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A NUMERICAL APPROACH TO THE DYNAMIC SOIL - PIPELINE INTERACTION UNDER DEGRADATING ENVIRONMENTAL EFFECTS

Dedicated to the memory of P.D. PANAGIOTOPOULOS (1950-1998),
Late Professor of Aristotle University of Tessaloniki, Greece, and RWTH Aachen, Germany

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- **Abstract**
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- The paper deals with a numerical approach for the dynamic soil-pipeline interaction, considered as an **inequality problem** of structural engineering. So, the **unilateral contact** conditions due to tensionless and elastoplastic softening/fracturing behaviour of the soil as well as due to **gapping** caused by earthquake excitations are taken into account.
- The numerical approach is based on a **double discretization** and on **mathematical programming**. First, in space the Finite Element Method (FEM) is used for the simulation of the pipeline and the unilateral contact interface, in combination with the boundary element method (BEM) for the soil simulation. Next, with the aid of **Laplace transform**, the problem equality conditions are transformed to convolutional ones involving as unknowns the unilateral quantities only. So the number of unknowns is significantly reduced. Then a marching-time approach is applied and finally a **nonconvex linear complementarity problem** is solved in each time-step.

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- **1. INTRODUCTION**
- Dynamic soil-pipeline interaction can be considered as one of the so-called **inequality problems** of structural engineering [1,2]. As wellknown [1-3], the governing conditions of these problems are equalities as well as **inequalities**. Indeed, for the case of the general dynamic soil-structure interaction, see e.g. [3], the interaction stresses in the transmitting interface between the structure and the soil are of compressive type only. Moreover, due to in general nonlinear, elastoplastic, tensionless, fracturing etc. soil behaviour, **gaps** can be created between the soil and the structure. Thus, during e.g. strong earthquakes, separation and **uplift** phenomena are often appeared, as the praxis has shown [4, 10-11].
- The mathematical treatment of the so-formulated inequality problems can be obtained by the variational or **hemivariational inequality approach** [1,2]. Numerical approaches for some dynamic inequality problems of structural engineering have been also presented, see e.g. [1-5].

The present paper deals with a **numerical** treatment for the inequality dynamic problem of soil-pipeline interaction where **second-order geometric effects** for the pile behaviour due to preexisting compressive loads are taken also into account. In the problem formulation, the above considerations about gapping as well as soil elastoplastic/softening behaviour are taken into account. The proposed numerical method is based on a double discretization and on **optimization methods** (nonlinear programming). So, in space the finite element method (FEM) coupled with the boundary element method (BEM), and in time a step-by-step method for the treatment of **convolutional** conditions are used. In each time-step a **non-convex** linear complementarity problem is solved with **reduced** number of unknowns. Finally, the presented procedure is applied to an example problem of dynamic pile-soil interaction, and some concluding remarks useful for the Civil Engineering praxis are discussed.

2. METHOD OF ANALYSIS

2.1 Coupling of FEM and BEM

- Interface soil-elements: Every such interface element consists of an elastoplastic-softening spring and a dashpot, connected in parallel (Figure 1), and appears a compressive force r(t) at the time-moments t only when the pipeline node comes in contact with the soil.
- Let v(t) denote the relative retirement displacement between the soil-element end and the pipeline -node, and g(t) the existing gap. Then the unilateral contact behaviour of the soil-pile interaction is expressed in the compact form of the following linear complementarity conditions:
- $$v \geq 0, \qquad r \geq 0, \qquad r.v = 0. \qquad (1)$$

The soil-element compressive force is in convolutional form [4]

$$r = S(t)*y(t), \qquad y = w - (g + v), \qquad (2a,b)$$

or in form used in Foundation Analysis [11]

$$r = c_s(\partial y/\partial t) + p(y). \qquad (2c)$$

By * is denoted the convolution operation.
S(t) is the dynamic stiffness coefficient for the soil and can be computed by the BEM [4].
Function p(y) is mathematically defined by the following, in general nonconvex and nonmonotone constitutive relation

$$p(y) \in \Theta P(y), \qquad (2d)$$

where Θ is Clarke's generalized gradient and P(.) the symbol of superpotential nonconvex functions [2]. So, (2d) expresses in general the elastoplastic-softening soil behaviour, where unloading-reloading, gapping, degrading, fracturing etc. effects are included.

