J^{-1}\right)\delta\left(\nabla_r u J^{-1}\right)\det\left(J\right)dV_r$$

J : Jacobian

$$\sum_{i=1}^{n}\int_{V_i}\rho\,\ddot{u}\,\delta u\,dV=\sum_{i=1}^{n}\int_{V_{ri}}\rho[u''-2\nabla_r u'\,J^{-1}\cdot X'-\left(\nabla_r u\,J^{-1}\right)\cdot X''+$$
$$+\nabla_r\left(\nabla_r u\,J^{-1}\right)J^{-1}X'\,X'+\nabla_r u\,J^{-1}\cdot\left(\nabla_r X'J^{-1}\right)X']\,\delta u\det\left(J\right)dV_r$$

$$\sum_{i=1}^{n}\int_{\Omega_i}t\,\delta u\,dA+\sum_{i=1}^{n}\int_{V_i}f\,\delta u\,dV=\sum_{i=1}^{n}\int_{\Omega_{ri}}t\,\delta u\det(\bar{J})\,d\Omega_r+\sum_{i=1}^{n}\int_{V_{ri}}f\,\delta u\det(J)\,dV_r$$



MOVING MESH METHOD: FOUNDAMENTAL EQUATIONS

ALE formulation to describe mesh motion

$$\nabla^2_{\tilde{X}} \Delta X_1 = 0, \qquad \nabla^2_{\tilde{X}} \Delta X_2 = 0.$$

$$\Delta X_1 = X_1 - r_1 \qquad \Delta X_2 = X_2 - r_2$$

"Mesh displacements of nodes Should be regular"

Mesh regularization technique
"Winslow Smoothing method"

Minimize the mesh warping

Boundary conditions

$$\left(\Delta X_1^j = 0,\ \Delta X_2^j = 0\right) \quad \text{on } \Omega_1 \cup \Omega_2,$$

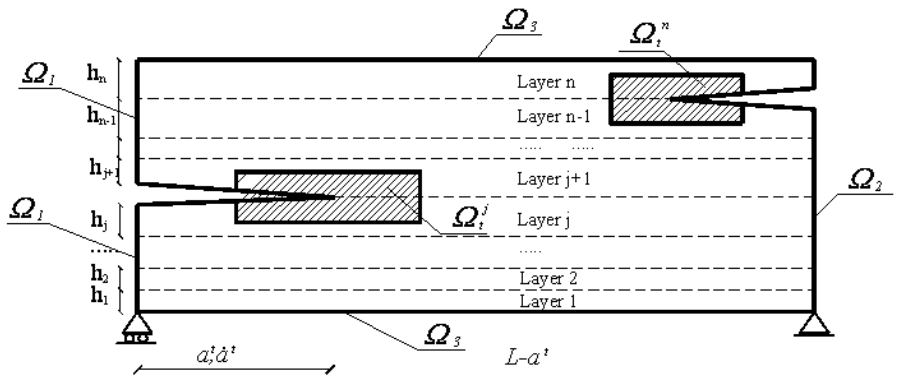
$$\Delta X_2^j = 0 \qquad \text{on } \Omega_3 \cup \Omega_4$$

$$\Delta X_1'^j = 0 \quad \Leftrightarrow if \;\; g_f^j < 0 \quad \text{on } \Omega_t^j,$$

$$\Delta X_1'^j = c_t \quad \Leftrightarrow if \;\; g_f^j \geq 0 \text{ on } \Omega_t^j,$$

$$\Delta X_2'^j = 0 \;\; \text{on } \Omega_t^j$$

$$\Delta X_1^j(0) = 0,\ \Delta X_2^j(0) = 0,\ \Delta \dot{X}_1^j(0) = 0,\ \Delta \dot{X}_2^j(0) = 0$$



Weak forms: coupled equations for the ALE and PS formulations:

PS

$$\sum_{i=1}^{n}\int_{V_{ri}} C\left(\nabla_r u J^{-1}\right)\delta\left(\nabla_r u J^{-1}\right)\det\left(J\right)dV_r+\sum_{i=1}^{n}\int_{V_{ri}}\rho[u''-2\nabla_r u' J^{-1}\cdot X'-\left(\nabla_r u J^{-1}\right)\cdot X''+$$

$$+\nabla_r\left(\nabla_r u J^{-1}\right)J^{-1}X' X'+\nabla_r u J^{-1}\cdot\left(\nabla_r X' J^{-1}\right)X']\delta u\det\left(J\right)dV_r$$

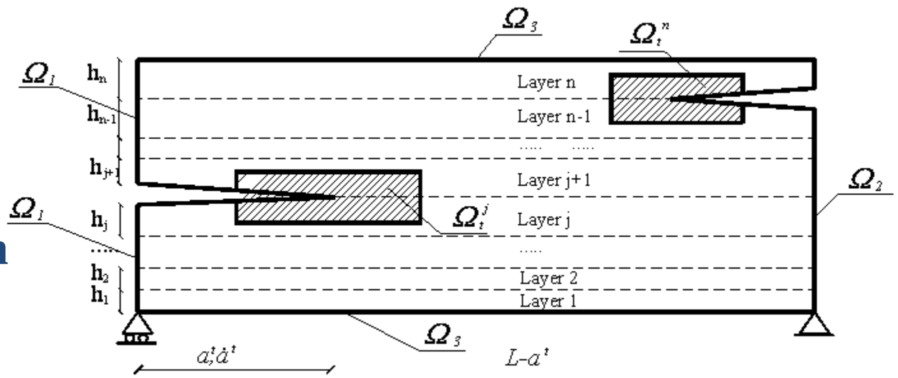
$$=\sum_{i=1}^{n}\int_{\Omega_{ri}} t\delta u\det(\bar{J})d\Omega_r+\sum_{i=1}^{n}\int_{V_{ri}} f\delta u\det(J)dV_r$$

ALE

$$\int_{V_r}\left(\nabla_r \Delta X J^{-1}\right)\cdot\left(\nabla_r w J^{-1}\right)\det\left(J\right)dV_r+\oint_{\Omega_r}\left[\delta\lambda\left(X'-c_t\right)i+\lambda\delta \dot{X}t\right]\left(\bar{\bar{J}}\right)ds=0,$$

Explicit equations for PS+ALE

Implicit equations for the crack growth



→ **Crack growth criterion**

VARIATIONAL FORMULATION AND FE IMPLEMENTATION

INDEX:
INTRODUCTION
MOTIVATIONS
ALE MODEL
FORMULATION
FE MODEL
RESULTS
CONCLUSIONS

FE approximation by "Comsol Multiphysics":

- Quadratic Lagrangian interpolation functions for displacements, velocity and acceleration fields
- Quadratic Lagrangian interpolation functions for mesh points displacements

Non Linear Equations System

FE equations

$$\sum_{i=1}^{n} \tilde{M}_i \tilde{U}_i'' + \sum_{i=1}^{n} \tilde{C}_i \tilde{U}_i' + \sum_{i=1}^{n} \left(\tilde{K}_i + \tilde{K}_{0i} + \tilde{K}_{1i} + \tilde{K}_{2i} \right) \tilde{U}_i + \sum_{i=1}^{n} \tilde{T}_i + \sum_{i=1}^{n} \tilde{P}_i = 0$$

