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Glued solids: a coupled predictive theory

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Motivations:

Reproduce different failure modes;

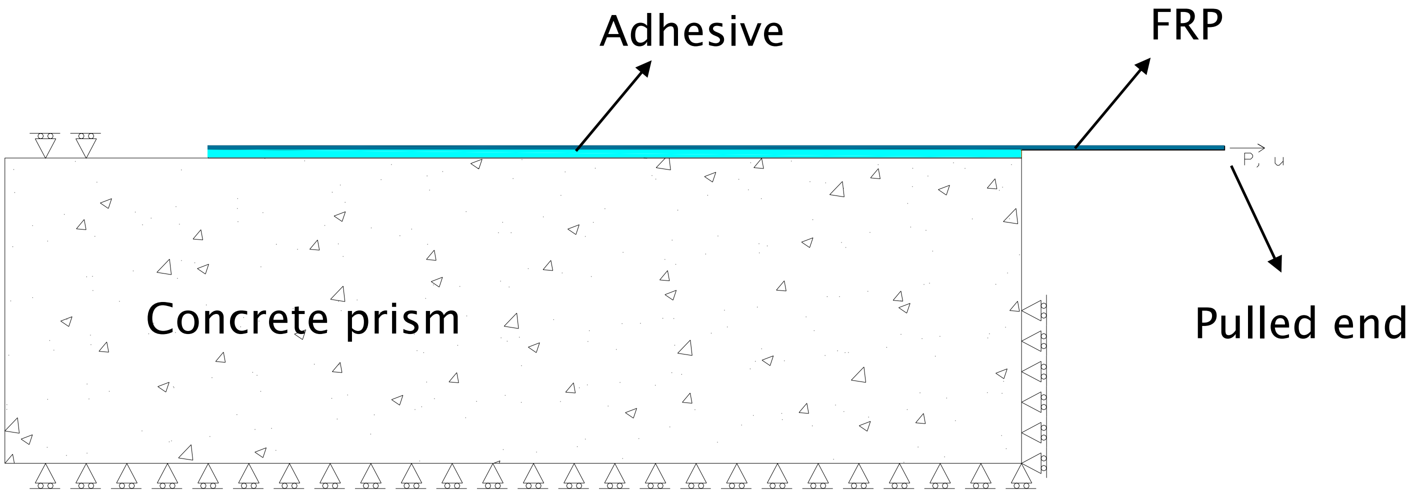
- In domains
- Between domains and adhesive layer
- In the adhesive layer
- In domains and interface

Application on FRP-Concrete debonding tests:

- Modelling debonding
- Influence of boundary conditions
- Preliminary results on accelerated ageing tests

