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Homogenized models of failure of defected composite microstructures with contact effects

Domenico Bruno, Fabrizio Greco, Paolo Lonetti, Paolo Nevone Blasi

Department of Structural Engineering
University of Calabria, Cosenza, ITALY

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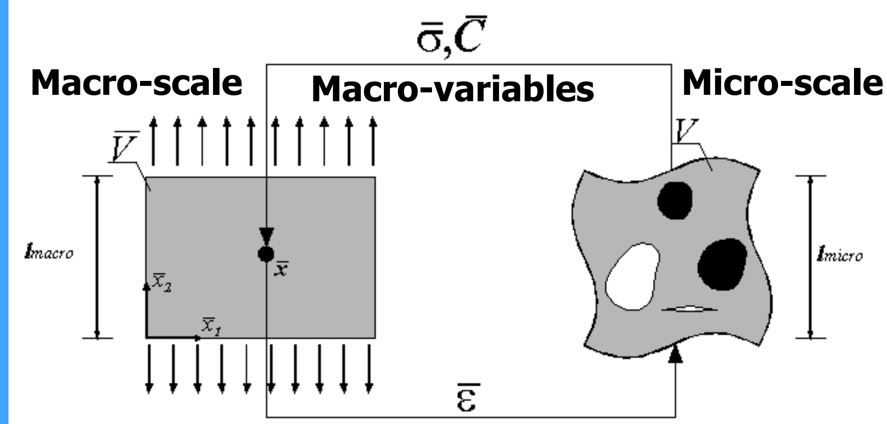
University of Calabria

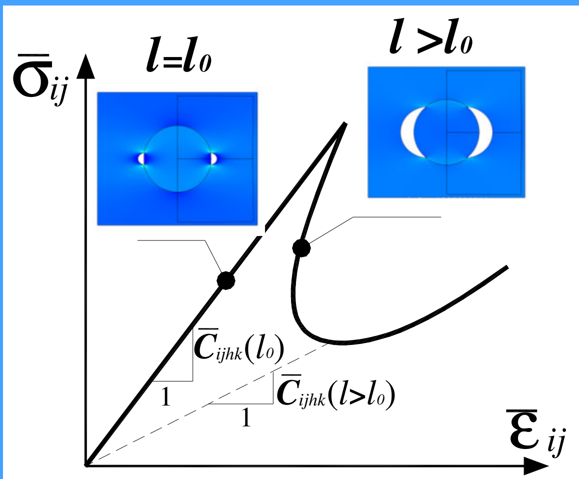
Introduction

ANALYSIS OF COMPOSITE MATERIALS: prediction of macroscopic properties as a function of the microstructure ⇒ ***Homogenization***

Strain-driven Micro-structures

Micro-cracks growing from the voids — **Interface debonding**





Presence of internal damage mechanisms (micro-cracks, voids, interphase debonding): the macroscopic behavior is *strongly non-linear* in contrast to linear elastic behavior of micro-constituents

Sources of non-linearity

Progressive stiffness loss due to damage accumulation, Contact between crack faces, Damage evolution criterion — ***Effects*** ⇒ ***Failure of the homogenized material (unstable crack propagation, localized deformation modes)***

It is of greater importance to determine a constitutive law at the macro-level representing microstructural damage mechanisms

Outline

- **1) Theoretical formulation of the model:** basic equations, variational formulation, macroscopic constitutive properties with evolving micro-cracking and contact;

- **2) Crack onset and propagation modelling:** *J*-integral formulation; Crack onset (Hybrid criterion); Propagation criterion with kinking competition;

- **3) Computational implementation by FEM:** interface model; micro-to-macro transition;

- **4) Numerical examples of homogenized constitutive laws** for two evolving microstructures.

  1) Cellular material with initial microcracks spreading from the voids: self-similar and non-self-similar crack propagation; macrostrain path effects;

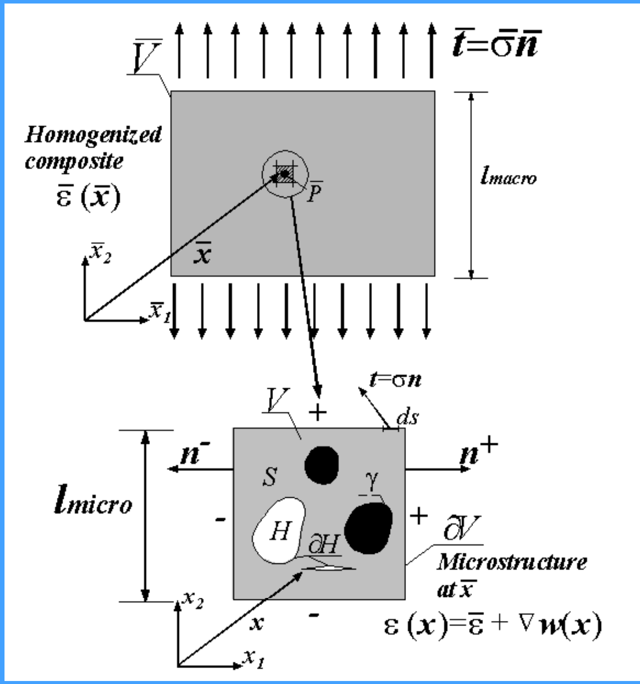
  2) Fiber reinforced composite with an originally damaged or undamaged fibre/matrix interface; Parametric studies of crack onset and propagation (macrostrain path; interface toughness; fiber diameter);

- **5) Conclusions and discussion**;

Problem formulation: basic equations

- RVE of a composite micro-structure, $V=S\cup H$, representative of a material point neighborhood of the homogenized composite.
- Solid part S and hole part H including discontinuities (crack and interface debonding) and voids. ∂H represents the union of micro-crack, and micro-void surfaces. Tractions $\boldsymbol{t}=0$ only on ∂V except contact ;
- Definition of macro-variables in terms of boundary data of tractions $\boldsymbol{t}$ and displacements $\boldsymbol{u}$:

$$\bar{\boldsymbol{\sigma}} = \frac{1}{|V|}\int_{\partial V} \boldsymbol{t}\otimes \boldsymbol{x}\,dA, \qquad \bar{\boldsymbol{\varepsilon}} = \frac{1}{|V|}\int_{\partial V} \boldsymbol{u}\otimes_s \boldsymbol{n}\,dA \qquad \frac{l_{micro}}{l_{macro}} = \varepsilon \ll 1$$



Deformation controlled by a **prescribed macro-strain**: coupling between micro- and macro-scale

Irregular materials: 1) and 3) provide upper and lower strain energy bounds converging as the RVE size increases;

Periodic materials: 2) yields exact results for a unit cell

$$\boldsymbol{u}(\boldsymbol{x})=\bar{\boldsymbol{\varepsilon}}\,\boldsymbol{x}+\boldsymbol{w}(\boldsymbol{x}) \quad \Rightarrow$$

Constraint on fluctuation field

$$\int_{\partial V} \boldsymbol{w}\otimes_s \boldsymbol{n}\,dA = \boldsymbol{0}$$

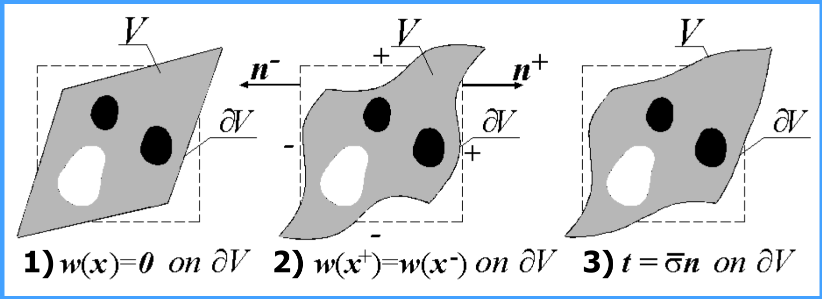
Alternative boundary conditions on ∂V

1) Linear deformations: $\boldsymbol{w}(\boldsymbol{x}) = \boldsymbol{0}$ on ∂V

2) Periodic fluctuations: $\boldsymbol{w}(\boldsymbol{x}^+) = \boldsymbol{w}(\boldsymbol{x}^-)$ on ∂V

3) Associated with uniform tractions: $\boldsymbol{t}=\bar{\boldsymbol{\sigma}}\boldsymbol{n}$ on ∂V

$$\bar{\boldsymbol{\sigma}}\bullet\bar{\boldsymbol{\varepsilon}}-\frac{1}{|V|}\int_{\partial V}\bar{\boldsymbol{\sigma}}\bullet(\boldsymbol{u}\otimes_s \boldsymbol{n})\,dA = 0$$



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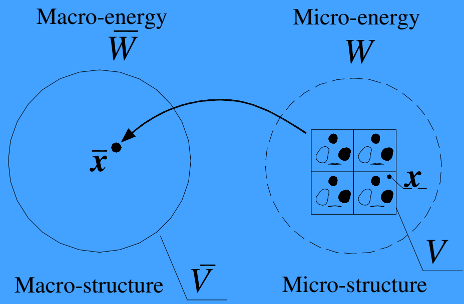
Problem formulation: variational formulation

Microscopic response: Linearly hyperelastic with convex microscopic strain energy $W(\boldsymbol{x})$



•Homogenization condition: $\overline{W}(\overline{\boldsymbol{\varepsilon}}) = \inf_{\boldsymbol{w}\in A(\overline{\boldsymbol{\varepsilon}})} \frac{1}{|V|}\int_S W(\boldsymbol{\varepsilon}(\boldsymbol{w}) = \overline{\boldsymbol{\varepsilon}}\boldsymbol{x} + \nabla_s \boldsymbol{w}, \boldsymbol{x})\, dV$

Minimization problem subjected to the three alternative constraints for a prescribed $\overline{\boldsymbol{\varepsilon}}$

$A(\overline{\boldsymbol{\varepsilon}})$ Closed subset of $H^1(S)$

From the macrostress potential ⇨ $\overline{\boldsymbol{\sigma}} = \frac{\partial \overline{W}}{\partial \overline{\boldsymbol{\varepsilon}}}, \quad \overline{\boldsymbol{C}} = \frac{\partial^2 \overline{W}}{\partial \overline{\boldsymbol{\varepsilon}}^2}$ **Macroscopic stresses and moduli**

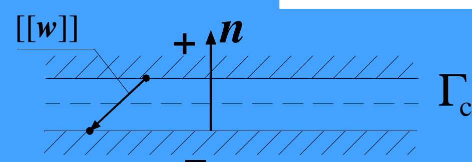
Linearity of the problem ⇨ $\overline{\boldsymbol{\sigma}} = \frac{1}{|V|}\int_S C(x)\boldsymbol{\varepsilon}(\boldsymbol{u})\, dV = \overline{C}\overline{\boldsymbol{\varepsilon}}, \quad \overline{C}_{ijhk} = \frac{1}{|V|}\int_S C(x)\boldsymbol{\varepsilon}(\boldsymbol{u}^{hk})\bullet\boldsymbol{\varepsilon}(\boldsymbol{u}^{ij})\, dV$

$\boldsymbol{u}^{ij}$ solution for $\overline{\boldsymbol{\varepsilon}}^{ij} = \boldsymbol{e}_i \otimes_s \boldsymbol{e}_j$

Crack face unilateral frictionless **contact** ⇨ **Macroscopic constitutive behavior non-linear but hyperelastic**

$K(\overline{\varepsilon}) = \{\boldsymbol{w} \in A(\overline{\varepsilon}) \mid [\![w_n]\!] \leq 0 \text{ on } \Gamma_c\}$

Admissible fluctuation space



$\overline{C} = \overline{C}(\overline{\varepsilon}) \qquad \overline{C}(\lambda\overline{\varepsilon}) = \overline{C}(\overline{\varepsilon}) \quad \lambda > 0$