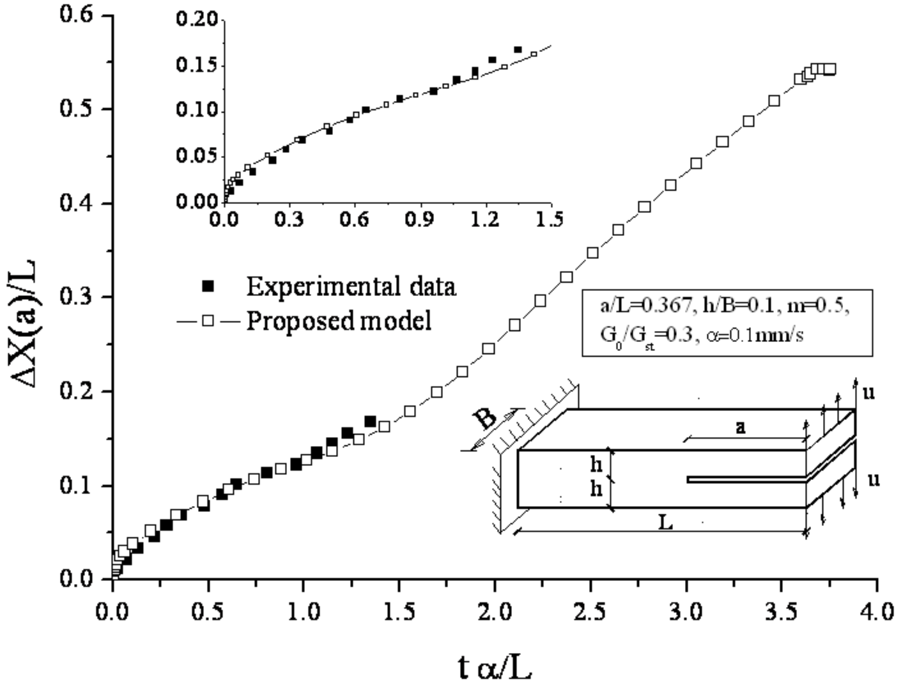
$$\tilde{W} \cdot \Delta \tilde{X} + \tilde{Q} \cdot \Delta \tilde{X}' + \tilde{L} = 0,$$

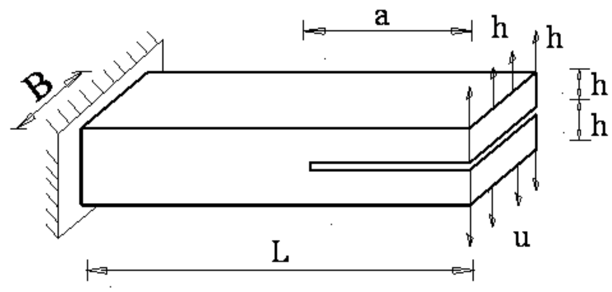
Solution Procedure

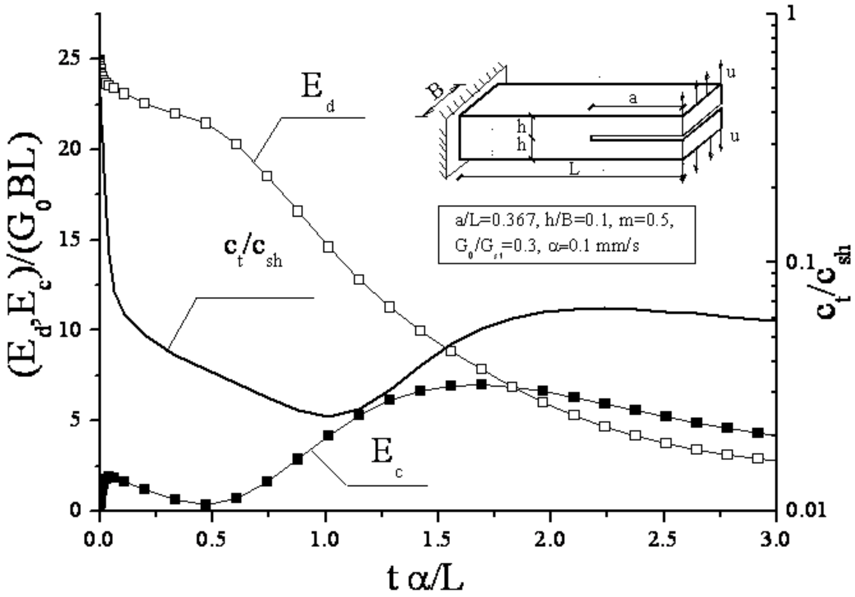
- Implicit time integration scheme based on variable-step-size backward differentiation formula

Iterative-incremental Solving procedure

RESULTS: VALIDATION OF THE STRUCTURAL MODEL



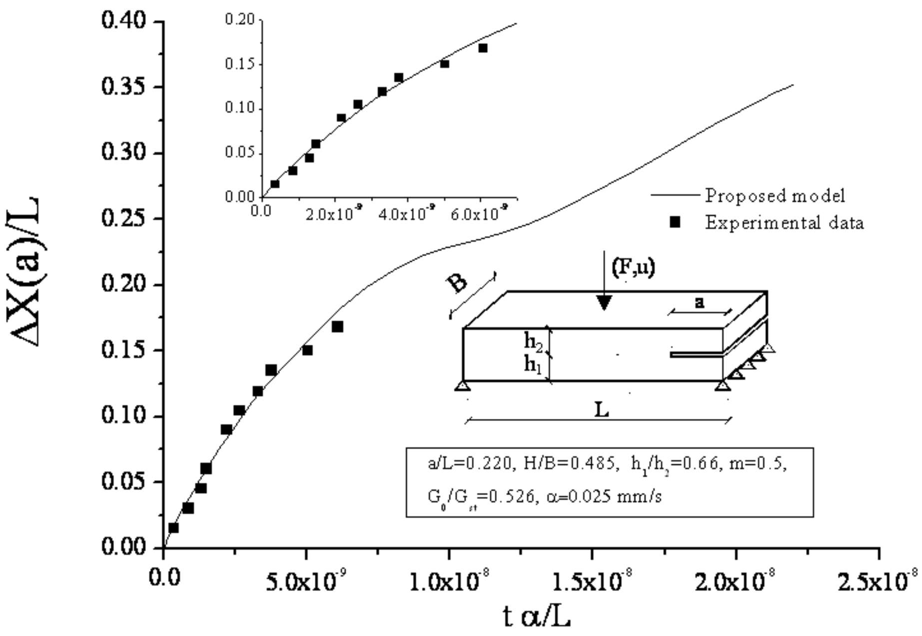


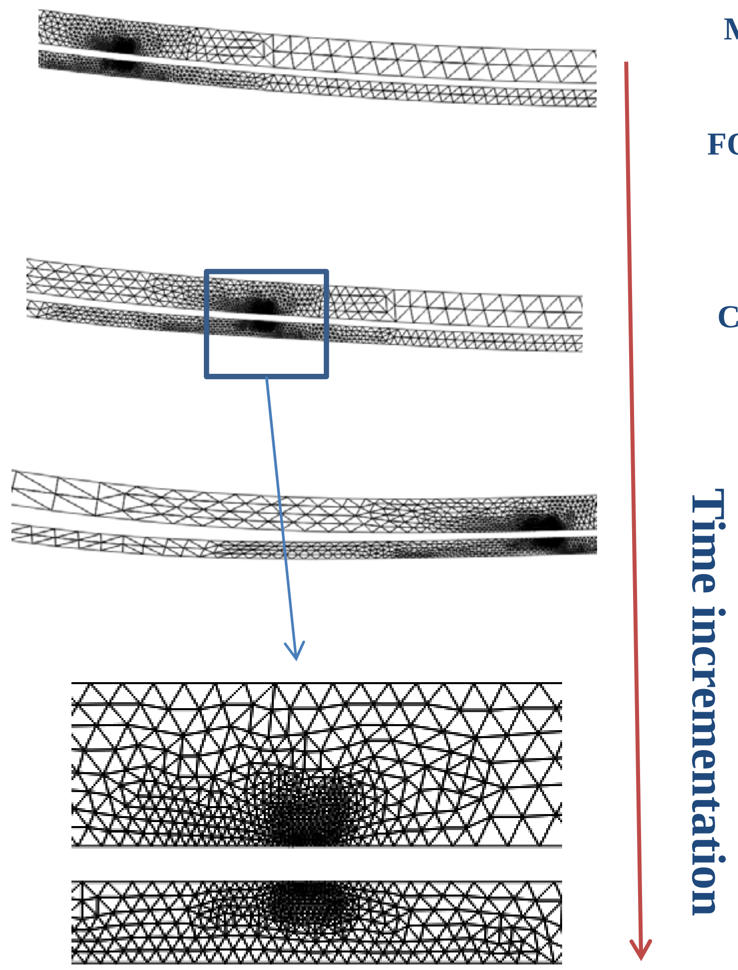


- **DCB mode I loading scheme**
- **Comparisons with experimental data**
- **AS 3501-6 Graphite/Epoxy**