Experimental pull tests

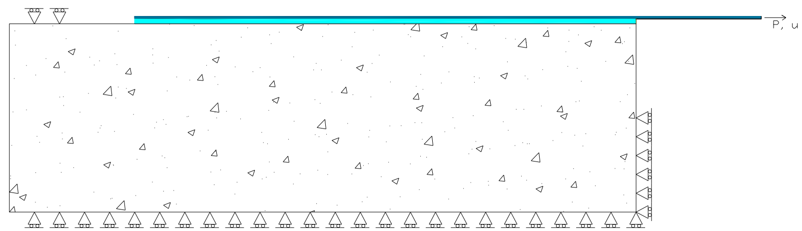
General information



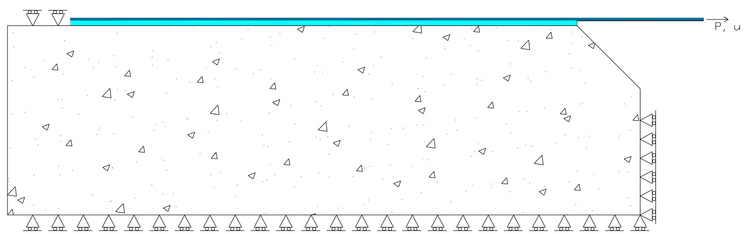
Experimental pull tests

General information

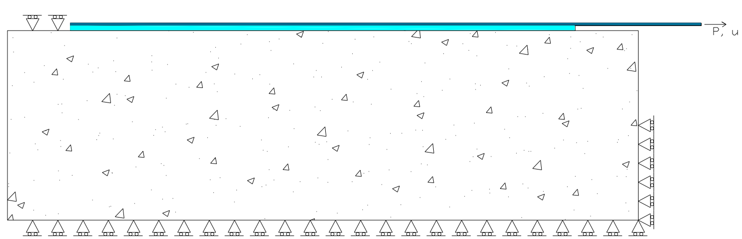
Test A



Test B



Test C



Anchorage Lenght 240 mm

Plate width 50 mm

Concrete: Dimension: 410*100*210 mm^3

E=32000 MPa

Mean value of tensile strenght = 2.47 MPa

Mean value of compressive strengh = 25.7 MPa

FRP: Thickness: 1.2 mm

E=165000 MPa

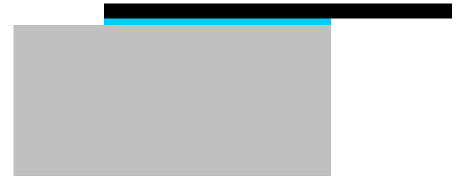
Maximum tensile stress = 3100 MPa

Adhesive: Thickness: 2 mm

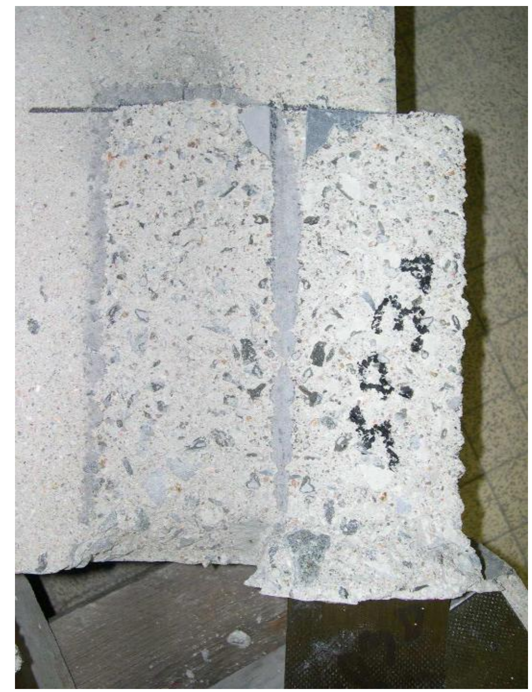
E=12800 MPa

Maximum tensile stress = 30 MPa

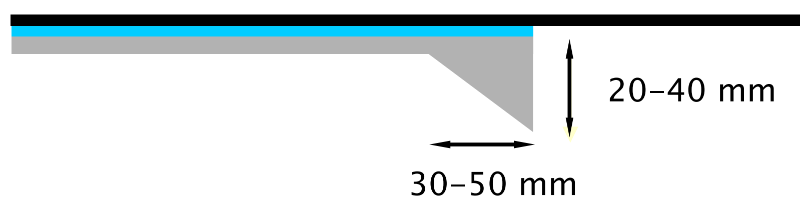
Experimental pull tests



Fracture Mode: Test A

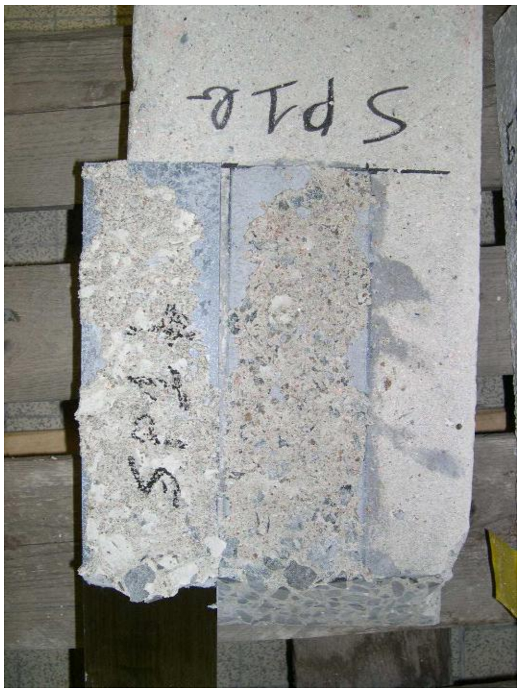


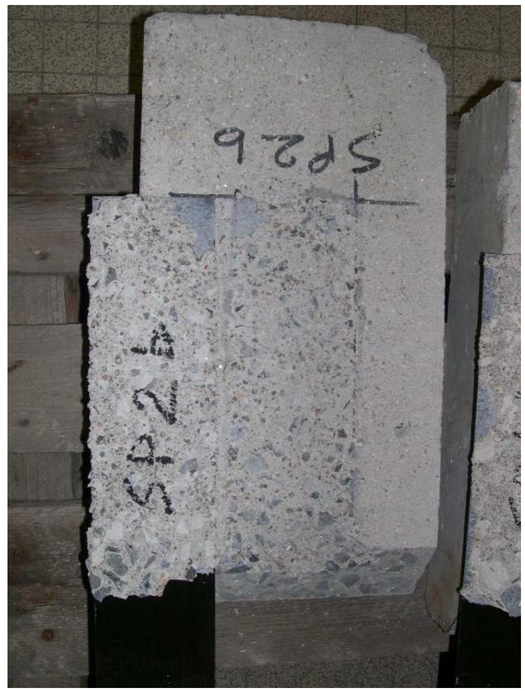
Wedge failure of concrete:



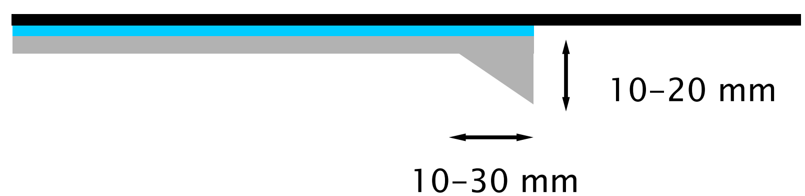
Experimental pull tests

Fracture Mode: Test B





Wedge failure of concrete:



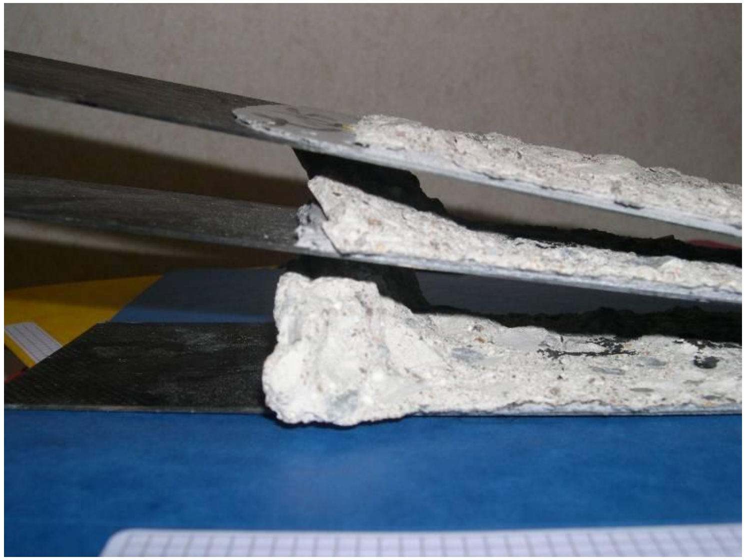
Experimental pull tests

Fracture Mode: Test C

No wedge

Experimental pull tests

Fracture Mode



Accelerated ageing tests

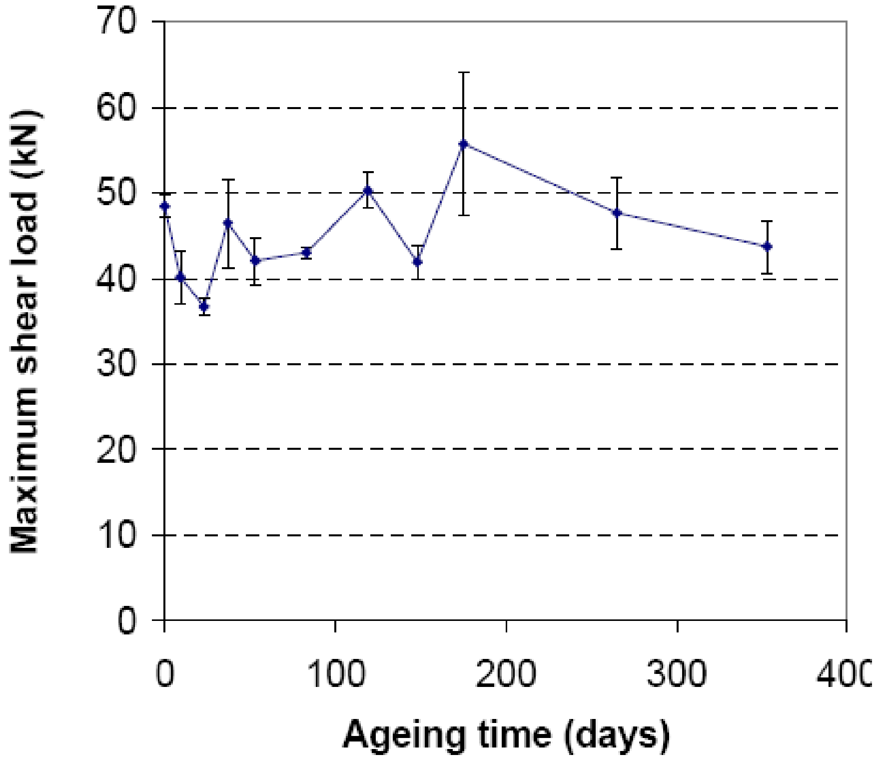
All specimens were exposed to saturated humidity in a climatic chamber (relative humidity of 95 %, at least), at a temperature of 40°C for several months.