$\overline{W}$ **C^1, Convex positively homogeneous of degree two**

Contact area is not a-priori known, depends on the direction of the imposed macro-strain $\hat{\overline{\varepsilon}} = \overline{\varepsilon}/\|\overline{\varepsilon}\|$

$\int_S \frac{\partial W[\boldsymbol{\varepsilon}(\boldsymbol{w})]}{\partial \boldsymbol{\varepsilon}} \bullet (\nabla_s \boldsymbol{v} - \nabla_s \boldsymbol{w})\, dV \geq 0 \quad \forall \boldsymbol{v} \in A(\overline{\varepsilon}) / K(\overline{\varepsilon})$

Euler-Lagrange equations: local equilibrium state in S, stress conditions, contact conditions on Γ_c

2): antiperiodic tractions on ∂V

3): incorporation of the weak constraint associated with the Lagrange multiplier $\overline{\boldsymbol{\sigma}}$ (saddle point)

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Problem formulation: macroscopic constitutive properties

Given damage configuration: $\bar{\boldsymbol{\sigma}} = \bar{\boldsymbol{C}}\left(\hat{\bar{\varepsilon}}, l\right)\bar{\boldsymbol{\varepsilon}}$

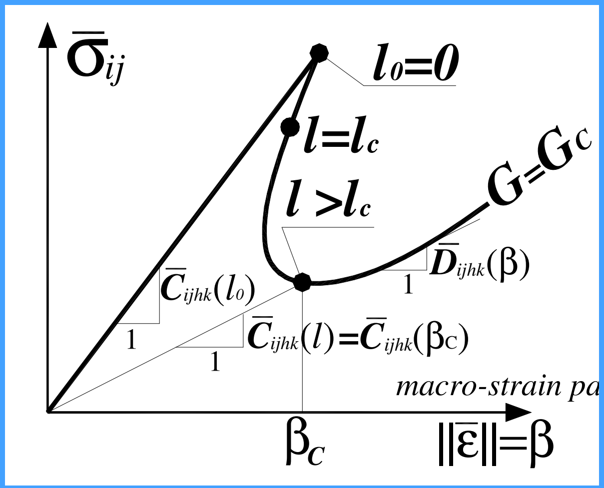
Moduli tensor depends on the macro-strain direction (contact) and on the crack length

$\bar{C}_{ijhk}(l) = \frac{1}{|V|}\int_V C_{ijmn}(\boldsymbol{x})\,\varepsilon_{mn}\left(\boldsymbol{u}^{hk}\right)dV$ *Contact excluded*

$\bar{C}\left(l, \hat{\bar{\varepsilon}}\right) = \frac{\partial^2 \bar{W}}{\partial \bar{\boldsymbol{\varepsilon}}^2}$ *Contact included*

Crack Onset and Mixed mode propagation (Kinking) criteria ⇨ **Damage evolution relation** $l = l\left(\bar{\boldsymbol{\varepsilon}}\right)$

<u>As a consequence</u>: macroscopic constitutive law becomes non-linear and dependent on the macro-strain history $\bar{C}\left(l(\bar{\boldsymbol{\varepsilon}}), \hat{\bar{\varepsilon}}\right)$



Incremental constitutive law ⇨

$$\dot{\bar{\boldsymbol{\sigma}}} = \bar{\boldsymbol{D}}(\bar{\boldsymbol{\varepsilon}})\,\dot{\bar{\boldsymbol{\varepsilon}}},$$

$$\bar{D}_{ijhk}(\bar{\boldsymbol{\varepsilon}}) = \bar{C}_{ijhk}(l(\bar{\boldsymbol{\varepsilon}})) + \begin{cases} \frac{d\bar{C}_{ijmn}(l(\bar{\boldsymbol{\varepsilon}}))}{d\bar{\boldsymbol{\varepsilon}}_{hk}}\bar{\boldsymbol{\varepsilon}}_{mn} & \dot{l} > 0 \\ 0 & \dot{l} \le 0 \end{cases}$$

$\bar{C}(l)$ Tangent macro-moduli with damage configuration unchanged

$\bar{D}(\bar{\boldsymbol{\varepsilon}})$ Tangent macro-moduli with fracture criterion imposed

<u>Macroscopic constitutive law incorporating damage evolution</u>

- Prescribed macro-strain path $\beta\hat{\bar{\varepsilon}}$, $\beta > 0$
- Monotonic crack growth $\dot{l} > 0$

Critical load factor for onset and subsequent propagation ⇨ β_c

Moduli as a function of the macro-strain $\bar{C}\left(\beta\hat{\bar{\varepsilon}}\right)$

Corresponding macro-stress $\bar{\sigma} = \bar{C}\left(\beta_c\hat{\bar{\varepsilon}}\right)\beta_c\hat{\bar{\varepsilon}}$

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Microstructure evolution: crack initiation

Crack onset point: onset point at an interface or within a constituent

- **Maximum principal stress criterion**

$\max\{\sigma_1/\sigma_c\}=1$ Within a homogeneous constituent

- **Ye's quadratic criterion**

$\max\left\{\left(\frac{\langle\sigma\rangle}{\sigma_c}\right)^2+\left(\frac{\tau}{\tau_c}\right)^2\right\}=1$ At an interface

Hybrid stress-energy criterion (Leguillon, 2002) for crack onset using FFM

Extension for mixed-mode initiation

At a neighboring point: (unknows β_c θ_c l_c)

$$\begin{cases} f(\sigma(\beta_c,l_c,\theta_c))=1 \\ E(\beta_c,l_c,\theta_c)/E_c=1 \\ \frac{\partial}{\partial\theta}\left(\frac{E}{E_c}\right)\Big|_{\theta=\theta_c}=0,\quad \frac{\partial^2}{\partial\theta^2}\left(\frac{E}{E_c}\right)\Big|_{\theta=\theta_c}<0 \end{cases}$$

Stress criterion:

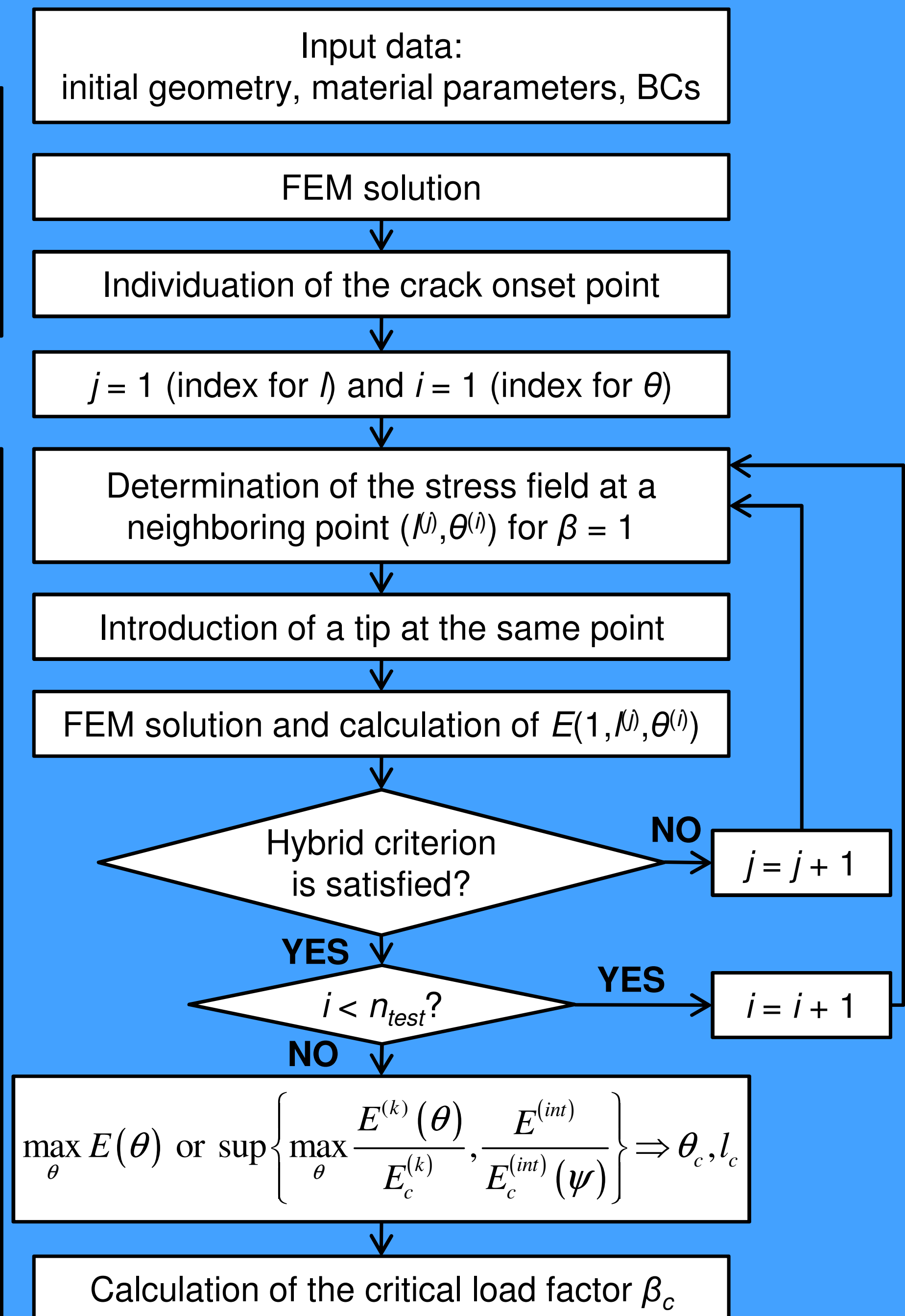
$$f(\sigma(\beta_c,l_c,\theta_c))=\begin{cases} \sigma_{\theta\theta}(\beta_c,l_c,\theta_c)/\sigma_c & \text{a)} \\ \left(\frac{\langle\sigma(\beta_c,l_c,\theta_c)\rangle}{\sigma_c}\right)^2+\left(\frac{\tau(\beta_c,l_c,\theta_c)}{\tau_c}\right)^2 & \text{b)} \end{cases}$$

Energy criterion (incremental Griffith's criterion)

Energy released for crack onset:

$$E(\lambda_c,l_c)=-\Delta\Pi(\lambda_c,l_c)=-\left[\Pi(\lambda_c,l_c)-\Pi(\lambda_c,0)\right]>0$$

Critical energy $E_c(l_c,\theta_c)=\begin{cases} G_c l_c(\theta_c) & \textbf{a)} \\ \int_0^{l_c} G_c(\psi(l))\,dl & \textbf{b) [Mantic, 2009]} \end{cases}$



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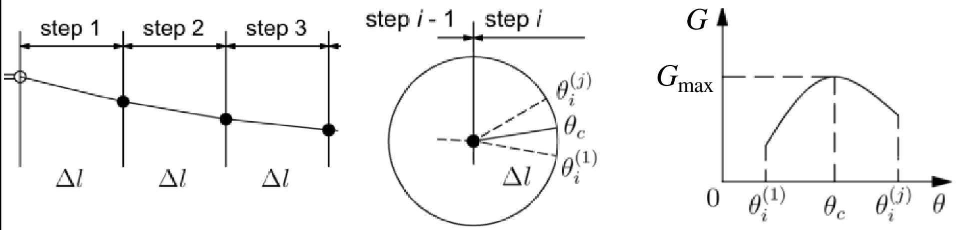
Microstructure evolution: crack propagation

Homogeneous constituent

Maximum energy release rate criterion (at a given l)