(Guo C, Sun CT. , 1998)

RESULTS: VALIDATION OF THE STRUCTURAL MODEL

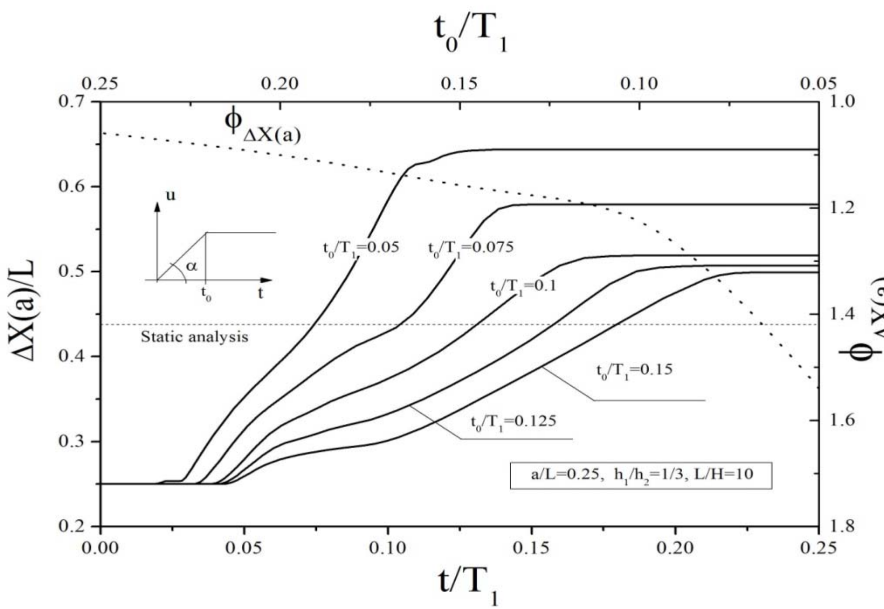


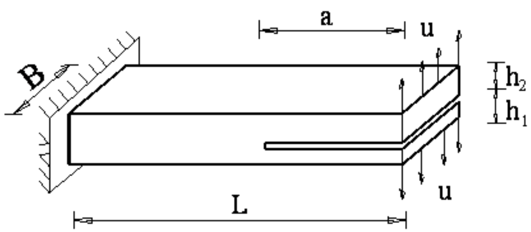


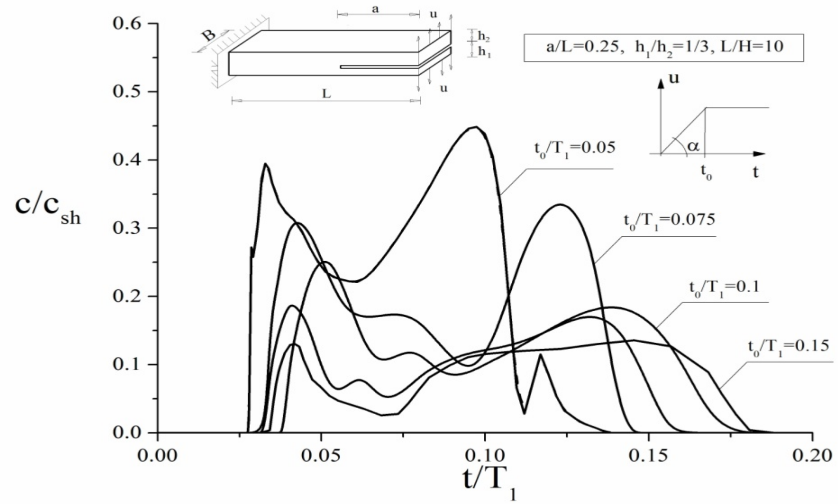
- **DCB mode I loading scheme**
- **Comparisons with experimental data**
- **AS 3501-6 Graphite/Epoxy**

(Tsai JL, Guo C, Sun CT. , 2001)

RESULTS : MIXED MODE ANALYSIS







- **Influence of the Loading Rate**
- **DAFs of the crack tip length**
- **Evolution of the crack tip speed**

RESULTS : MULTIPLE DELAMINATIONS

INDEX:

INTRODUCTION

MOTIVATIONS

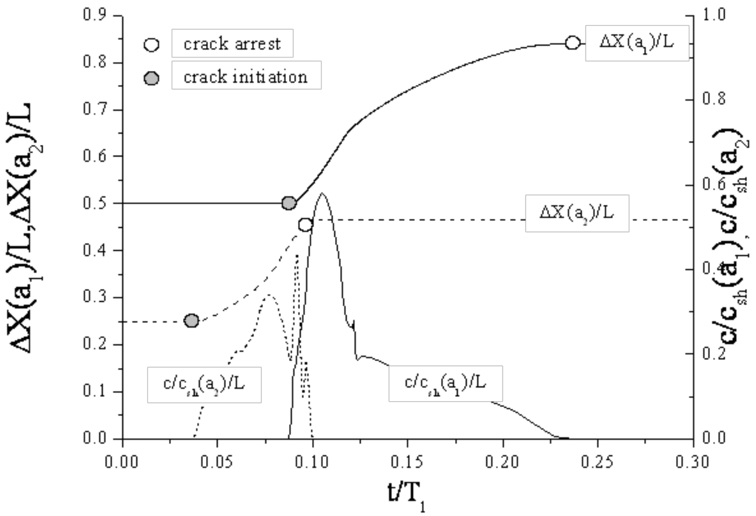
ALE MODEL

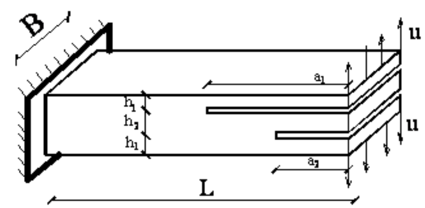
FORMULATION

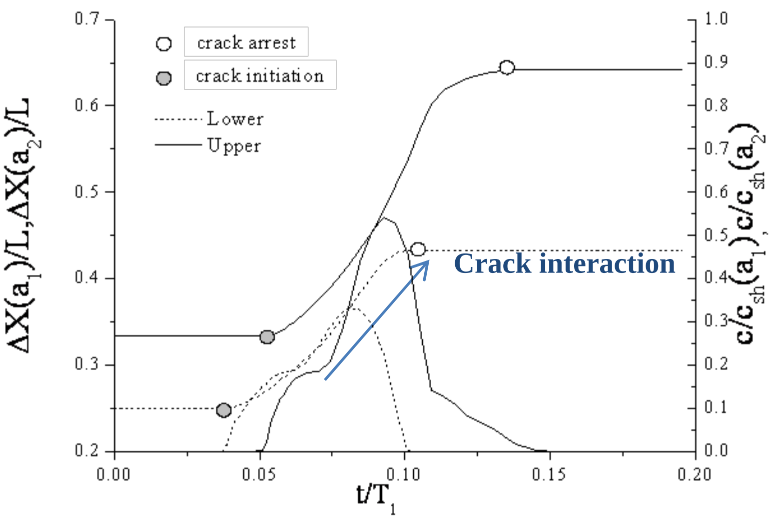
FE MODEL

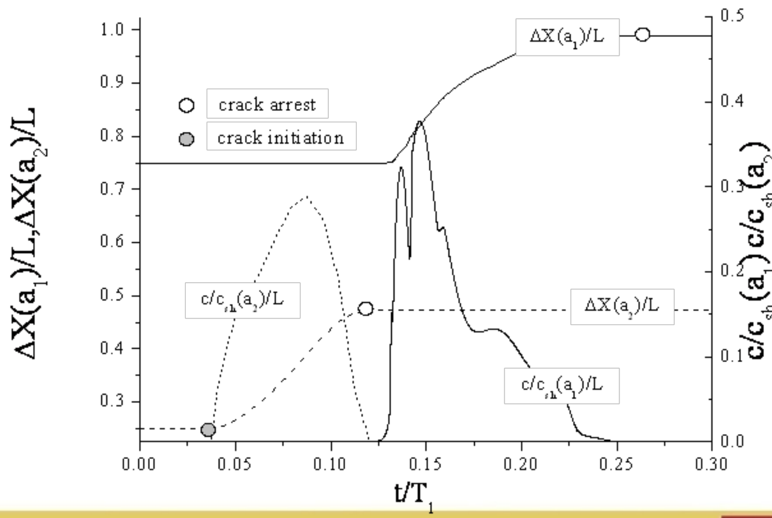
RESULTS

CONCLUSIONS



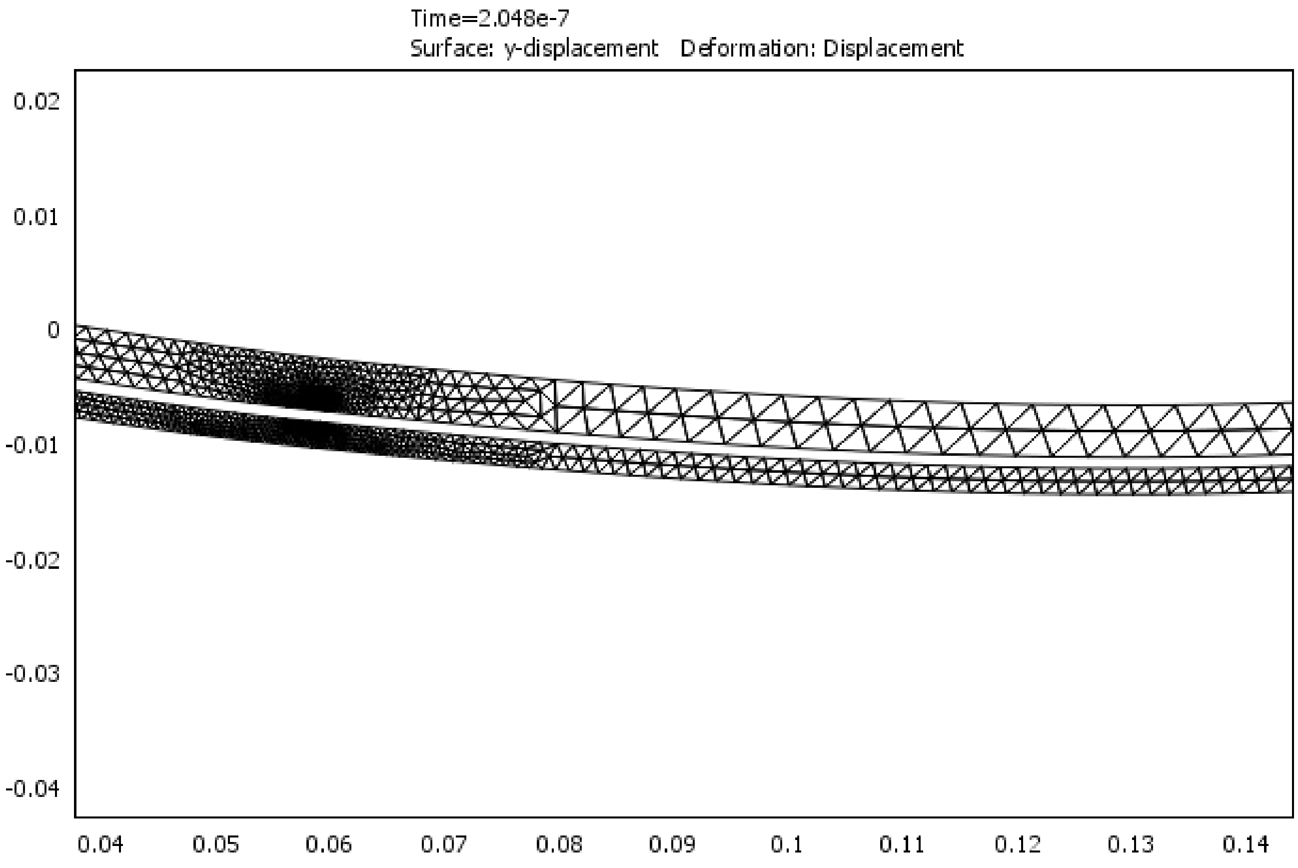






CONCLUDING REMARKS

- **A delamination model for general loading conditions based on moving mesh methodology and fracture mechanics is proposed.**
- **New expressions of the ERR mode components based on the J-integral decomposition procedure are derived.**
- **Comparisons with experimental data are proposed to validate the delamination modelling**
- **The analyzed parametric study shows that delamination phenomena are quite influenced by the loading rate, inertial effects leading to high amplifications in the ERR prediction and the crack growth.**


Time=2.048e-7
Surface: y-displacement Deformation: Displacement
0.02
0.01
0
-0.01
-0.02
-0.03
-0.04
0.04
0.05
0.06
0.07
0.08
0.09
0.1
0.11
0.12
0.13
0.14

GOVERNING EQUATIONS OF THE ALE-FM MODEL

Variational form of the structural model:

$$\sum_{i=1}^{n}\int_{V_{ri}} C\left(\nabla_r u J^{-1}\right)\delta\left(\nabla_r u J^{-1}\right)\det(J)\,dV_r + \sum_{i=1}^{n}\int_{V_{ri}} \rho\Big[u'' - 2\nabla_r u' J^{-1}\cdot X' - \left(\nabla_r u J^{-1}\right)\cdot X'' +$$

$$+\nabla_r\left(\nabla_r u J^{-1}\right)J^{-1}X'X' + \nabla_r u J^{-1}\cdot\left(\nabla_r X' J^{-1}\right)X'\Big]\delta u \det(J)\,dV_r = \sum_{i=1}^{n}\int_{\Omega_{ri}} t\delta u \det(\bar{J})\,d\Omega_r + \sum_{i=1}^{n}\int_{V_{ri}} f\delta u \det(J)\,dV_r$$

Unknown quantities: $(u, \Delta X, X, J)$ → **depend on the mesh configuration**

Variational formulation of the ALE equation

$$\int_V \nabla_X \Delta X_1 \nabla_X w_1\, dA + \oint_{\Omega}\left[\delta\lambda\left(X_1' - c_t\right) + \lambda\delta\dot{X}_1\right] ds = 0,$$