Change in the failure mode

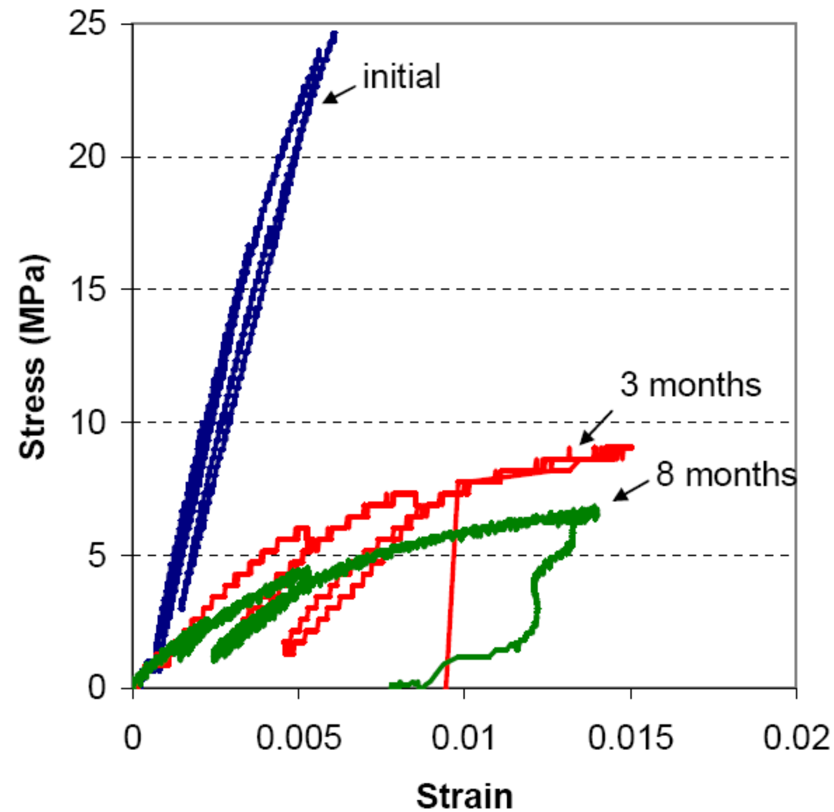
BEFORE AGEING

AFTER AGEING

Maximum transmitted force is almost constant



Decrease in the adhesive bond strength



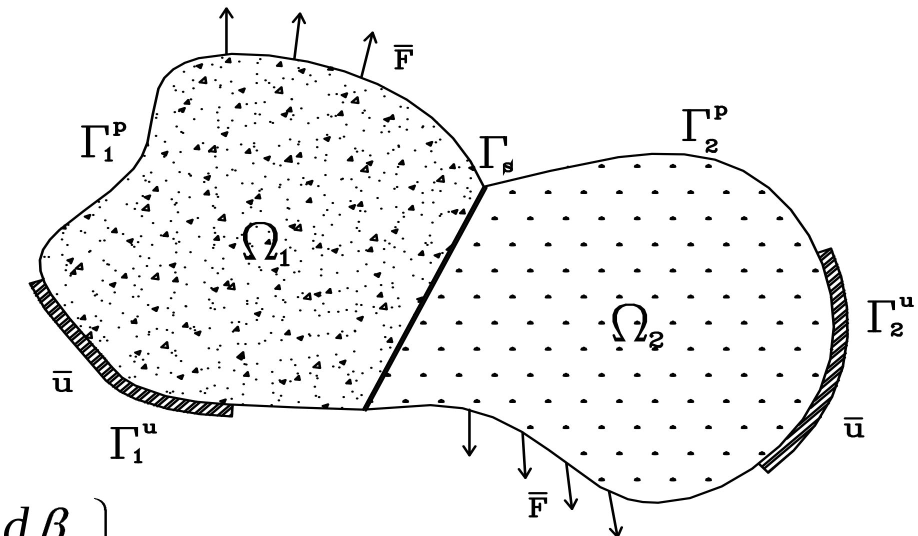
Problem Formulation

State quantities: Freddi & Fremond [2006]

Domains

$$E_i = \varepsilon_i, \beta_i, grad\,\beta_i$$

$$\delta E_i = \left\{ \frac{d\varepsilon_i}{dt}, \frac{d\beta_i}{dt}, grad\,\frac{d\beta_i}{dt} \right\}$$



Interface

$$E_s = u_2 - u_1, \beta_s, grad_s\,\beta_s, \beta_1, \beta_2$$

$$\delta E_s = \left\{ \frac{du_2}{dt} - \frac{du_1}{dt}, \frac{d\beta_s}{dt}, grad_s\,\frac{d\beta_s}{dt}, \frac{d\beta_1}{dt}, \frac{d\beta_2}{dt} \right\}$$

Differences with respect to the original model Freddi & Fremond [2006]

No non-local term

Different constitutive laws

Problem Formulation

Principle of virtual power

Virtual power of the internal, exterior and acceleration forces

$$P_{\text{int}} = -\int_{\Omega_i} \sigma_i : \frac{d\varepsilon_i}{dt} - \int_{\Omega_i} B_i \frac{d\beta_i}{dt} + H_i \cdot grad \; \frac{d\beta_i}{dt} \, d\Omega_i$$

$$- \int_{\partial\Omega_1 \cap \partial\Omega_2} \vec{R} \;\; U_2 - U_1 \;\; d\Gamma_s - \int_{\partial\Omega_1 \cap \partial\Omega_2} B_s \frac{d\beta_s}{dt} + H_s \cdot grad \; \frac{d\beta_s}{dt} + B_{i,s}\left(\frac{d\beta_i}{dt} - \frac{d\beta_s}{dt}\right) d\Gamma_s$$

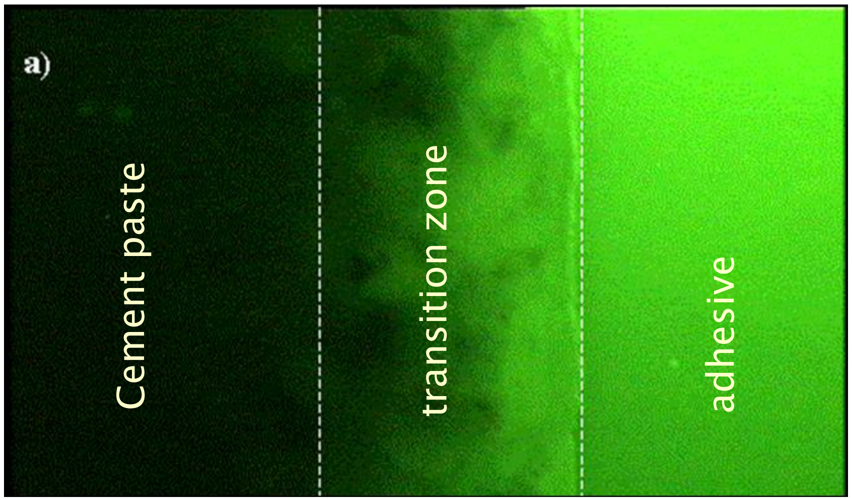
B: internal work of damage **H**: flux vector of internal work of damage