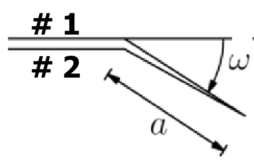
$$\begin{cases} \left.\dfrac{\partial G(\beta_c,l,\theta)}{\partial\theta}\right|_{\theta=\theta_c}=0, \quad \left.\dfrac{\partial^2 G(\beta_c,l,\theta)}{\partial\theta^2}\right|_{\theta=\theta_c}<0 \\ G(\beta_c,l,\theta_c)=G_c \end{cases} \quad \text{then } \dot{l}\geq 0$$



Heterogeneous material: interface and phases

Kinking out of the interface will be favored over interface cracking if

$$G^{(int)}/G_c^{(int)}(\psi)<G_{\max}^{(k)}/G_c^{(k)}$$



[Hutchinson and Suo, 1992]

Toughness function $\quad G_c(\psi)=G_{1c}\left\{1+\tan^2\left[(1-\lambda)\psi\right]\right\}$

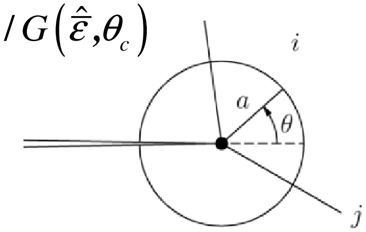
$$\tan\psi=\frac{K_2}{K_1}=\frac{\Im\left(Kl^{i\varepsilon}\right)}{\Re\left(Kl^{i\varepsilon}\right)}$$ measure of the relative amount of mode 2 to mode 1

λ (adjusts the influence of mode 2 contribution)

Critical load factor

$$G\left(\beta\hat{\bar{\varepsilon}},\theta_c\right)=\beta^2 G\left(\hat{\bar{\varepsilon}},\theta_c\right) \qquad \beta_c=\sqrt{G_c/G\left(\hat{\bar{\varepsilon}},\theta_c\right)}$$

$$\sup\left\{\max_{\varphi}\frac{G^{(k)}(\theta)}{G_c^{(k)}},\frac{G^{(int)}}{G_c^{(int)}(\psi)}\right\}\Rightarrow\theta_c$$



Input data: initial geometry, material parameters, BCs, initial crack, increment length

↓

Solution and calculation of $G(\beta=1,l_0,\theta_0)$

↓

Calculation of the critical load factor β_c

↓

$i=1$ (index for step) and $j=0$ (index for θ)

↓

Trial update of geometry: crack advance in direction $\theta^{(j)}$ ($l^{(i)}=l_0+i\cdot\Delta l$ and $\theta^{(j)}=\theta_0+j\cdot\Delta\theta$)

↓

Solution and calculation of $G(1,l^{(i)},\theta^{(j)})$

↓

$j<n_{test}$? — YES → $j=j+1$ (back to trial update) — NO ↓

$$\max_{\theta} G(\theta) \text{ or } \sup\left\{\max_{\theta}\frac{G^{(k)}(\theta)}{G_c^{(k)}},\frac{G^{(int)}}{G_c^{(int)}(\psi)}\right\}\Rightarrow\theta_c$$

↓

Calculation of the critical load factor β_c

↓

Update of geometry

↓

$i<n_{step}$? — YES → $i=i+1$ (back to trial update) — NO → Stop

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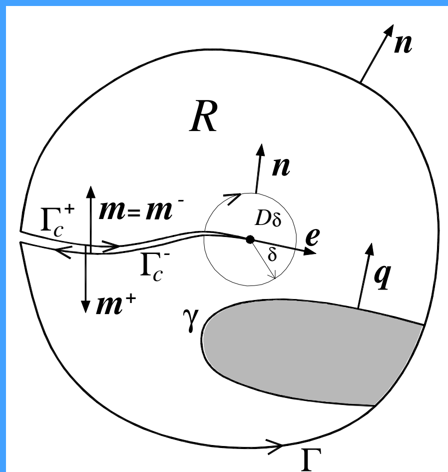
Microstructure evolution: *J*-integral formulation and SIFs

$G(\hat{\bar{\varepsilon}},l)$ evaluated by *J*-integral
- ***Homogeneous body***: $G = J$ for any path enclosing crack tip
- ***Inhomogeneous body***: $G = J$ for a path approaching crack tip



Path independence property: <u>very attractive</u> in FEM

Hyperelastic heterogeneities and contact influence the path independence property

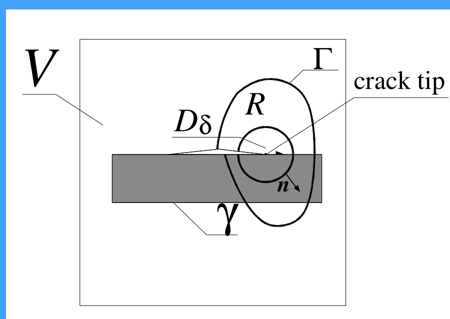
Transport theorem, traction continuity conditions at material interface γ: $[\![\boldsymbol{\sigma}]\!]\boldsymbol{n} = \boldsymbol{0}$

$$G(\bar{\boldsymbol{\varepsilon}},l) = \lim_{\delta\to 0} \boldsymbol{e}\bullet\int_{\partial D_\delta}\left(W\boldsymbol{n} - \nabla\boldsymbol{u}^T\boldsymbol{\sigma}\boldsymbol{n}\right)ds$$

Divergence theorem to *R* with material interface discontinuities
Curved crack and material interfaces not aligned with *e*

$$G(\bar{\boldsymbol{\varepsilon}},l) = \lim_{\delta\to 0} \boldsymbol{e}\bullet\left[\int_{\Gamma}\left(W\boldsymbol{n} - \nabla\boldsymbol{u}^T\boldsymbol{\sigma}\boldsymbol{n}\right)ds + \int_{\Gamma_c}[\![W\boldsymbol{1} - \nabla\boldsymbol{u}^T\boldsymbol{\sigma}]\!]\boldsymbol{m}\,ds + \int_{\gamma}[\![W\boldsymbol{1} - \nabla\boldsymbol{u}^T\boldsymbol{\sigma}]\!]\boldsymbol{q}\,ds\right]$$

Singular field integration ->0 near crack tip



Restricted path-independence property: straight cracks and material interfaces inside *R* aligned with *e*

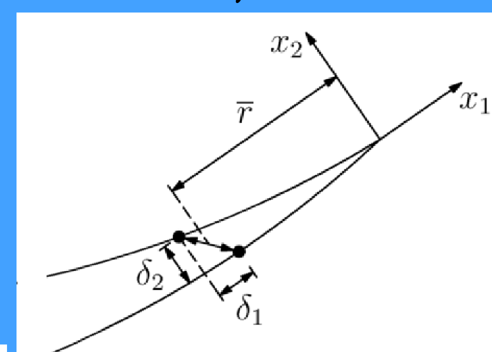
$$G(\bar{\boldsymbol{\varepsilon}},l) = J(\Gamma,\bar{\boldsymbol{\varepsilon}},l) = \boldsymbol{e}\bullet\int_{\Gamma}\left(W\boldsymbol{n} - \nabla\boldsymbol{u}^T\boldsymbol{\sigma}\boldsymbol{n}\right)ds$$

$$\int_{\Gamma_c}\left[[\![W\boldsymbol{1}]\!]\boldsymbol{m}\bullet\boldsymbol{e} - [\![(\nabla\boldsymbol{u}\,\boldsymbol{e})\bullet(\boldsymbol{\sigma}\boldsymbol{m})]\!]\right]ds + \int_{\gamma}\left[[\![W\boldsymbol{1}]\!]\boldsymbol{n}\bullet\boldsymbol{e} - [\![(\nabla\boldsymbol{u}\,\boldsymbol{e})\bullet(\boldsymbol{\sigma}\boldsymbol{n})]\!]\right]ds = 0$$

m*,*n perpendicular to ***e***; $\partial\boldsymbol{u}/\partial\boldsymbol{e}$ and ***t*** continuous across γ and Γ

Extraction of the mixed-mode SIFs from the *J*-integral

- Elimination of the singular terms in the CODs by choosing a special length ⟹ $\bar{l} = \bar{r}\big/e^{\varepsilon^{-1}\tan^{-1}(2\varepsilon)}$
- Accurate evaluation of the ratio of the SIFs ⟹ $\bar{K}_2/\bar{K}_1 = \delta_1/\delta_2$
- Explicit formulas for the component separation method ⟹ $\begin{Bmatrix}\bar{K}_1\\ \bar{K}_2\end{Bmatrix} = \begin{Bmatrix}\delta_2\\ \delta_1\end{Bmatrix}\sqrt{\dfrac{JE_*\big/(1-\beta^2)}{\delta_1^2+\delta_2^2}}$
- Transformation rule to obtain SIFs with the reference length *l* ⟹ $\begin{Bmatrix}K_1\\ K_2\end{Bmatrix} = \begin{bmatrix}\cos\omega & -\sin\omega\\ \sin\omega & \cos\omega\end{bmatrix}\begin{Bmatrix}\bar{K}_1\\ \bar{K}_2\end{Bmatrix}$



$\omega = \varepsilon\ln\left(l/\bar{l}\right)$

Interface model and micro-to-macro transition

Unilateral frictionless contact bewteen crack faces: interface model

$$\boldsymbol{t} = \boldsymbol{k}[\![\boldsymbol{u}]\!]$$

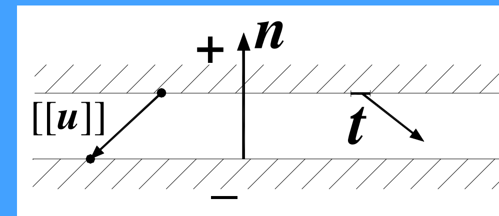
Traction vector $\boldsymbol{t} = \{t_n, t_t\}$

Displacement jump vector $[\![\boldsymbol{u}]\!] = \{[\![u_n]\!], [\![u_t]\!]\}$

Interface stiffness: *k* penalty parameter

$$\boldsymbol{k} = diag\{k_n, k_t\}$$

$$k_t = 0,\ k_n = \frac{k}{2}\left(1 - sign[\![u_n]\!]\right)$$



Continuation strategy: solution for the previous *k* as initial guess

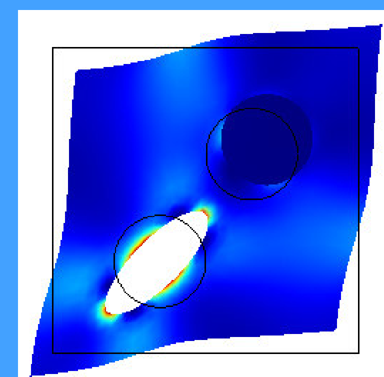
Micro-to-macro transition

Displacement type FE approximation

FE model developed by using COMSOL MULTIPHYSICS™

Boundary conditions of homogenization by point-wise constraints

$$\boldsymbol{u}(\boldsymbol{x}^+) = \boldsymbol{u}(\boldsymbol{x}^-) + \bar{\boldsymbol{\varepsilon}}(\boldsymbol{x}^+ - \boldsymbol{x}^-)$$



Periodic fluctuations: *extrusion coupling variables* methodology

On the destination boundary $\Longrightarrow$ $\boldsymbol{u}(\boldsymbol{x}^-)$ **Extruded displacement from ∂V^-**

Accurate stresses evaluation: point constraints -> weak constraints via Lagrangian multiplier λ

$$-\frac{1}{|V|}\int_{\partial V^+} \boldsymbol{\lambda}\cdot\left([\![\boldsymbol{u}]\!] - \bar{\boldsymbol{\varepsilon}}[\![\boldsymbol{x}]\!]\right) dA = 0$$

• Non-linear constitutive law reproduced by a program in Matlab

Crack extension simulated by a remeshing technique with discrete variation of *l*

Post-processing of FE solution:

- **Macroscopic variables by integration coupling variables**
- ***J*-integral: integration coupling variable**
- **Numerical determination of homogenized moduli**

$$\bar{C}_{ijhk}\left(\beta\hat{\bar{\varepsilon}}, l\right) = \frac{\partial\bar{\sigma}_{ij}}{\partial\bar{\varepsilon}_{hk}}\left(\beta\hat{\bar{\varepsilon}}, l\right) \simeq \frac{\bar{\sigma}_{ij}\left(\beta\hat{\bar{\varepsilon}} + \boldsymbol{e}_h \otimes \boldsymbol{e}_k \Delta\bar{\varepsilon}_{hk}\right) - \bar{\sigma}_{ij}\left(\beta\hat{\bar{\varepsilon}} - \boldsymbol{e}_h \otimes \boldsymbol{e}_k \Delta\bar{\varepsilon}_{hk}\right)}{2\Delta\bar{\varepsilon}_{hk}}$$

In practice: reference to the unstressed configuration with bilateral constraint along the actual contact zone determined solving the RVE driven by $\hat{\bar{\varepsilon}}$ (moduli dependence on the macro-strain direction)

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Numerical examples: homogenized constitutive laws

2D micro-structures:

h **side of the RVE**

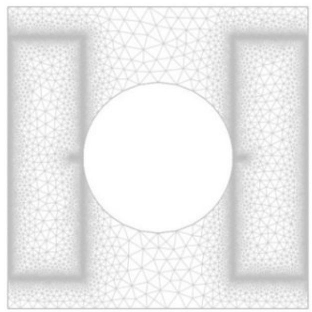
plane-strain conditions

initial crack length:

a) l_0/h=0.02; b) $2l_0/h=0.04$ or no initial crack

⇨

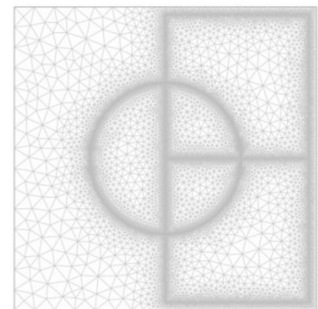
a) Cellular material with micro-cracks spreading from the void



36,816 elements

h = 1.0 mm
d = 0.5h,
E_m = 30 GPa,
ν_m = 0.17
G_c = 100 J/m^2

b) Continuous fiber-reinforced composite material with interface debonding



30,284 elements

h = 30 μm
d_f = 0.5h
E_m = 2.79 GPa
ν_m = 0.33
E_f = 70.8 GPa
ν_f = 0.22
σ_c = 90 Mpa
τ_c = 120 Mpa
G_{1c} = 2 J/m^2 λ=0,3
G_c = 100 J/m^2

FE discretization:

unstructured mesh of quadratic triangular elements, mesh refinement along J-integral contours

Macro-strain directions

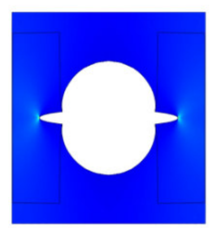
Periodic fluctuations;

No crack face overlap $\hat{\varepsilon}_2^{+}\left(\hat{\varepsilon}_1^{+}\right)$

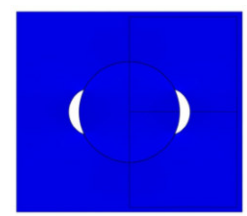
Contact occurs for $\hat{\varepsilon}_2^{-}\left(\hat{\varepsilon}_1^{-}\right)$ crack completely closed

b) Multiple Contact for $\hat{\varepsilon}_S^{\pm}$

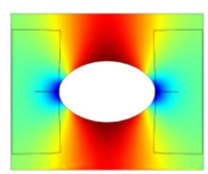
Symmetric J except in shear for case b) with an initial defect



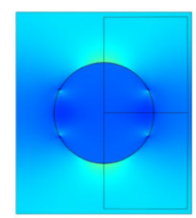
$\hat{\varepsilon}_2^{+}=e_2\otimes e_2$



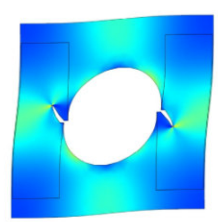
$\hat{\varepsilon}_1^{+}=e_1\otimes e_1$



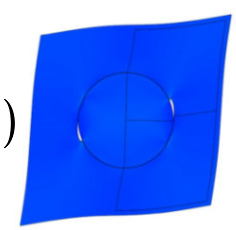
$\hat{\varepsilon}_2^{-}=-e_2\otimes e_2$



$\hat{\varepsilon}_1^{-}=-e_1\otimes e_1$



$\hat{\varepsilon}_S^{\pm}=\pm 1/\sqrt{2}\,(e_1\otimes_S e_2)$

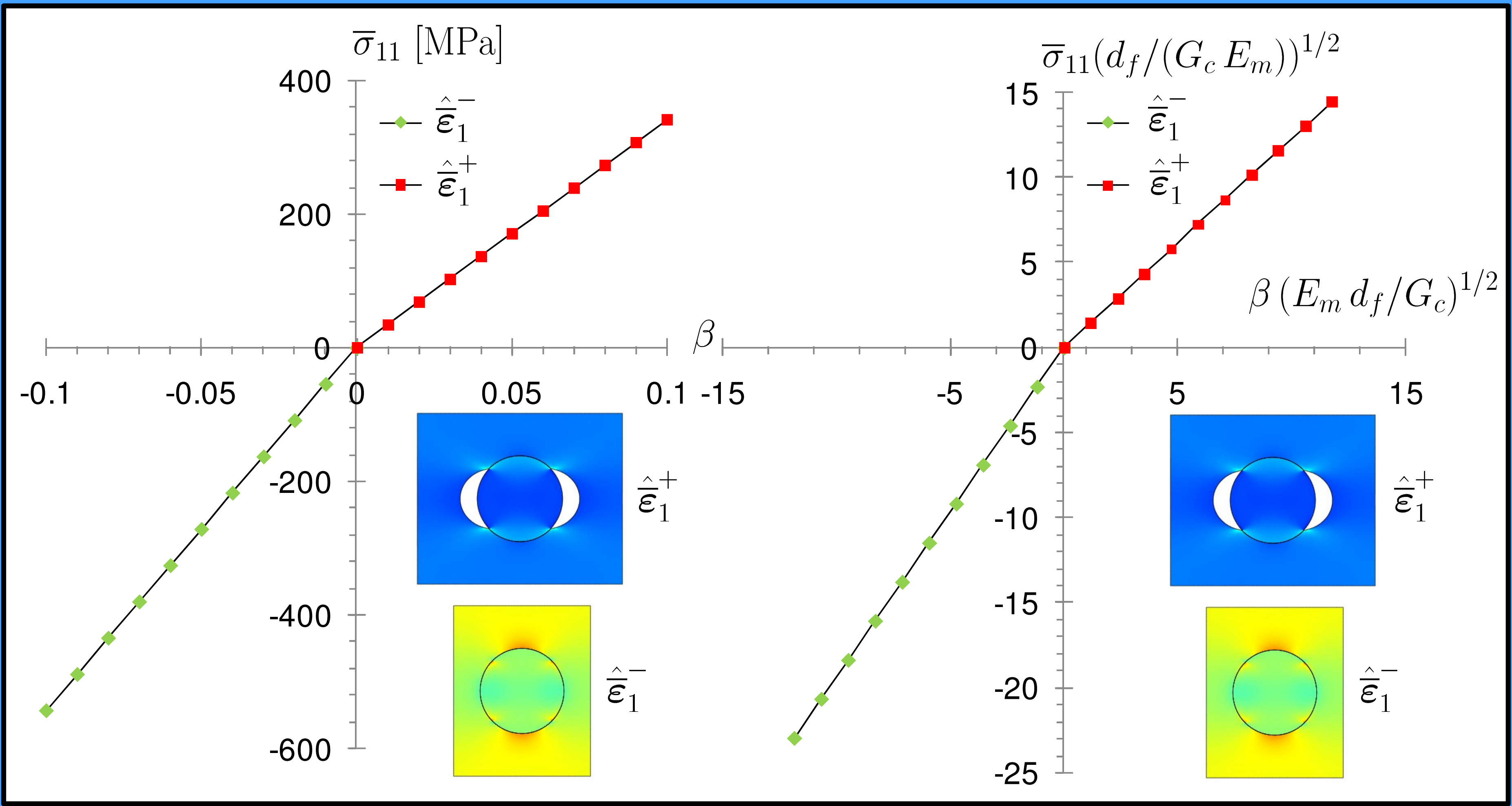


$\hat{\varepsilon}_S^{\pm}=\pm 1/\sqrt{2}\,(e_1\otimes_S e_2)$

Numerical examples: homogenized constitutive laws

Homogenized constitutive law at a fixed crack length

Uniaxial macrostrain path $\overline{\varepsilon}_1^{\pm} = \beta \hat{\overline{\varepsilon}}_1^{\pm}$



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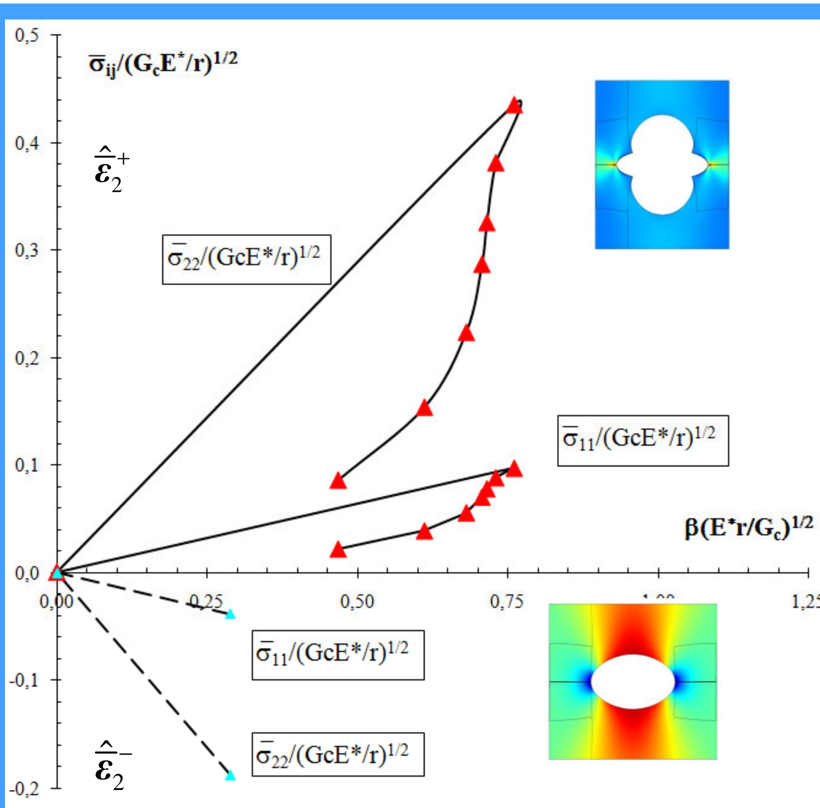
Uniaxial mode case a)



Dimensionless $\bar{\sigma}_{ij}\sqrt{l_c/(G_c E_m)}$ **vs** $\beta\sqrt{E_m l_c / G_c}$