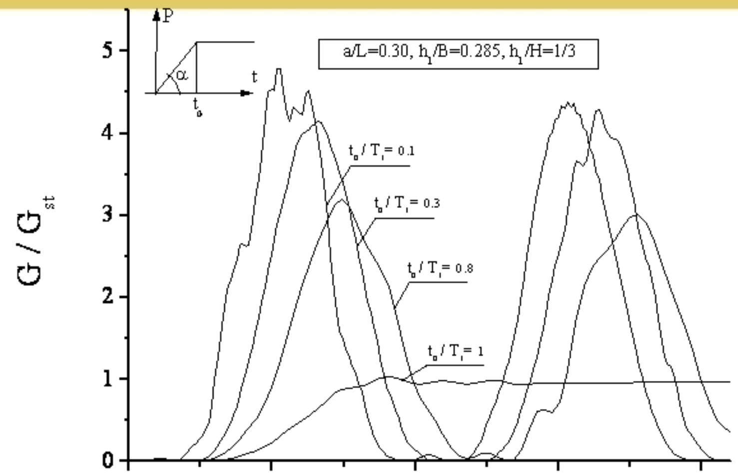
$$\int_V \nabla_X \Delta X_2 \nabla_X w_2\, dA = 0,$$

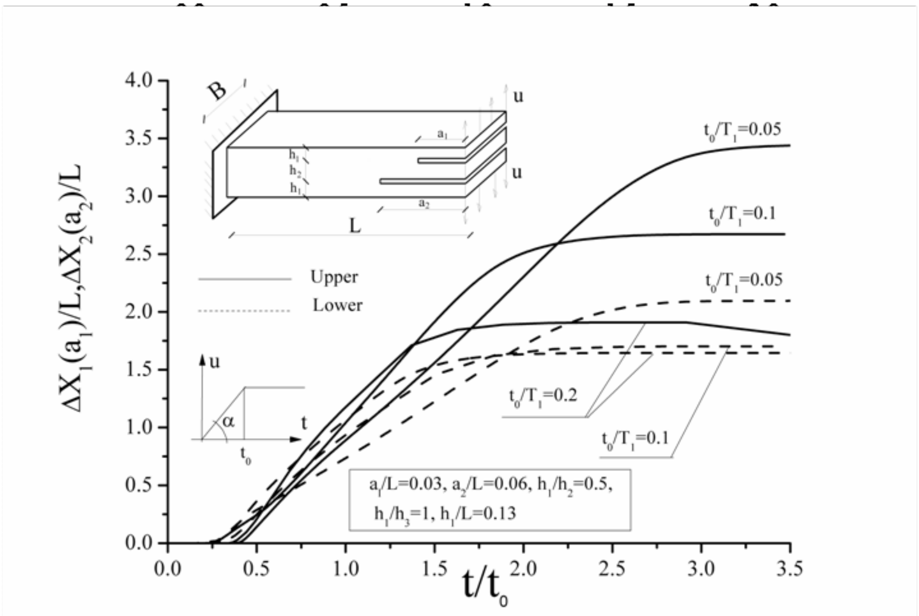
↓ **Crack growth criterion**

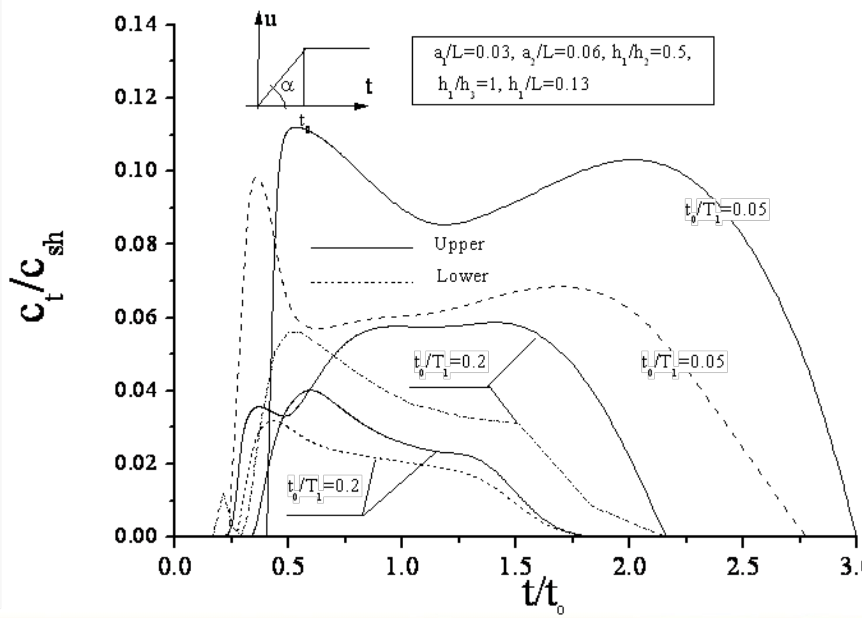
→ **LMM is utilized to impose the mesh speed at the delamination plane**

RESULTS : MULTIPLE DELAMINATIONS

Influence of the Loading Rate







INDEX:

BASICS OF MOVING MESH STRATEGY: ARBITRARY-LAGRANGIAN EULERIAN FORMULATION

Physical fields in ALE formulation

$$\dot{\tilde{u}} = \tilde{u}' - \tilde{X}' \frac{d}{d\tilde{X}} \tilde{u}\left(\tilde{X}, t\right)$$

"Material velocity"

$$\ddot{\tilde{u}} = \tilde{u}'' - 2\nabla_x \tilde{u}' \cdot \tilde{X}' - \nabla_x \tilde{u}\tilde{X}'' + \nabla_x\left(\nabla_x \tilde{u}\right)\tilde{X}'\tilde{X}' + \nabla_x \tilde{u}\nabla_x \tilde{X}'\tilde{X}'$$

"Material accel."

"Material Configuration"

Fundamental relationships between Material and Referential configurations

$$\nabla_x \tilde{u} = \nabla_r \tilde{u}\,\tilde{J}^{-1}$$

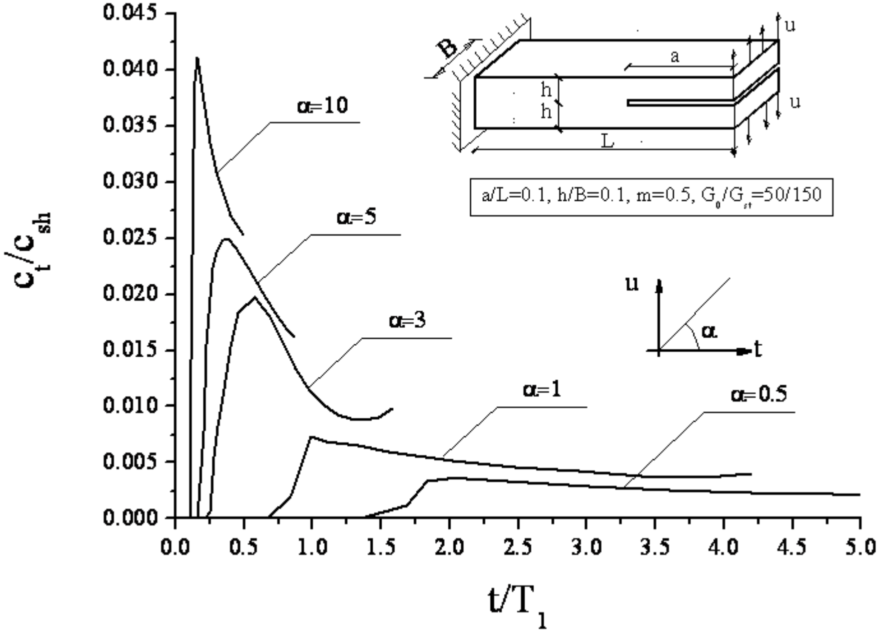
$$\tilde{J} = \begin{bmatrix} \frac{dX_1}{dr_1} & \frac{dX_1}{dr_2} \\ \frac{dX_2}{dr_1} & \frac{dX_2}{dr_2} \end{bmatrix}$$