- Problem conditions for the assembled soil-pipeline system, written in matrix form according to the finite element method:
-
- $$\underline{M}\,(t)\,\underline{\ddot{u}} + \underline{C}\,(t)\,\underline{\grave{u}} + (\underline{K} + \underline{G})\,\underline{u}(t) = \underline{f}(t) + \underline{A}^T\,\underline{r}(t) \qquad (3)$$
-
- $$\underline{y} = \underline{A}^T\underline{u} - \underline{u}_g - \underline{g} - \underline{B}\,\underline{z}, \qquad \underline{r} = \underline{S}^*\underline{y}, \qquad (\text{or } \underline{r} = \underline{E}\,\underline{y}), \qquad (4),(5)$$
-
- $$\underline{\omega} = \underline{B}^T\underline{r} - \underline{H}\,\underline{z} - \underline{k}, \qquad \underline{\omega} \leq 0, \qquad \underline{z} \geq \underline{0}, \qquad \underline{z}^T.\underline{\omega} = 0, \qquad (6)$$
-
- $$\underline{u}(t{=}0) = \underline{u}_o, \qquad \underline{\grave{u}}\,(t{=}0) = \underline{\grave{u}}_{o}, \qquad \underline{g}(t{=}0) = \underline{g}_o \qquad (7)$$

- $\underline{G}$ is the geometric stiffness matrix depending linearly on pre-existing stress state [6,7]; $\underline{u}$, $\underline{f}$ are the displacement and the force vectors, respectively; $\underline{u}_g(t)$ is the vector of (possible) seismic ground displacement; $\underline{A}$, $\underline{B}$ are kinematic transformation matrices; $\underline{z}$, $\underline{k}$ are the nonnegative multiplier and the unilateral capacity vectors; and $\underline{E}$, $\underline{H}$ are the elasticity and unilateral interaction square matrices, symmetric and positive definite the former, positive semidefinite the latter for the elastoplastic soil case. In the case of soil softening, some diagonal entries of $\underline{H}$ are nonpositive [7].
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- For the case of nonlinear pile behaviour, either the linear terms $\underline{C\dot{u}}$ and $\underline{K}\ \underline{u}$ can be replaced by the nonlinear matrix functions $\underline{C}(\underline{\dot{u}})$ and $\underline{K}(\underline{u})$, or the local nonlinearities (e.g. elastoplasticity) are included in appropriate internal unilateral constraints [7-9].
- Thus the so-formulated problem is to find ($\underline{u}$,$\underline{r}$,$\underline{g}$,$\underline{z}$) satisfying (1)-(7) when ($\underline{f}$, $\underline{u}_g$,
- $\underline{\dot{u}}_g$, $\underline{u}_o$, $\underline{g}_o$) are given.

2.2 Time discretization. The Convolutional LCP.

- Assuming that the unilateral quantities $\underline{z}$ and $\underline{T}$ include all local nonlinearities and unilateral behaviour, the procedure of Liolios [9] can be used. So, applying the Laplace transform to (3)-(7), except $(6)_4$, and after suitable elimination of unknowns and back transforming to time domain, we arrive eventually at
-
- $$\underline{\omega}(t) = \underline{D}(t) * \underline{z}(t) + \underline{d}(t). \qquad (8)$$
-
- Thus, at every time-moment the problem of rels. $(6)_{2,3,4}$ and (8) is to be solved. This problem is called here Convolutional Linear Complementarity Problem (CLCP), has a reduced number of unknowns and is solved by time discretization.