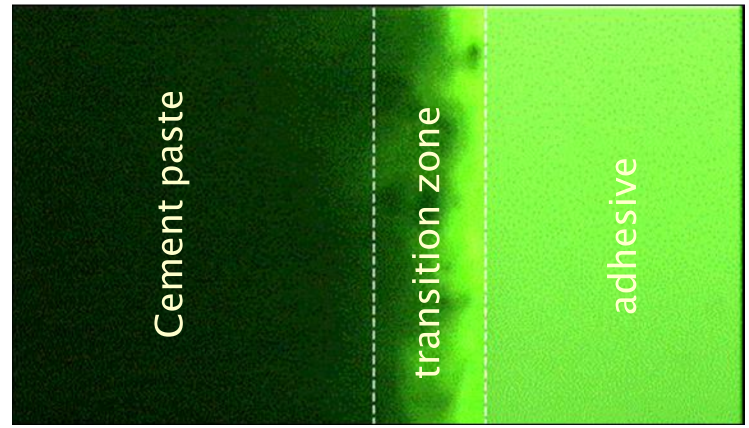
$$P_{\text{ext}} = -\int_{\Omega_i} f_i U_i d\Omega_i - \int_{\partial\Omega_i \setminus \partial\Omega_1 \cap \partial\Omega_2} g_i U_i d\Gamma_i$$

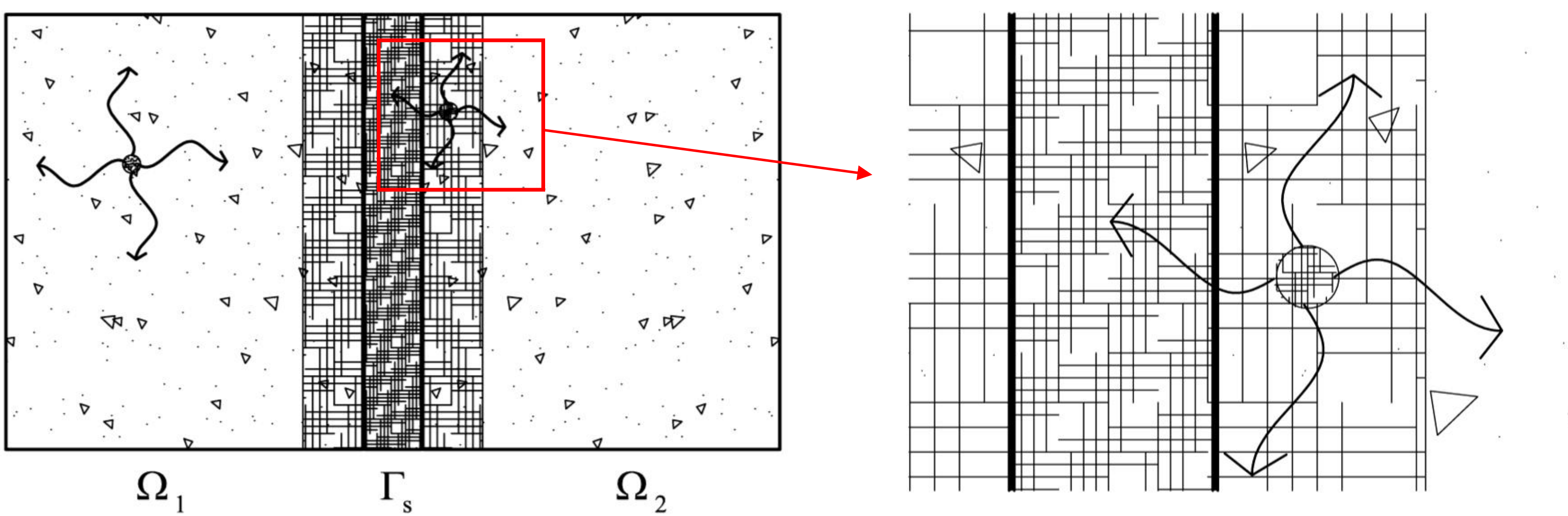
$$P_{\text{acc}}\left(U, \frac{d\beta}{dt}\right) = P_{\text{ext}}\left(U, \frac{d\beta}{dt}\right) + P_{\text{int}}\left(U, \frac{d\beta}{dt}\right)$$