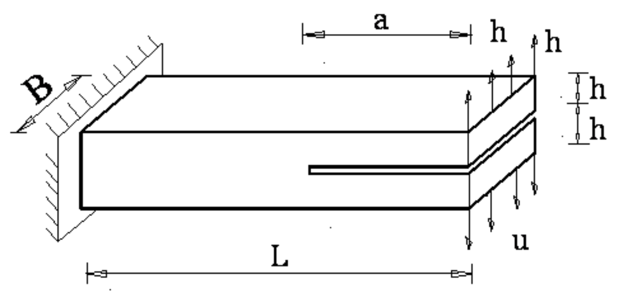
"Referential Configuration"

$$\det \tilde{J} \neq 0$$

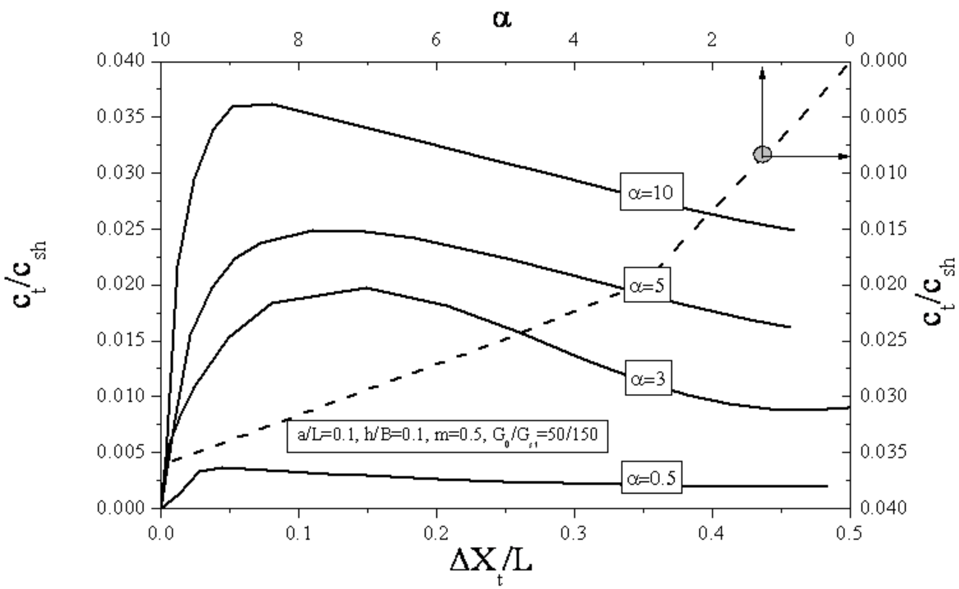
"one-to-one relationship"