So, for the time moment $t_n = n.\Delta t$, where Δt is the time step, we arrive eventually at a non-convex linear complementarity problem [6]:

$$\underline{\omega}_n = \underline{D}\, \underline{z}_n + \underline{d}_n, \qquad \underline{z}_n \geq \underline{0}, \qquad \underline{\omega}_n \leq \underline{0}, \qquad \underline{z}^T . \underline{\omega}_n = 0. \qquad (9)$$

Solving problem (9) by available computer codes of nonlinear mathematical programming [1,2,12], we compute which of the unilateral constraints are active and which not in each time-step Δt. Due to soil softening, matrix $\underline{D}$ is not strictly positive definite in general. But as numerical experiments have shown, in most civil engineering applications of soil-pile interaction this matrix is P-copositive. Thus the existence of a solution is assured [7].

3. NUMERICAL EXAMPLE

An empty horizontal steel circular pipeline of length L = 200 m, outside diameter 1 m, thickness 1.5 cm, elastic modulus $21*10^7$ KN/m^2 and yield stress 50 KN/cm^2 is considered. As depicted in Figure 1, the pipeline is clamped by the two anchor blocks A and B imbedded into a rock soil. The soil, into which the horizontal pipeline is buried, has an elastoplastic behaviour as in Figure 2 and was initially the same along the length L = 200 m. Due to environmental effects, the soil consists finally of two regions: the first (I), the degraded one, is soft with a shear modulus G_i = 5000 KN/m^2, the second (II) remains hard with a shear modulus G_{ii} = 100000 KN/m^2. The parameters for the elastoplastic behaviour in Figure 2 are taken to be $a = p_u.b$, $b = 100\ m^{-1}$, where it is p_u = 100 KN/m^2 for the soft region (I) and p_u = 2000 KN/m^2 for the hard region (II).

Further, the seismic ground velocity excitation is assumed to be a sinusoidal horizontal wave propagation parallel to the pipeline axis (Figure 1), with mean speed $v_g = 0.4$ km/sec in the soft region (I) and $v_g = 0.8$ km/sec in the hard one (II), frequency $f_g = 10$ rad/sec, duration $T = 2\pi/f_g$ and maximum ground displacement $w_0 = 5$ cm. Thus the horizontal ground motion, perpendicular to the pipeline axis x, is expressed mathematically by the following relation (6), where H(t) is the Heaviside function:

$$u_g(x,t) = w_0 \sin(t-x/v_g) . \{H(t-x/v_g)-H(t-x/v_g -T)\}. \qquad (6)$$

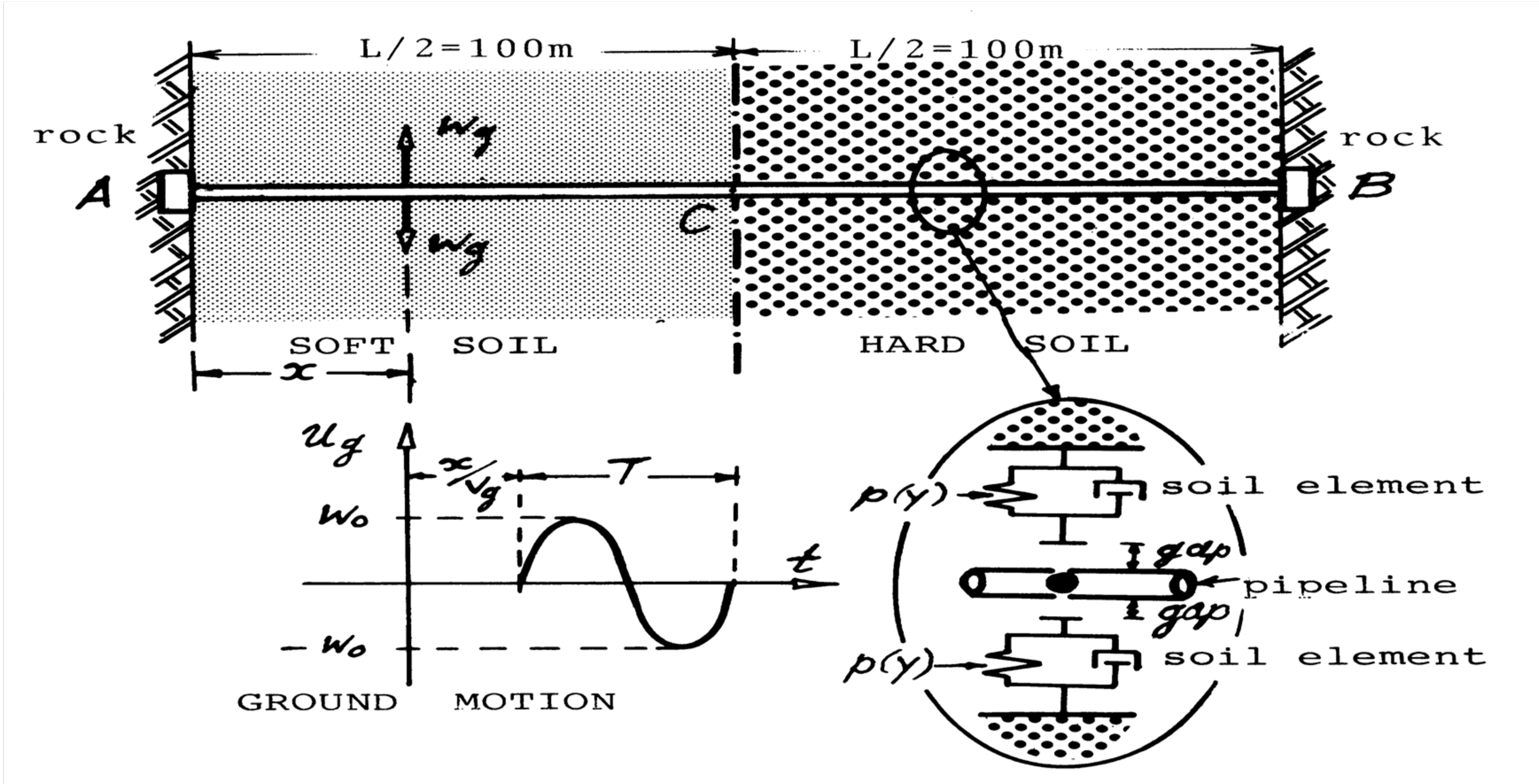

Figure 1: Soil-pipeline system, horizontal wave travelling ground motion and soil-pipeline interaction modelization.