Problem Formulation

Phenomena: Interaction between the concrete and the glue

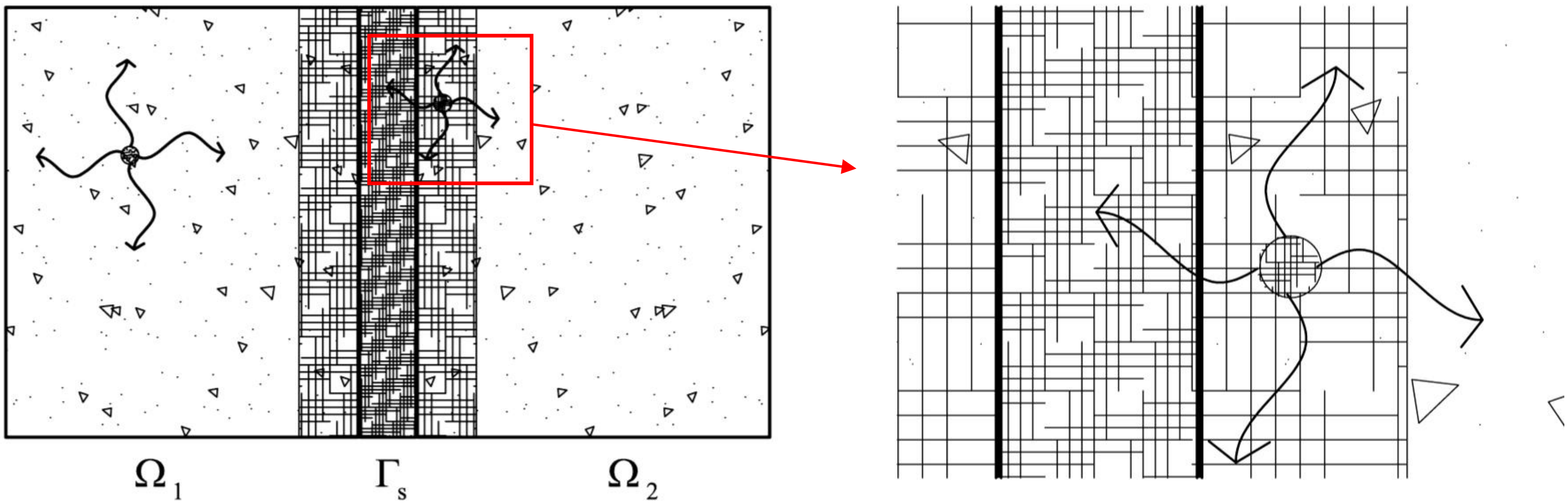






Problem Formulation

Local interaction



$\Rightarrow$ $\beta_s - \beta_i$

- is a boundary source for each domain
- is a source for the interface

Formulation

Constitutive laws

Domains

$$\Psi_i\left(\boldsymbol{\varepsilon}_i, \beta_i, grad\,\beta_i\right) = \frac{1}{2}\beta_i \boldsymbol{\varepsilon}_i : \mathbf{C}_i : \boldsymbol{\varepsilon}_i + \frac{k_i}{2}\left|grad\,\beta_i\right|^2,$$

$$\Phi_i\left(\boldsymbol{\varepsilon}_i, \dot\beta_i;\right) = \frac{c}{2}\dot\beta_i^2 + \dot\beta_i\left(-\frac{1}{2}\boldsymbol{\varepsilon}_i : \mathbf{C}_i : \boldsymbol{\varepsilon}_i + \beta_i F\left(\boldsymbol{\varepsilon}_i\right)\right) + I_-\left(\dot\beta_i\right)$$

Interface

$$\Psi_s\left(\mathbf{u}_2 - \mathbf{u}_1, \beta_s, grad\,\beta_s, \beta_1, \beta_2\right) = \frac{1}{2}\beta_s \hat{k}_s\left|\mathbf{u}_2 - \mathbf{u}_1\right|^2 + \frac{k_s}{2}\left|grad\,\beta_s\right|^2 + I_-\left(\left(\mathbf{u}_2 - \mathbf{u}_1\right)\cdot \mathbf{N}_2\right) +$$

$$+ \frac{k_{s,1}}{2}\left(\beta_1 - \beta_s\right)^2 + \frac{k_{s,2}}{2}\left(\beta_2 - \beta_s\right)^2$$

$$\Phi_s\left(\mathbf{u}_2 - \mathbf{u}_1, \dot\beta_s;\right) = \frac{c_s}{2}\dot\beta_s^2 + \dot\beta_s\left(-\frac{1}{2}\hat{k}_s\left|\mathbf{u}_2 - \mathbf{u}_1\right|^2 + \beta_s G\left(\mathbf{u}_2 - \mathbf{u}_1\right)\right) + I_-\left(\dot\beta_s\right)$$

Internal constraints

$\beta_{i,s} \leq 0,$ Irreversibility of damage quantities

$\left(\mathbf{u}_2 - \mathbf{u}_1\right)\cdot \mathbf{N}_2 \leq 0$ Impenetrability condition

Problem Formulation

Equations

Mechanical equation + damage equations

Domains equations

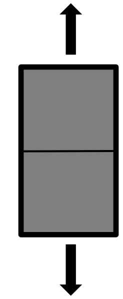
$$\begin{cases} c_i \dot{\beta}_i - k_i \Delta \beta_i + \partial I \quad \dot{\beta}_i \ni \beta_i F \quad \boldsymbol{\varepsilon}_i \\ k_i \dfrac{\partial \beta_i}{\partial n_i} - k_{s,i} \quad \beta_s - \beta_i \quad = 0 \\ \beta_i \quad \mathbf{x}, 0 \quad = \beta_{i,0} \quad \mathbf{x} \end{cases}$$

Flux damage contributions

Interface equation

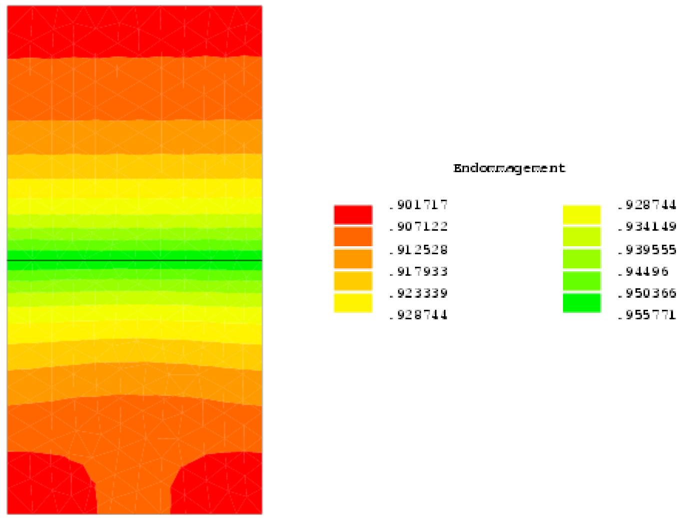
$$\begin{cases} c_s \dot{\beta}_s - k_s \Delta \beta_s + \partial I \quad \dot{\beta}_s \ni \beta_s G \quad \mathbf{u}_1 - \mathbf{u}_2 \quad - k_{s,1} \quad \beta_s - \beta_1 \quad - k_{s,2} \quad \beta_s - \beta_2 \\ k_s \dfrac{\partial \beta_s}{\partial n_s} = 0 \quad , \quad \beta_s \quad \mathbf{x}, 0 \quad = \beta_{s,0} \quad \mathbf{x} \end{cases}$$