Dimensionless $G/(E_m l_c)$

- Initial behavior characterized by $\bar{C}(l_0)$

$\bar{\varepsilon}$ **control**

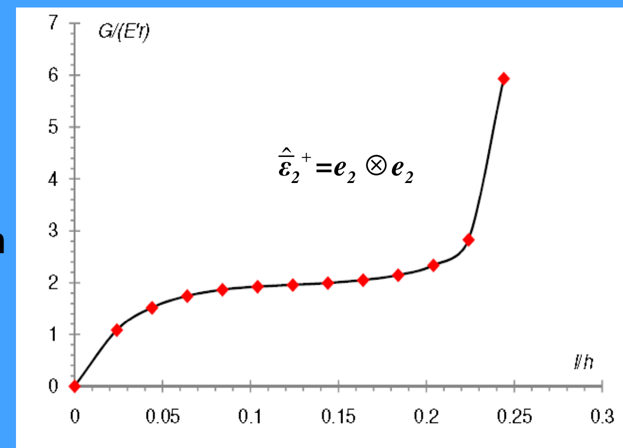
$G\left(\hat{\bar{\varepsilon}}_2^+, l\right)$

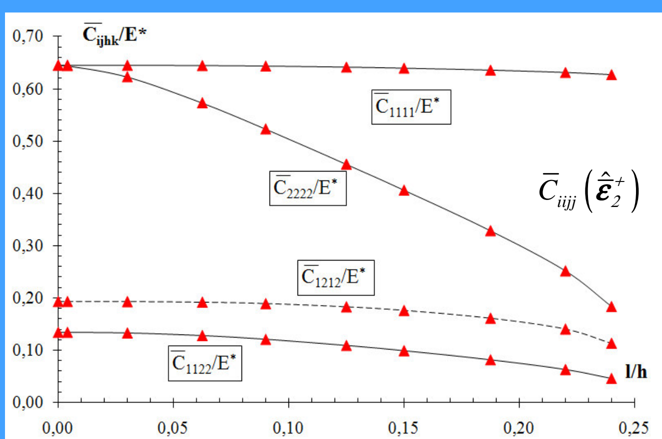
Increasing function of l
unstable propagation

- Snap-backs

$G\left(\hat{\bar{\varepsilon}}_2^-, l\right) = 0$

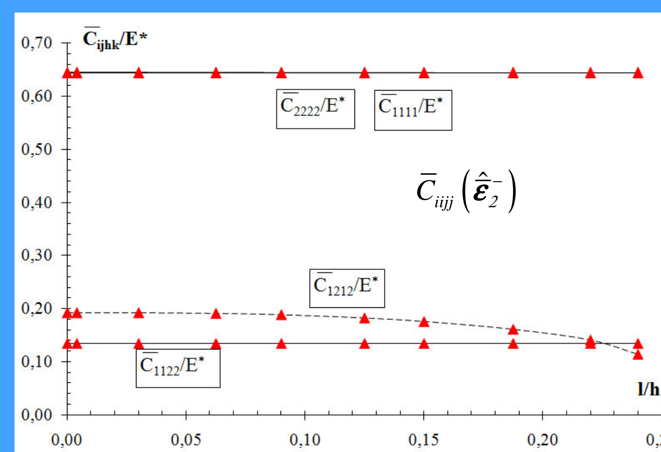
Crack **does not propagate**





$\bar{C}_{12jj} \simeq 0$

- Moduli dependent on l for extension mode
- Moduli independent on l for compression mode due to contact
- Largest loss in stiffness for $\bar{C}_{2222}$



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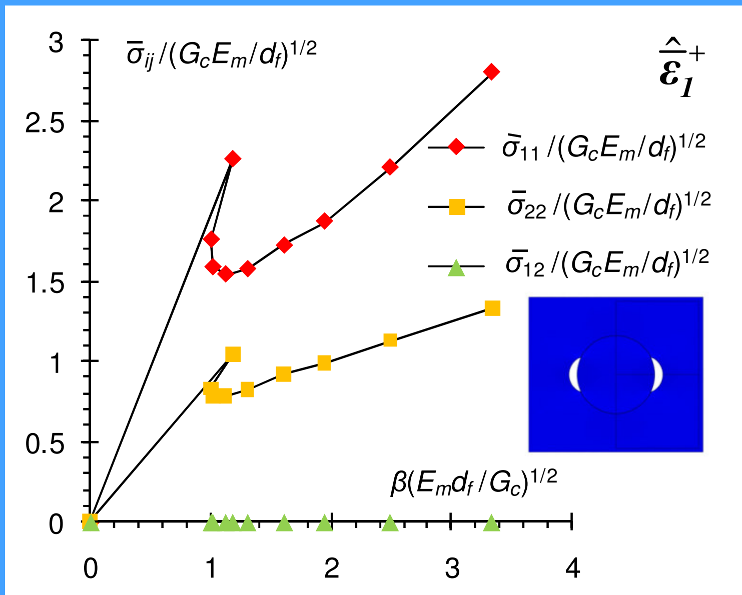
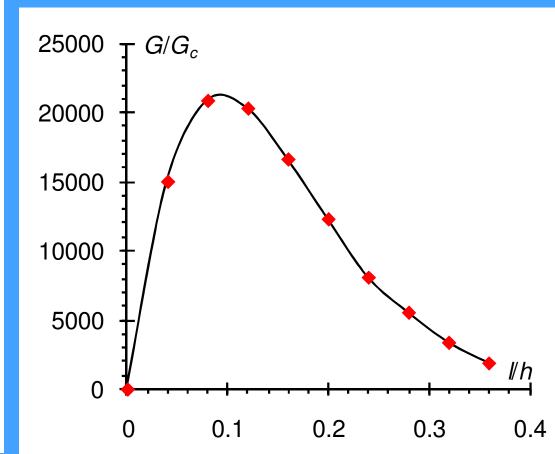
Uniaxial mode case b) with initial debonding

Dimensionless $\bar{\sigma}_{ij}\sqrt{l_c/(G_c E_m)}$ **vs** $\beta\sqrt{E_m l_c/G_c}$



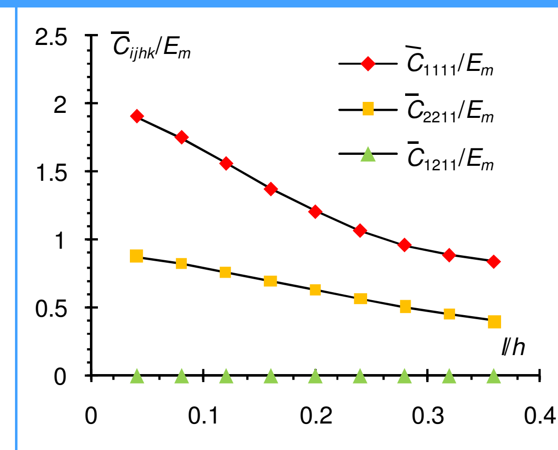
Energy release rate: $G/G_c(\psi)$

- snap-back and snap-through
- unstable/stable propagation

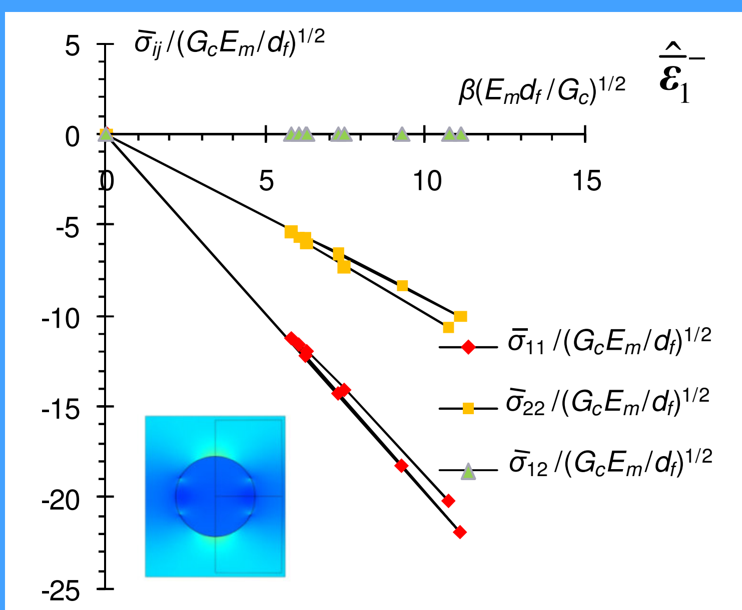


Moduli: $\bar{C}_{ijhk}/E_m$

- initial behavior characterized by $\bar{C}(l_0)$
- dependence of moduli on l
- orthotropic behavior

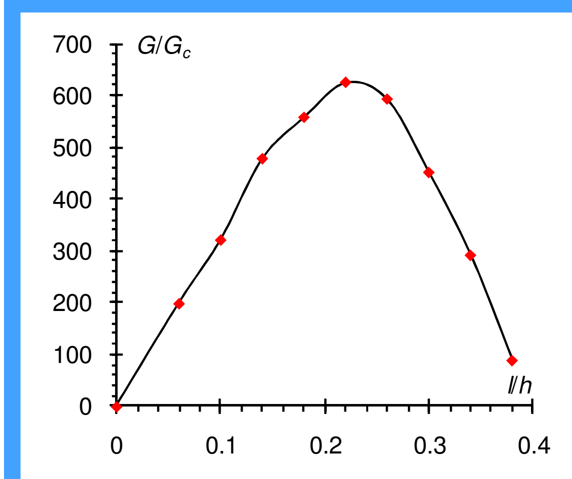


Dimensionless $\bar{\sigma}_{ij}\sqrt{l_c/(G_c E_m)}$ **vs** $\beta\sqrt{E_m l_c/G_c}$



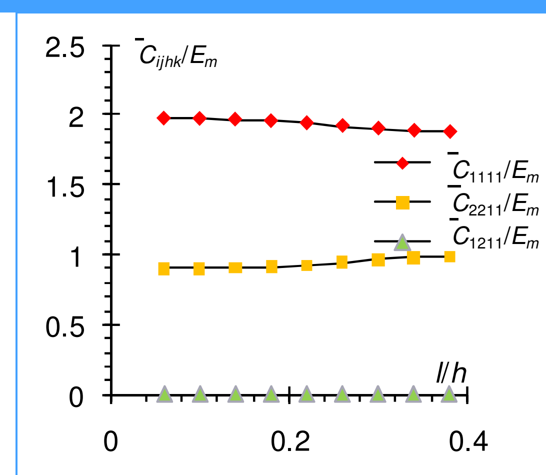
Energy release rate: $G/G_c(\psi)$

- snap-back and snap-through
- unstable/stable propagation



Moduli: $\bar{C}_{ijhk}/E_m$

- initial behavior characterized by $\bar{C}(l_0)$
- orthotropic behavior
- scarce dependence of moduli on l