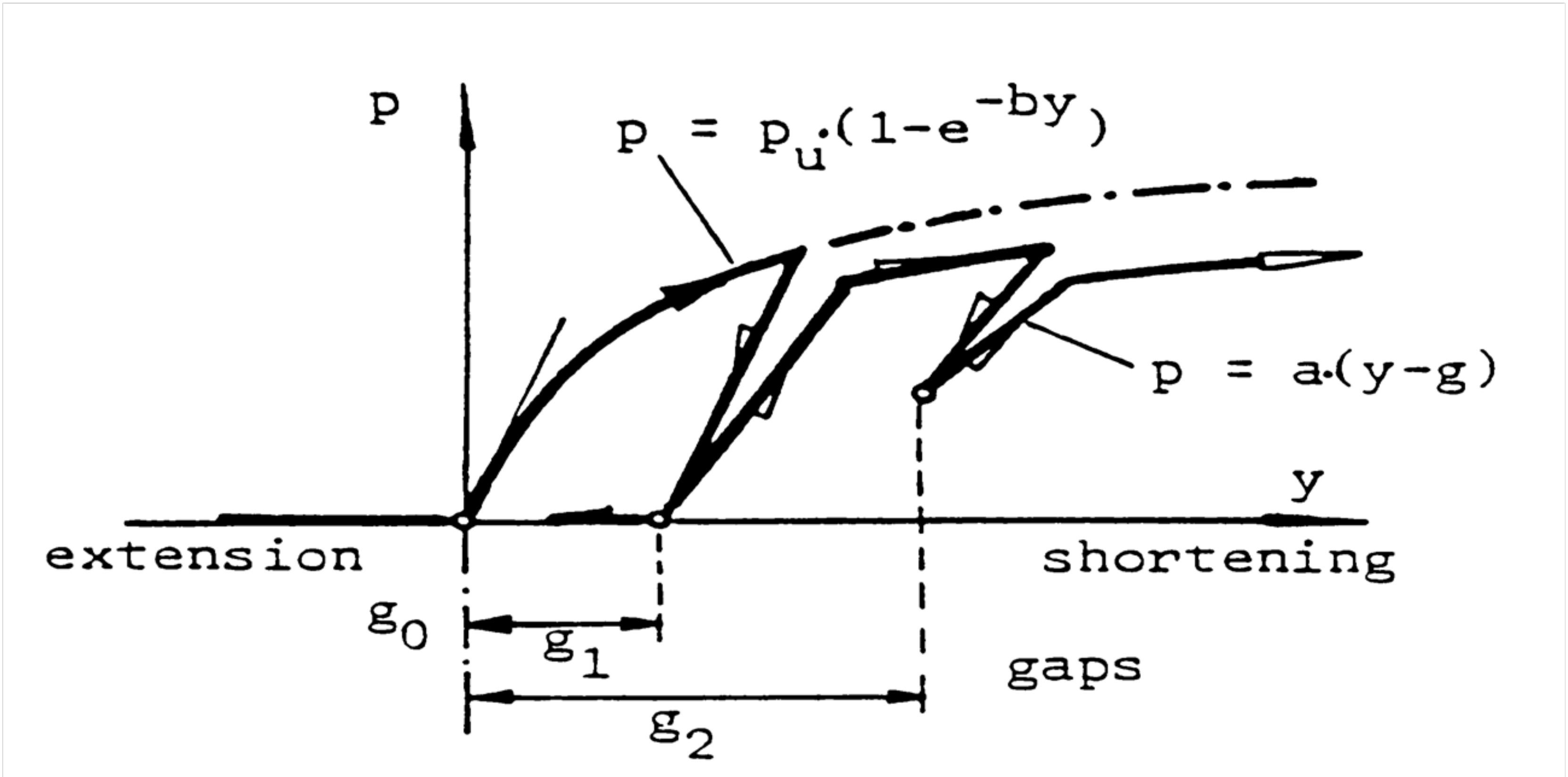

Figure 2: Unilateral, degradating soil behaviour in loading-unloading with remaining gaps.

Some response results from the ones obtained by applying the herein presented numerical procedure are indicatively reported.
So they are shown:

in Figure 3: Gaps along the pipeline at times $t_1 = 0.6$ sec and $t_2 = 2.1$ sec,
in Figure 4: Soil-pressure distribution at the time $t_1 = 0.6$ sec.