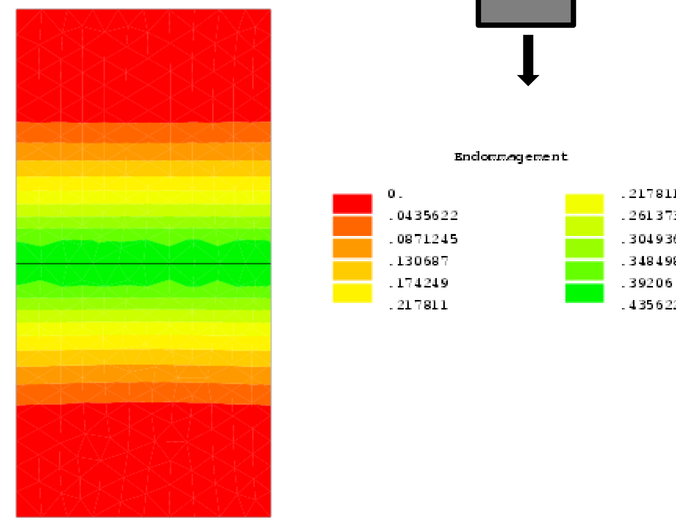
Effect of damage interaction



Weak interaction

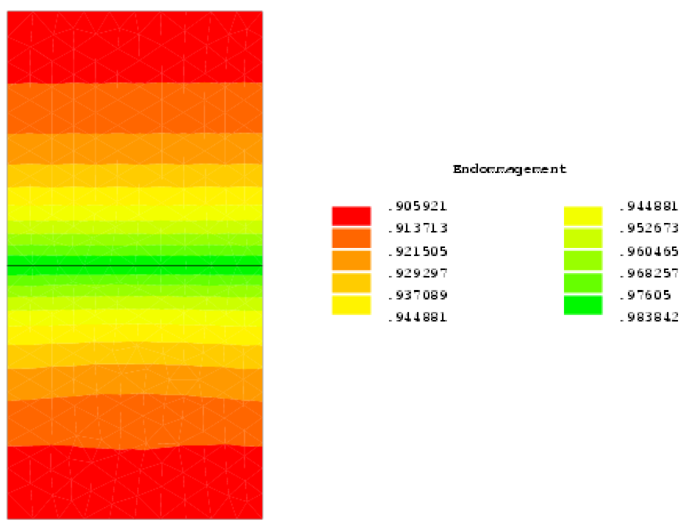
$$k_{s,1} = k_{s,2} = 0.05$$

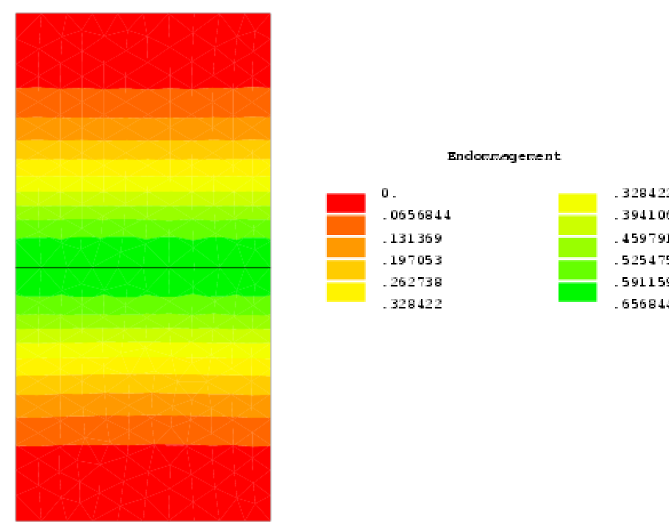




Strong interaction

$$k_{s,1} = k_{s,2} = 0.2$$





Problem Formulation

Adopted Model: the simplest case

Rate-independent

Domain equations

$$\cancel{c_i \dot{\beta}_i} - k_i \Delta\beta_i + \partial I \;\; \dot{\beta}_i \ni \beta_i F \;\; \boldsymbol{\varepsilon}_i$$

k_i Extension parameter

$$F \;\; \boldsymbol{\varepsilon}_c \;\; = \frac{w_c}{\beta_c} - \frac{1}{2}\boldsymbol{\varepsilon}_c^+ : \mathbf{C} : \boldsymbol{\varepsilon}_c^+$$

Concrete

FRP

$$F \;\; \boldsymbol{\varepsilon}_p \;\; = \frac{w_p}{\beta_p} - \frac{1}{2}\boldsymbol{\varepsilon}_p : \mathbf{C} : \boldsymbol{\varepsilon}_p$$

Interface equation

$$\cancel{c_s \dot{\beta}_s} - k_s \Delta\beta_s + \partial I \;\; \dot{\beta}_s \ni \beta_s G \;\; \mathbf{u}_1 - \mathbf{u}_2 \;\; - k_{s,1} \;\; \beta_s - \beta_1 \;\; - k_{s,2} \;\; \beta_s - \beta_2$$

$$G \;\; \mathbf{u}_1 - \mathbf{u}_2 \;\; = \frac{w_s}{\beta_s} - \frac{1}{2}\hat{k}_s \;\; \mathbf{u}_1 - \mathbf{u}_2 \;\; ^2$$