RESULTS: EFFECT OF THE LOADING RATE

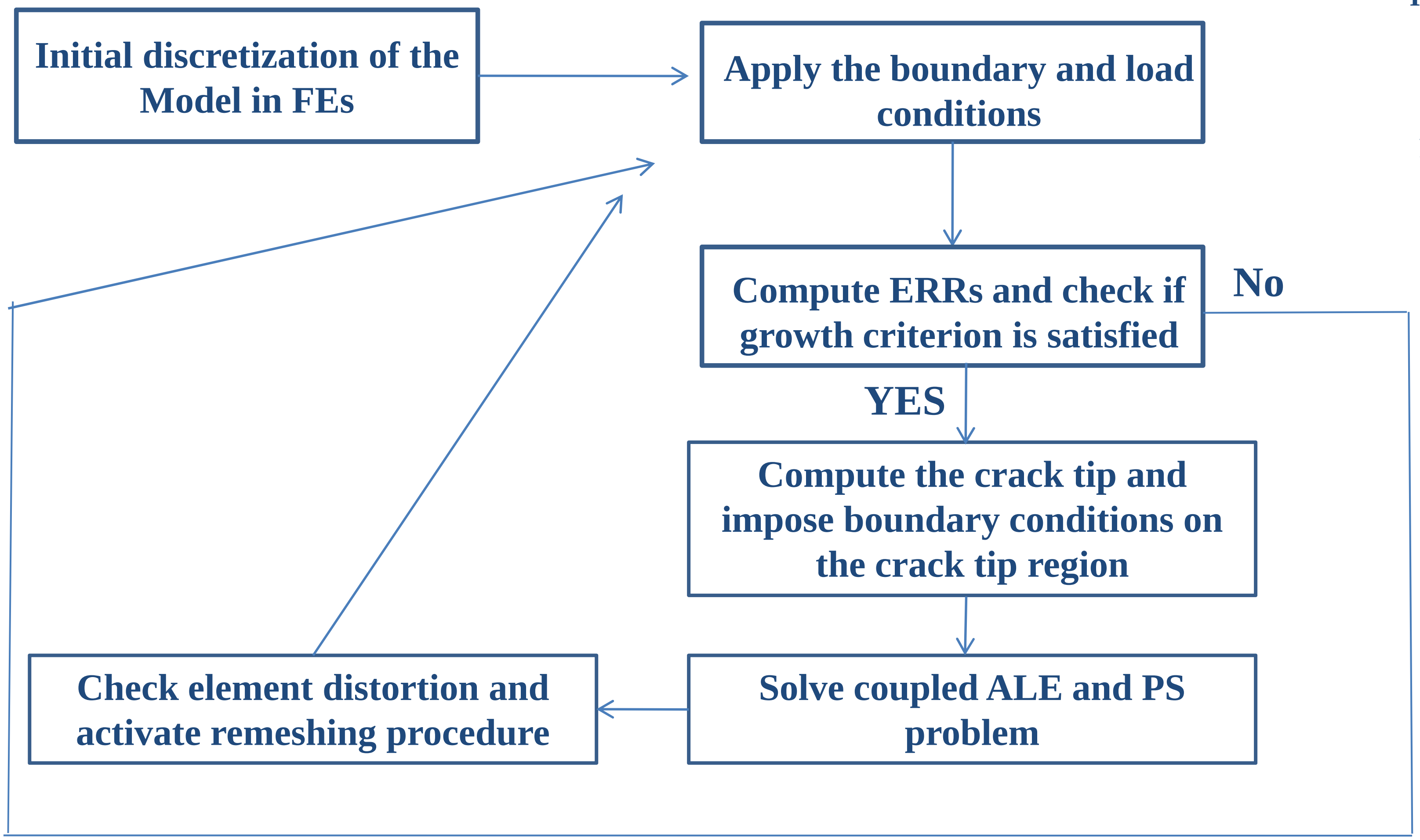




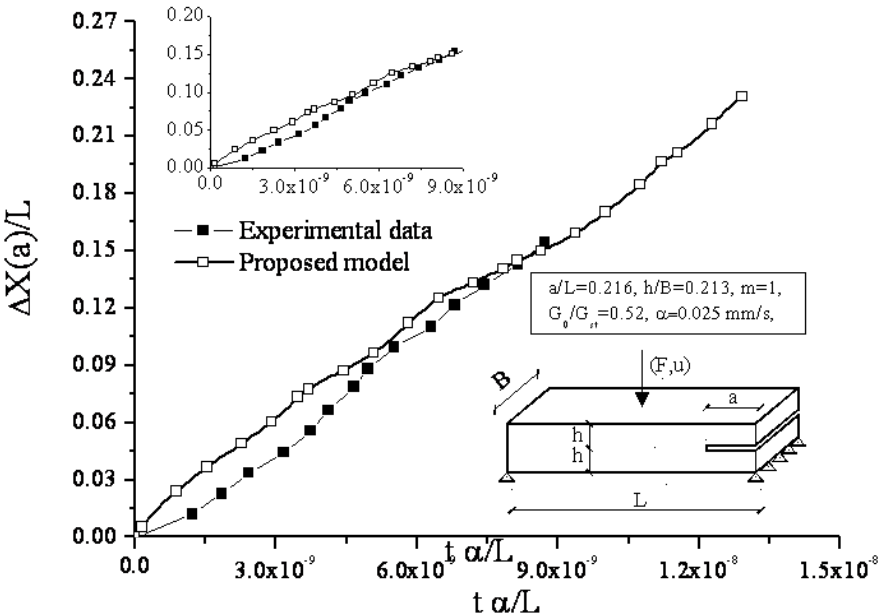
- **DCB mode I loading scheme**
- **Influence of the loading rate**
- **Evolution of the crack tip speed**

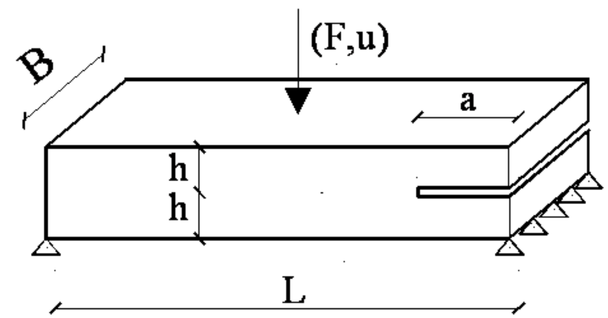


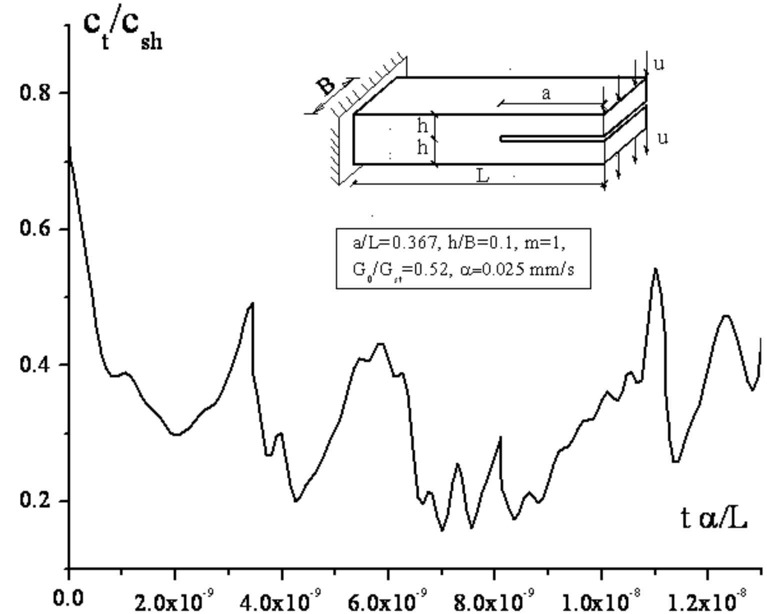
SYNOPTICAL RAPRESENTATION OF THE FEM ALGHORITM



RESULTS : MODE II ENF SCHEME



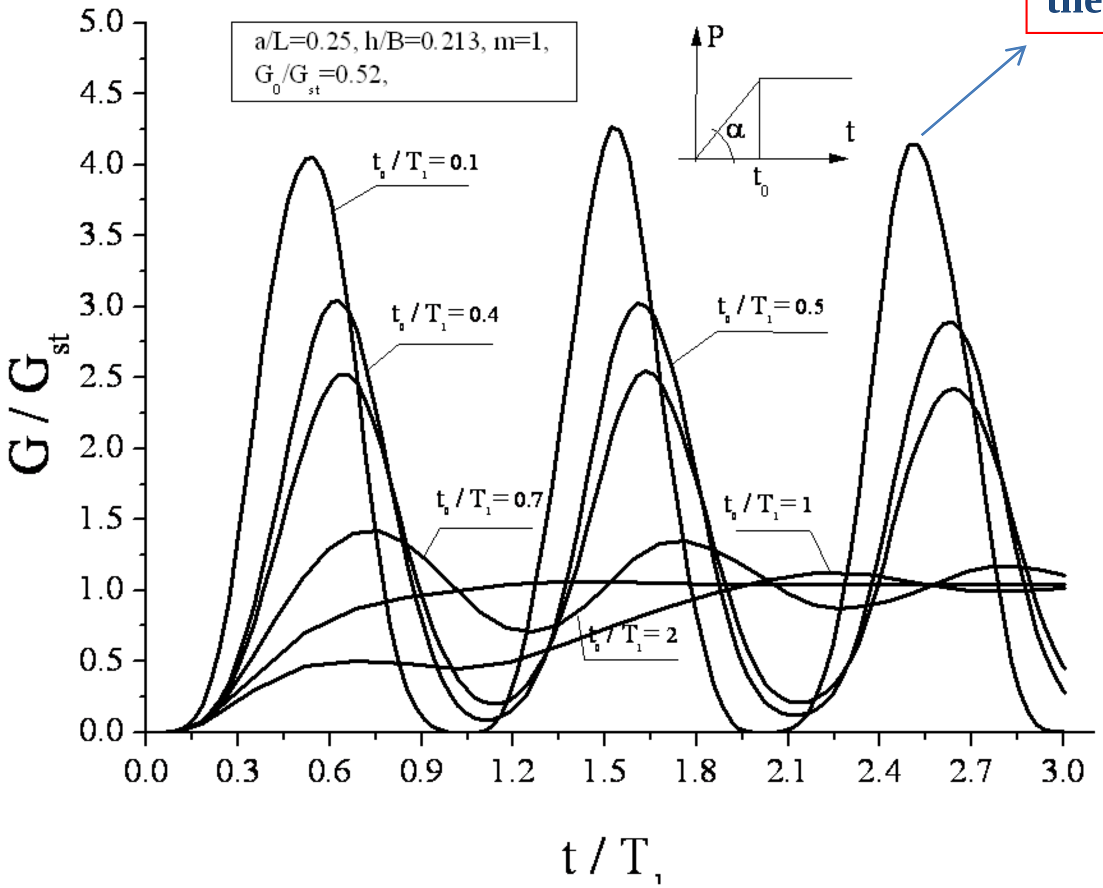


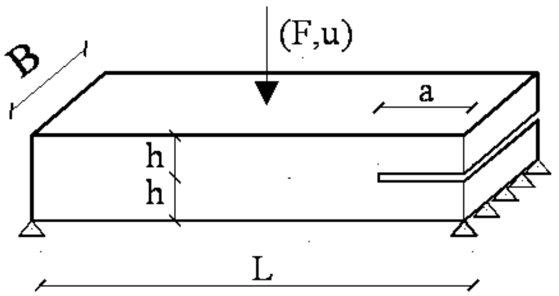


- **ENF mode II loading scheme**
- **Comparisons with experimental data**
- **S2/8553 Glass/Epoxy**