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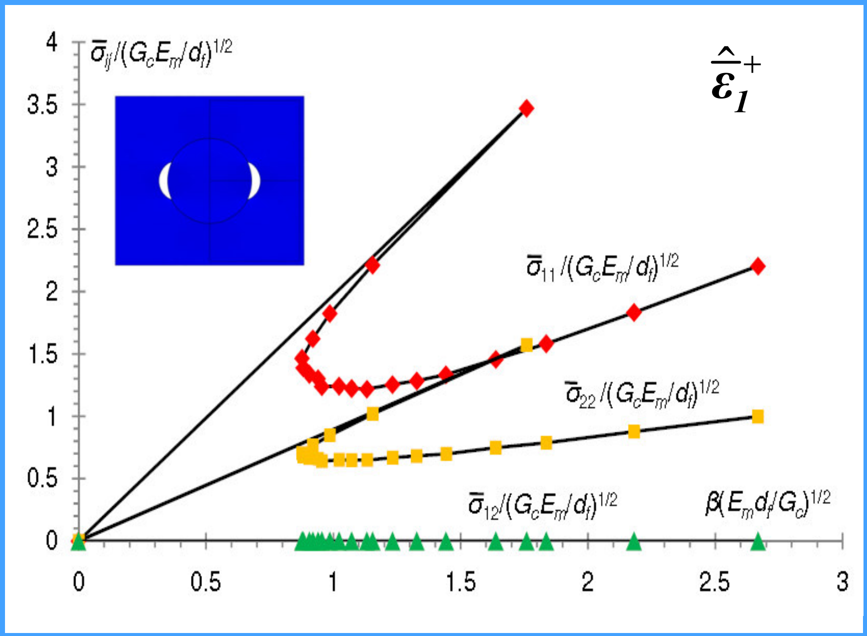
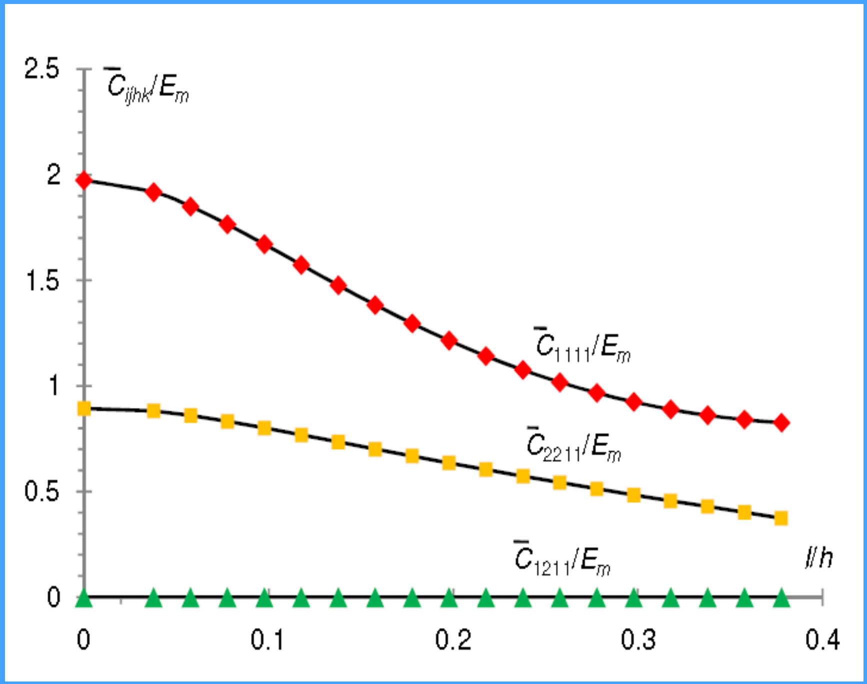
Uniaxial mode case b) no initial debonding



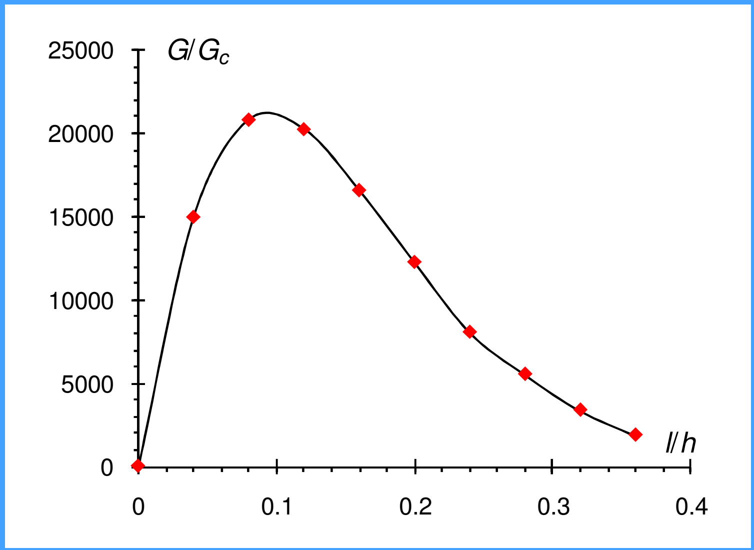


Dimensionless $\bar{\sigma}_{ij}\sqrt{l_c/(G_cE_m)}$ **vs** $\beta\sqrt{E_ml_c/G_c}$

Increase of strength

Energy release rate: $G/G_c(\psi)$

- snap-back and snap-through
- unstable/stable propagation



Moduli: $\bar{C}_{ijhk}/E_m$

- initial behavior characterized by $\bar{C}(l_0=0)$
- dependence of moduli on *l*
- orthotropic behavior

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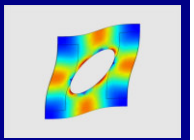
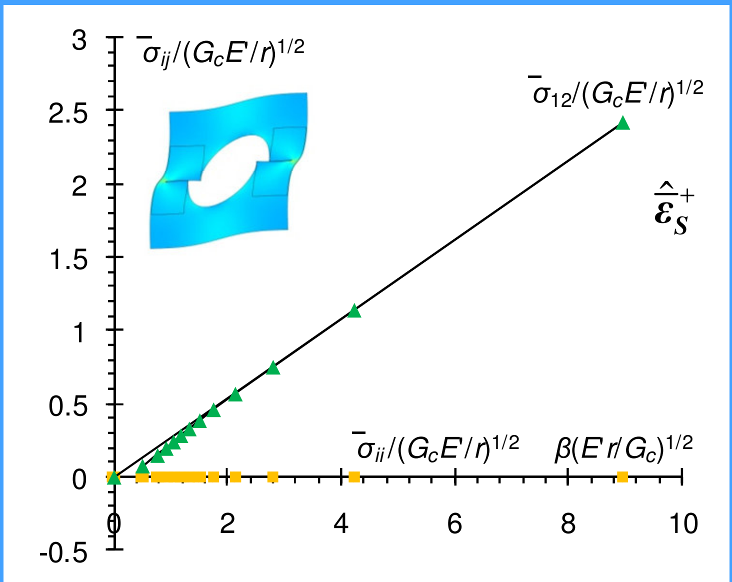
Shear mode case a)



• Prescribed crack path

Dimensionless $\bar{\sigma}_{ij}\sqrt{l_c/(G_c E_m)}$ **vs** $\beta\sqrt{E_m l_c/G_c}$



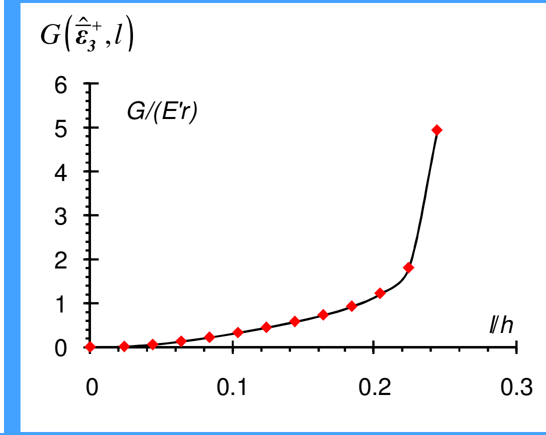
Energy release rate: $G/(E_m l_c)$

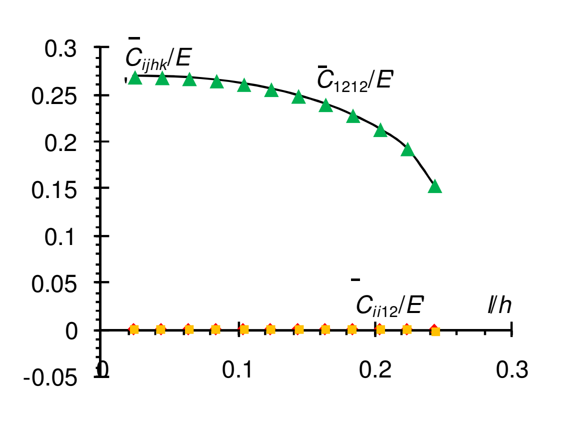
- severe snap-back
- unstable propagation

Moduli: $\bar{C}_{ijhk}/E_m$

- initial behavior characterized by $\bar{C}(l_0)$
- dependence of shear modulus on l

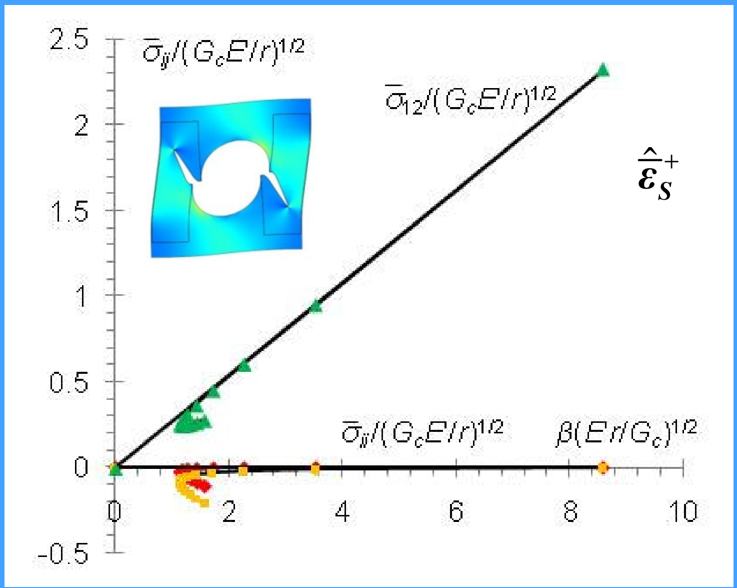
$\bar{\varepsilon}$ control





• Crack path prediction by MERR criterion: stabilizing effect

Dimensionless $\bar{\sigma}_{ij}\sqrt{l_c/(G_c E_m)}$ **vs** $\beta\sqrt{E_m l_c/G_c}$

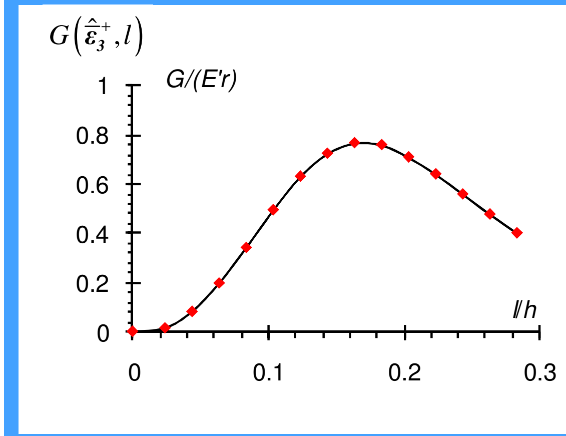


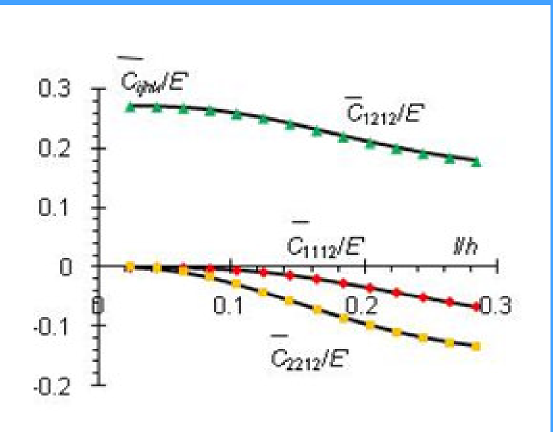
Energy release rate: $G/(E_m l_c)$

- severe snap-back and small snap-through
- unstable/stable propagation

Moduli: $\bar{C}_{ijhk}/E_m$

- dependence of moduli on l
- **loss of orthotropy**





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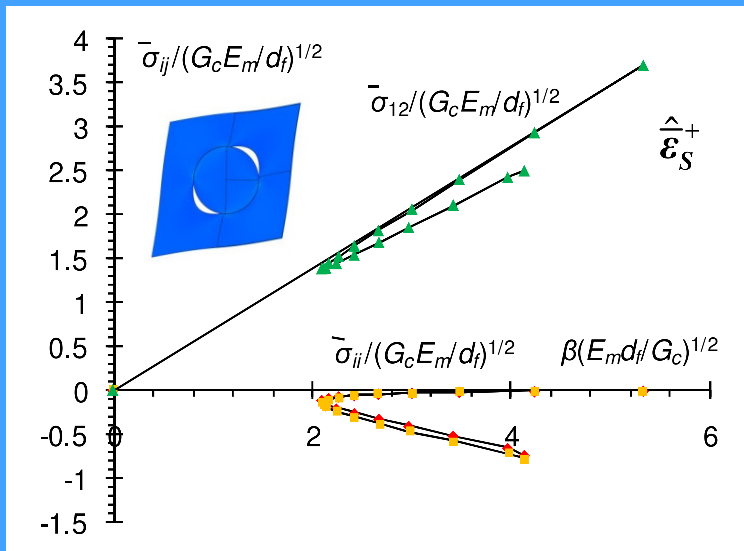
Shear mode case b) with and without an initial debonding