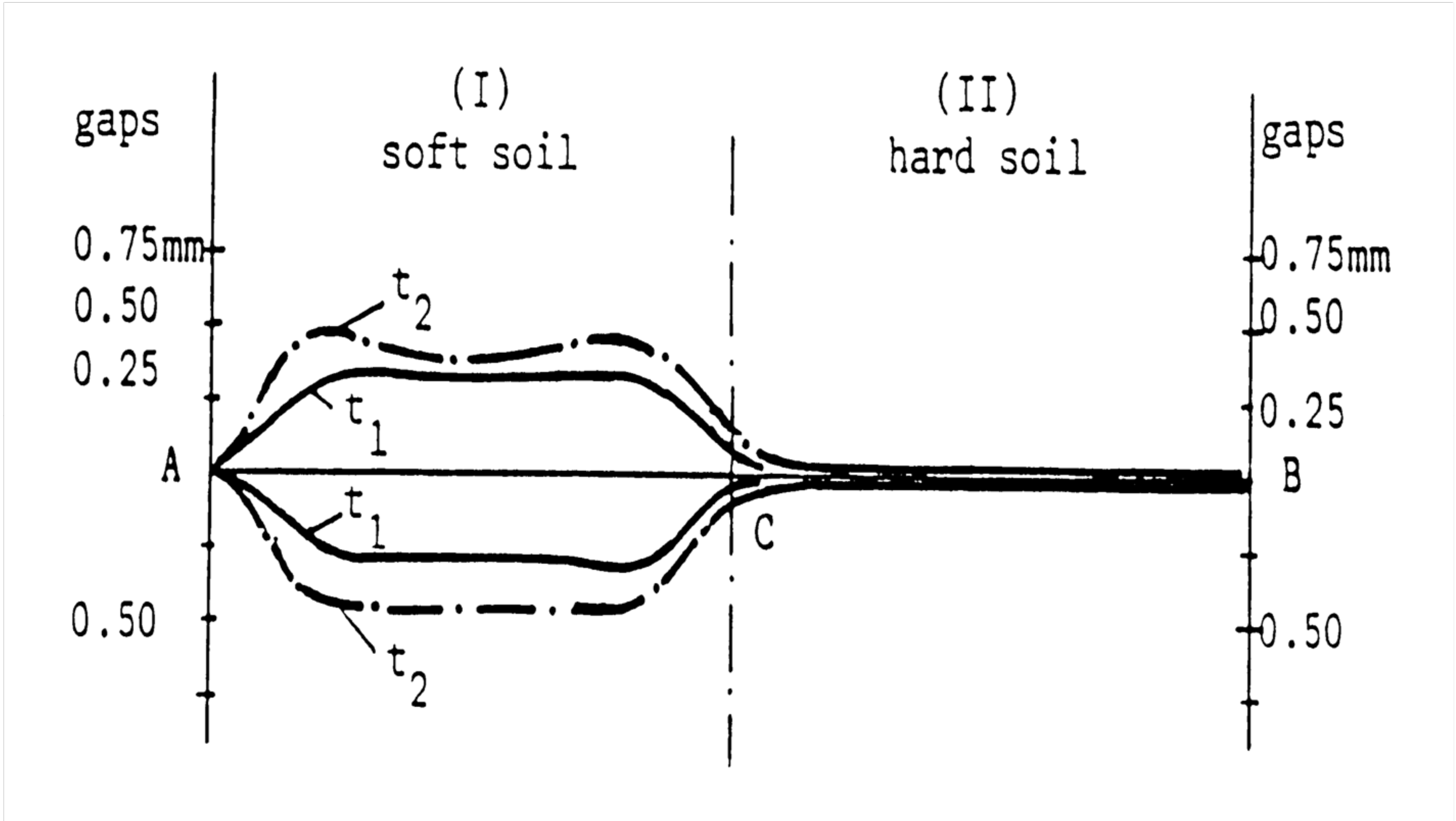

Figure 3: Gaps along the pipeline at times $t_1 = 0.6$ sec and $t_2 = 2.1$ sec

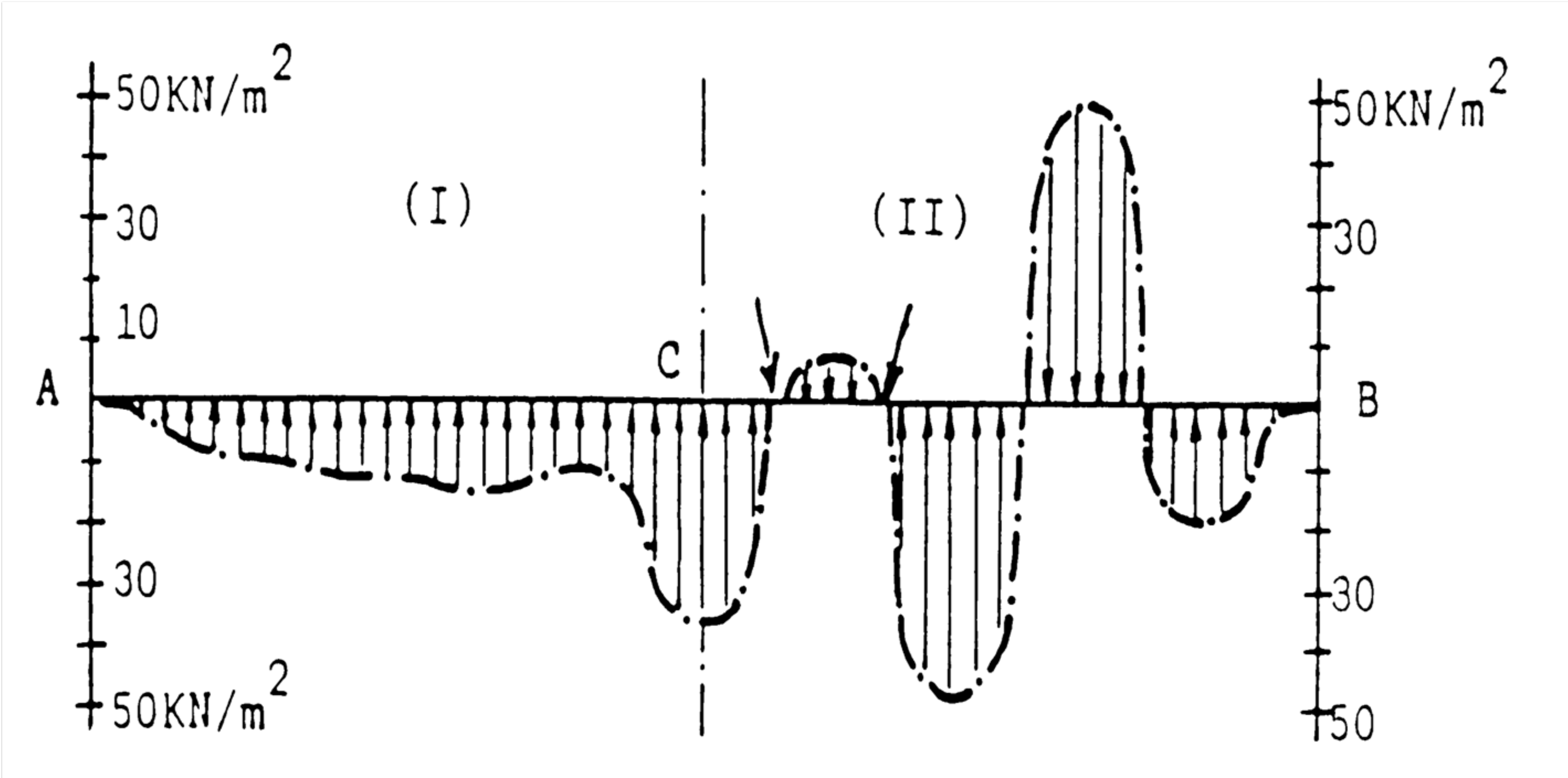

Figure 4: Soil-pressure distribution at the time t1 = 0.6 sec

- **4. CONCLUDING REMARKS**
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- As the above indicative results of the numerical example show, **unilateral contact** effects due to tensionless soil capacity and to **gapping** may be significant and have to be taken into account for the dynamic soil-pipelinee interaction. Also environmental effects can cause degradation to soil behaviour.
- All these effects can be numerically estimated by the herein presented procedure, which is realizable on computers by using existent codes of coupling the FEM and BEM as well as **optimization algorithms**. Thus, the presented approach can be useful in the praxis for the earthquake resistant construction, design and control of piles.