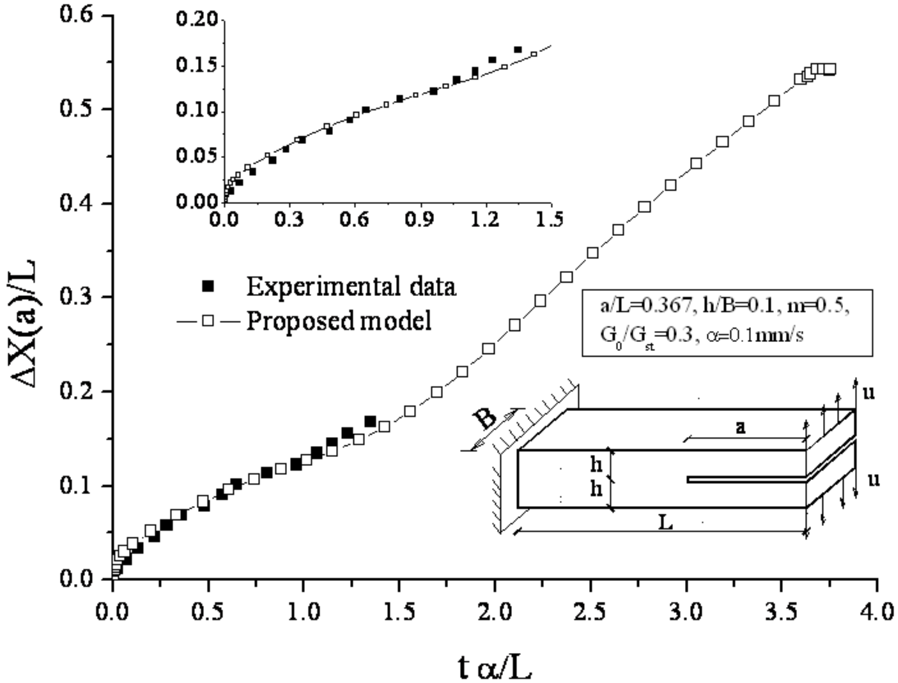
RESULTS : MODE II ENF SCHEME

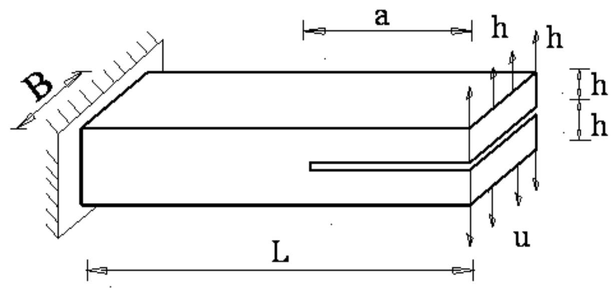
High amplifications in the ERR prediction

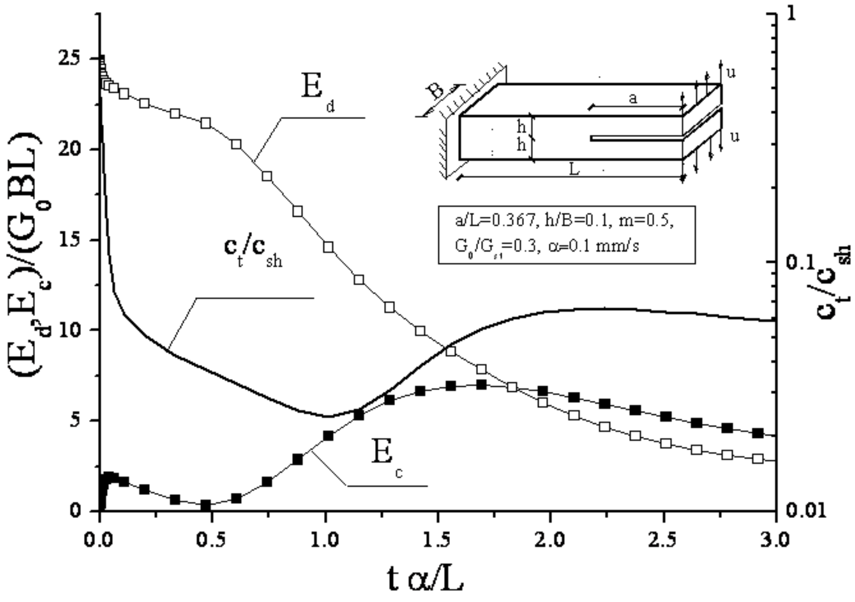




RESULTS: VALIDATION OF THE STRUCTURAL MODEL

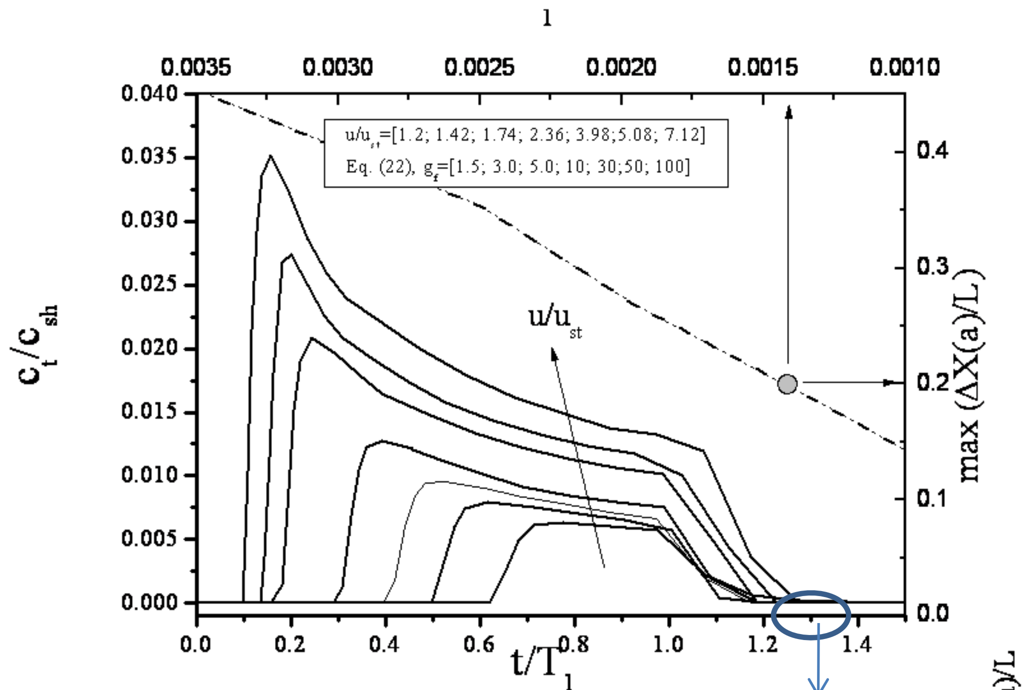


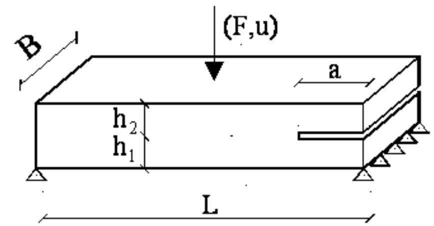


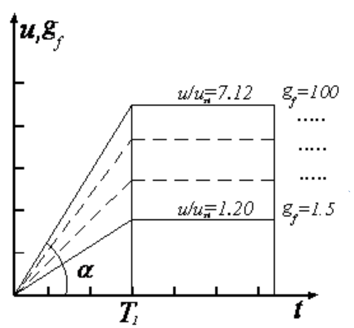


- **DCB mode I loading scheme**
- **Comparisons with experimental data**
- **AS 3501-6 Graphite/Epoxy**

RESULTS : MIXED MODE ANALYSIS







Crack arrest phenomenon

Loading curves