1) Formulation
2) Crack onset and propagation
3) Computational implementation
4) Numerical examples
5) Conclusions and discussion

- **Prescribed initial damage as interface crack:** unsymmetric growth

Dimensionless $\bar{\sigma}_{ij}\sqrt{l_c/(G_c E_m)}$ **vs** $\beta\sqrt{E_m l_c/G_c}$

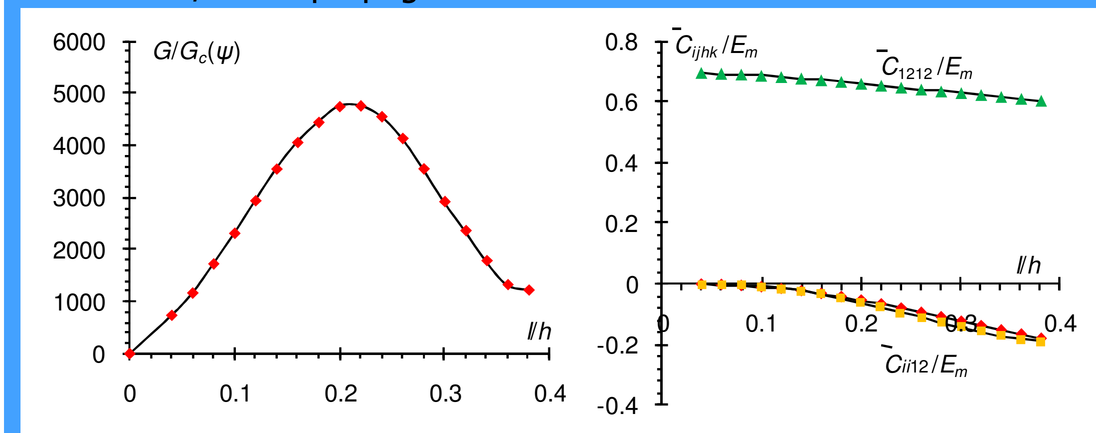


Energy release rate: $G/G_c(\psi)$

- severe snap-back and snap-through
- unstable/stable propagation

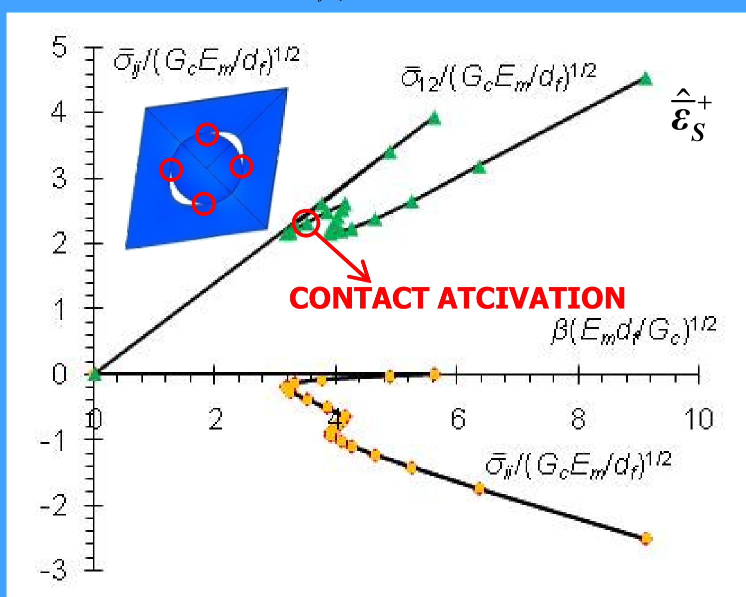
Moduli: $\bar{C}_{ijhk}/E_m$

- initial behavior characterized by $\bar{C}(l_0)$
- moderate dependence of moduli on l



- **Crack onset and propagation analysis: symmetric growth**

Dimensionless $\bar{\sigma}_{ij}\sqrt{l_c/(G_c E_m)}$ **vs** $\beta\sqrt{E_m l_c/G_c}$

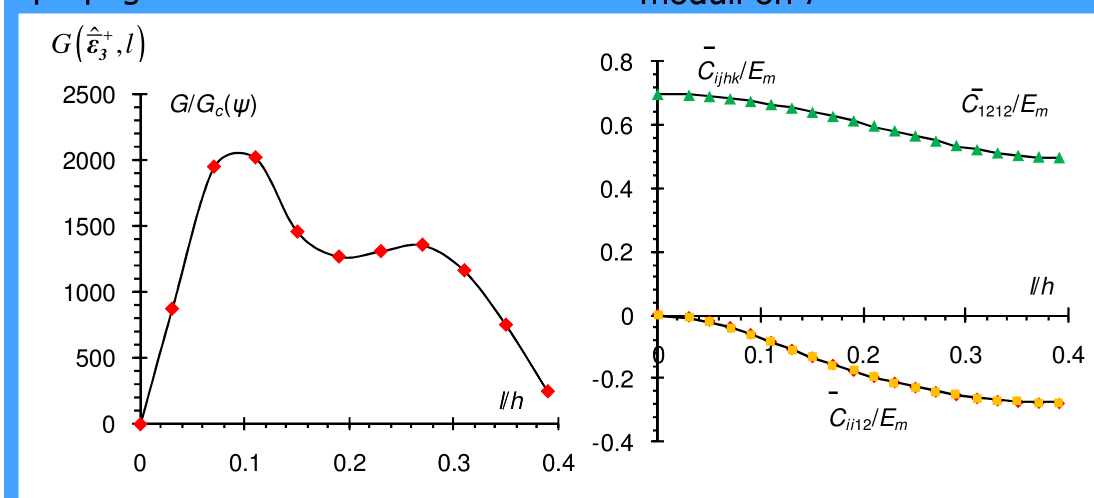


Energy release rate: $G/G_c(\psi)$

- double snap-back and snap-through
- unstable/stable/unstable/stable propagation

Moduli: $\bar{C}_{ijhk}/E_m$

- onset study: initial behavior characterized by $\bar{C}(l_0=0)$
- stronger dependence of moduli on l



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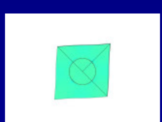
Shear mode case b): competition between kinking and debonding

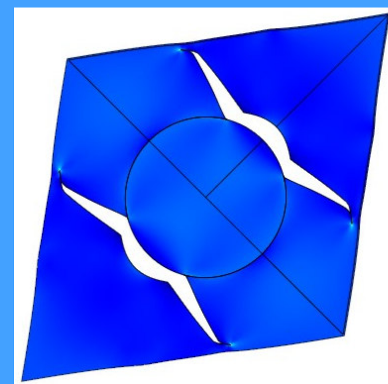


Influence of interfacial toughness on macroscopic constitutive laws: competition between crack advance within the interface and kinking out of the interface

Epoxy matrix (G_c = 100 J/m²)
Weak interface (G_{1c} = 2 J/m²)

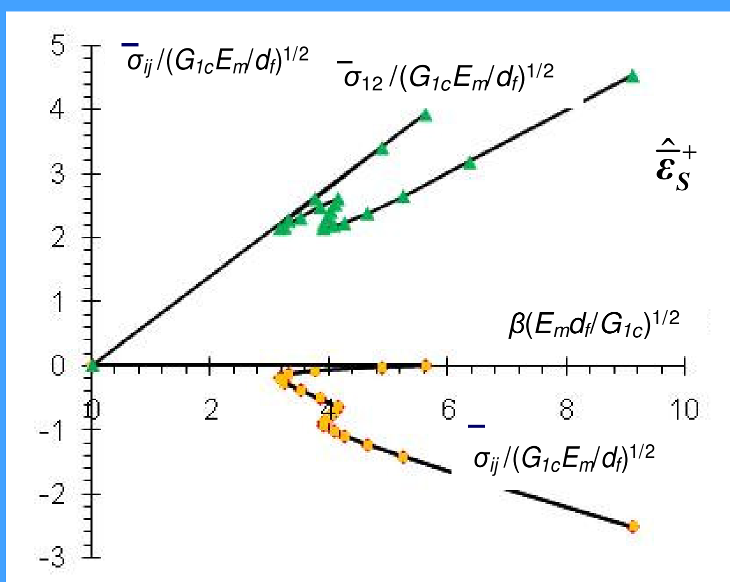
- initial behavior characterized by $\bar{C}(l_0 = 0)$
- crack advance within the interface involving contact
- double snap-back and snap-through
- dependence of moduli on *l*

Epoxy matrix (G_c = 100 J/m²)
Strong interface (G_{1c} = 20 J/m²)

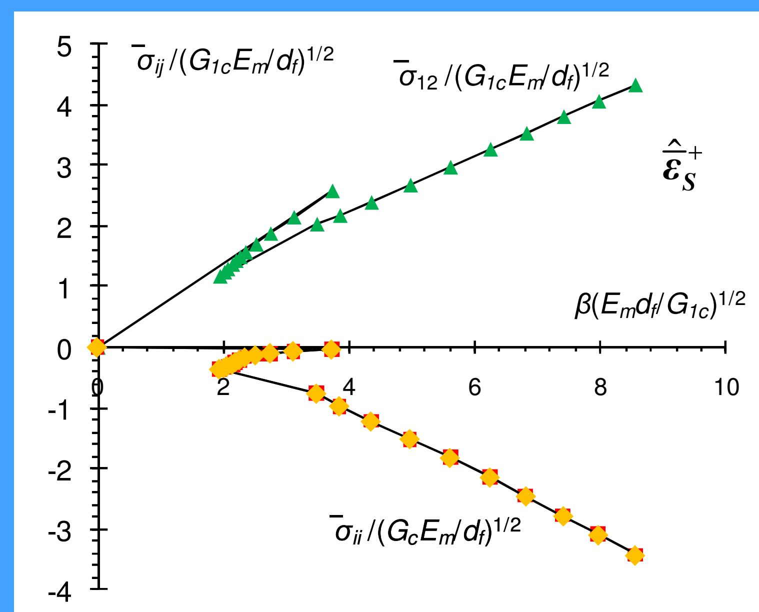


- same initial behavior characterized by $\bar{C}(l_0 = 0)$
- kinking out of the interface
- snap-back and snap-through
- Increase of strength

Dimensionless $\bar{\sigma}_{ij}\sqrt{l_c/(G_c E_m)}$ **vs** $\beta\sqrt{E_m l_c / G_c}$



Dimensionless $\bar{\sigma}_{ij}\sqrt{l_c/(G_c E_m)}$ **vs** $\beta\sqrt{E_m l_c / G_c}$

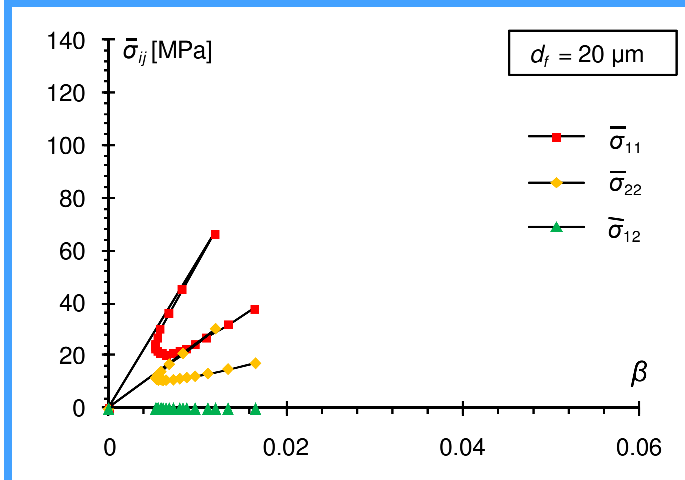


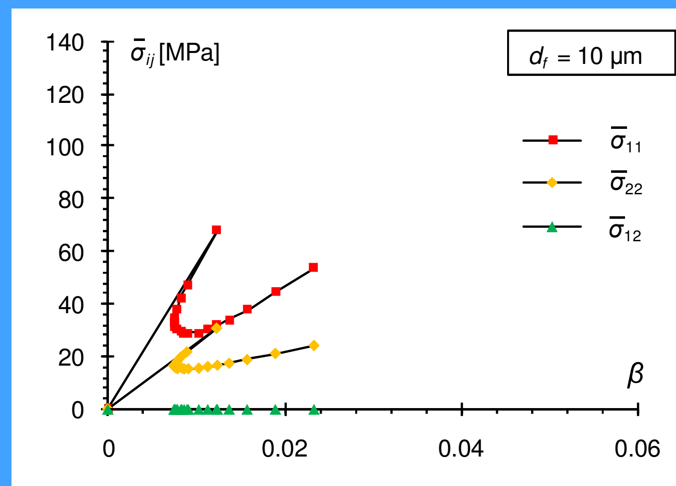
Uniaxial mode case b): influence of fiber diameter

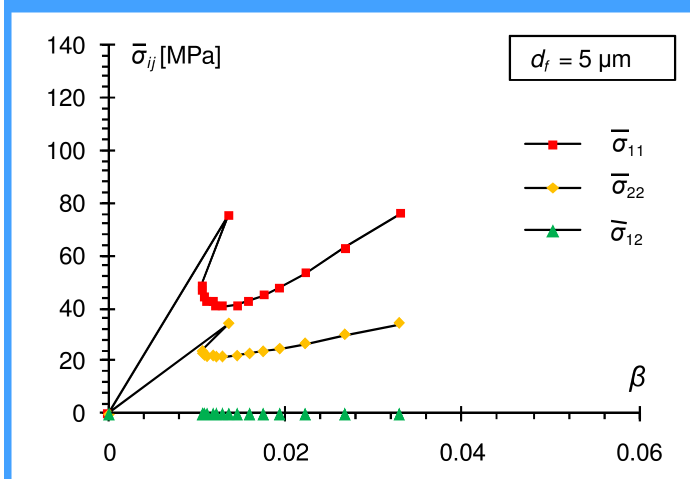
Influence of RVE size on macroscopic constitutive laws

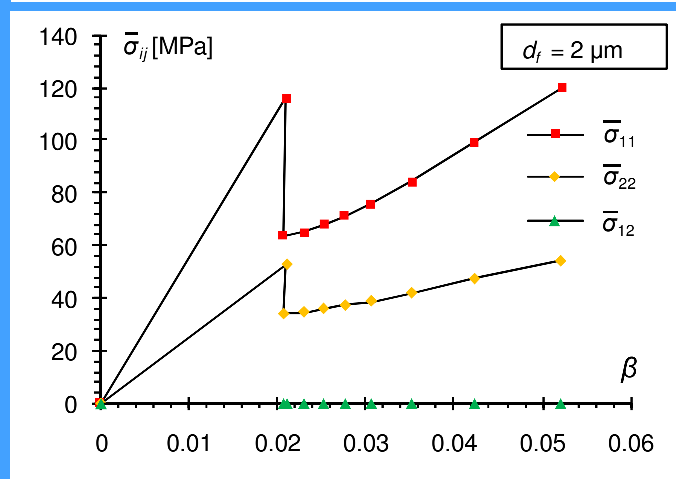
- The critical load factor at onset is increasing for decreasing and small d_f
- The dimensionless crack semilength l_d/h at onset increases for decreasing and small d_f
- Transition from snap-back instability in case of large fiber diameters to a stable mechanical response for smaller reinforcements

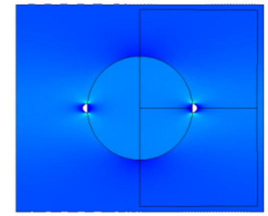
Leguillon's criterion -> *size effects* in composites



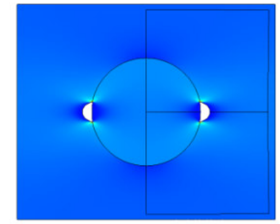




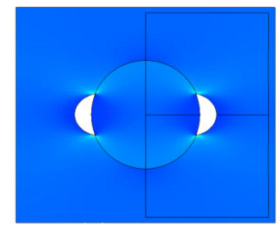




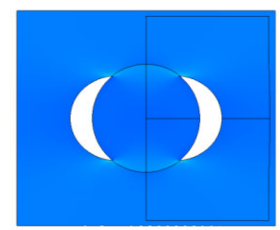
d_f = 20 μm



d_f = 10 μm



d_f = 5 μm



d_f = 2 μm

d_f = 20 μm ⇨ SEVERE SNAP-BACK

d_f = 2 μm ⇨ NO SNAP-BACK

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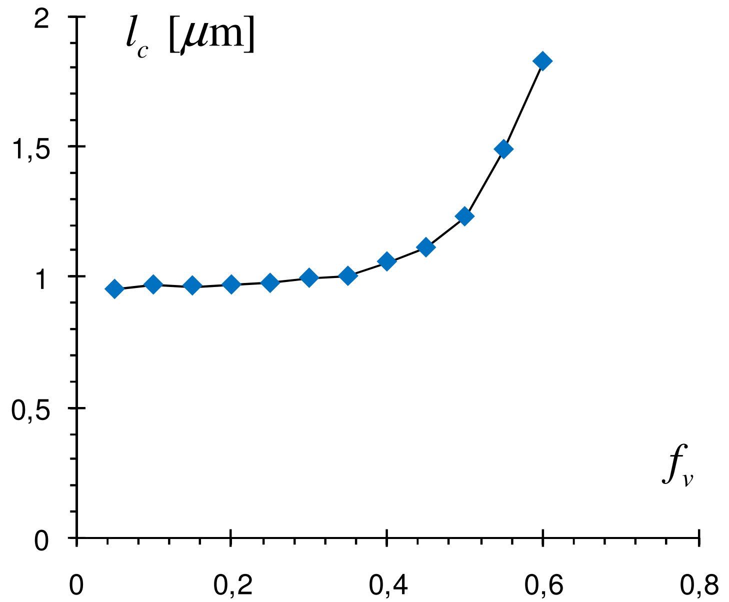
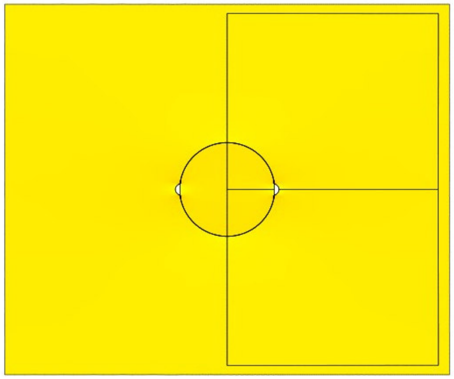
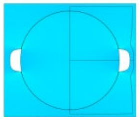
Uniaxial mode case b): influence of volume fraction

Influence of <u>fiber volume fraction</u> on macroscopic constitutive laws



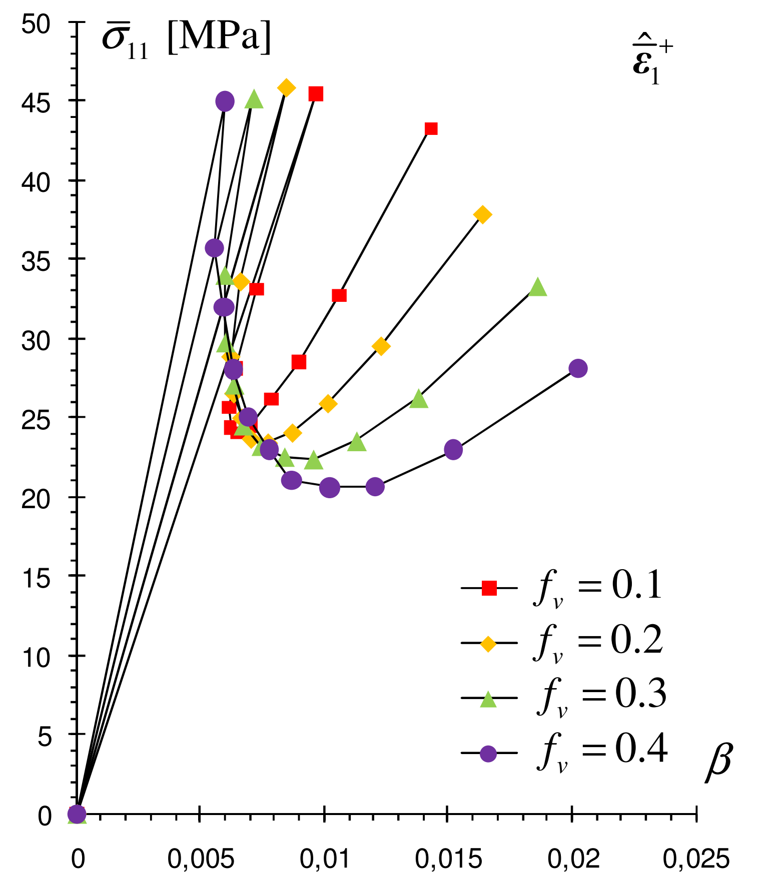


$f_v = 0.05$
$d_f = 15$ μm
$h_{RVE} = 59.4$ μm



$f_v = 0.6$
$d_f = 15$ μm
$h_{RVE} = 17.2$ μm

- The crack semilength l_c at onset increases for increasing f_v
- Transition from snap-back instability in case of small volume fraction to a stable mechanical response for larger volume fractions (f_v > 40%)
- Loss in stiffness is larger for increasing fiber volume fraction



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Conclusions and discussion

A notable problems in the context of homogenization theory of elastic composite materials: Study of the **effects of mixed mode micro-cracking onset and evolution** on macroscopic properties including contact ;

Proposed micromechanical model provides constitutive laws for a composite micro-structure taking into account for **micro-cracking and contact** by using FE method and interface models; A **novel *J*-integral formulation** is incorporated; **Crack onset and curved crack propagation** is modelled;

Analyzed 2D microstructures: a) **porous material** with edge cracks and b) **short fiber reinforced** material with a damaged or undamaged fiber/matrix interface;

Uniaxial and shear macro-strain paths: damage evolution and contact give rise to a **strong non-linearity** in the macroscopic response (severe snap back and snap through); **Notable dependence** on the macro-strain path;

Influence of the type of simulation (self similar crack propagation, crack curving) on macroscopic constitutive law (orthotropy), Stabilizing effects of general crack growth;

Competition between interface debonding and kinking -> favorable effect of increasing interface toughness;

Prediction of size effect due to Hybrid criterion for crack onset (fiber volume fraction, RVE size)