Adhesive

Problem Formulation

Parameter evaluation

Domains

$$-k_i \Delta \beta_i + \partial I \quad \dot{\beta}_i \qquad \beta_i F \quad \boldsymbol{\varepsilon}_i$$

$$F \quad \boldsymbol{\varepsilon}_c \quad = \frac{w_c}{\beta_c} - \frac{1}{2} \boldsymbol{\varepsilon}_c^+ : \mathbf{C} : \boldsymbol{\varepsilon}_c^+ \qquad\qquad F \quad \boldsymbol{\varepsilon}_p \quad = \frac{w_p}{\beta_p} - \frac{1}{2} \boldsymbol{\varepsilon}_p : \mathbf{C} : \boldsymbol{\varepsilon}_p$$

$$w_c = \frac{1}{2} \frac{\bar{\sigma}_t^2}{E_C} \qquad w_p = \frac{1}{2} \frac{\bar{\sigma}_0^2}{E_P}$$

k_i, k_s Extension parameter depends on the size of the grains

Interface

$$-k_s \Delta \beta_s + \partial I \quad \dot{\beta}_s \ni \beta_s G \quad \mathbf{u}_1 - \mathbf{u}_2 \quad - k_{s,1} \quad \beta_s - \beta_1 \quad - k_{s,2} \quad \beta_s - \beta_2$$

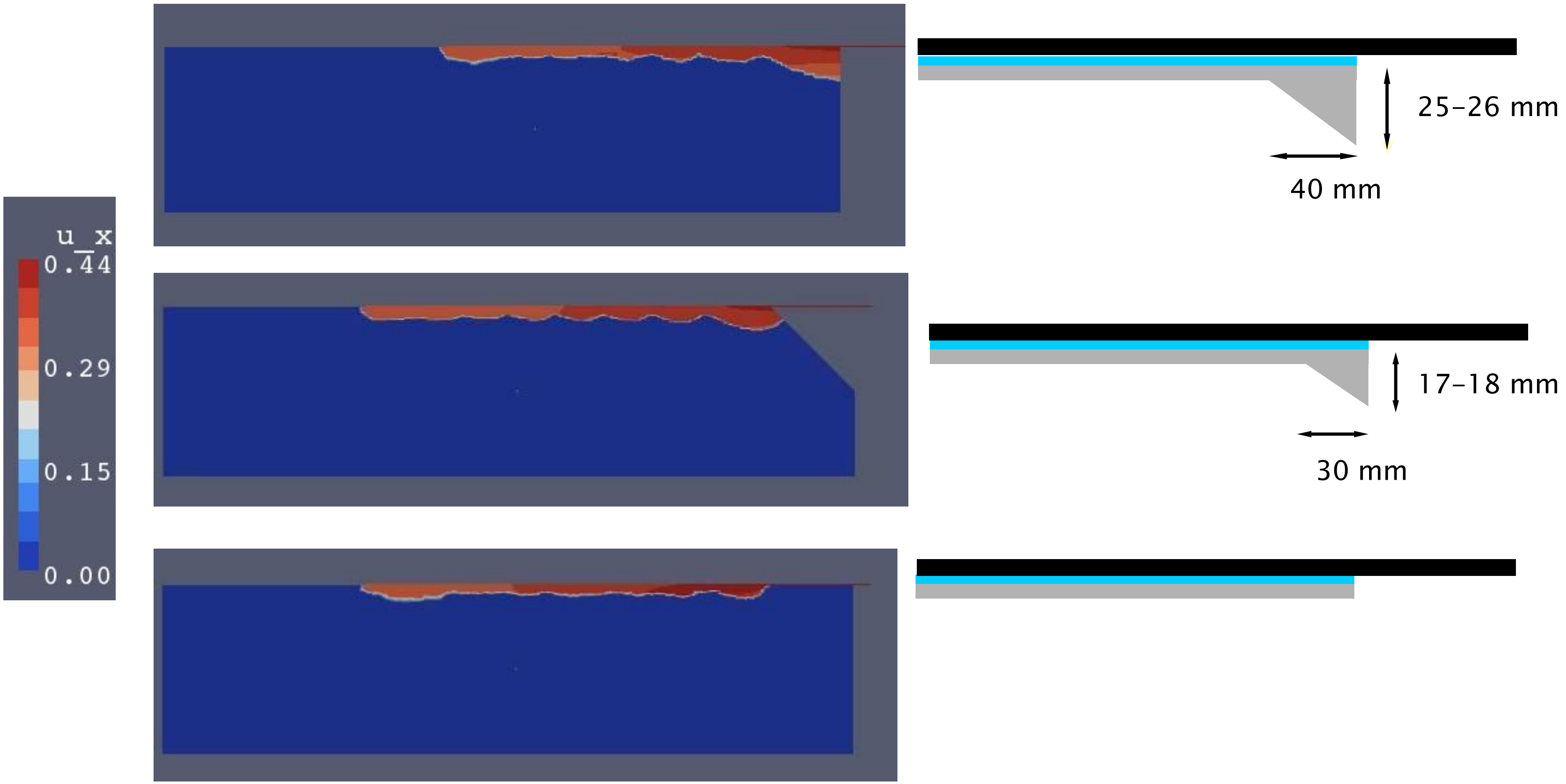
$\hat{k}_s$ Stiffness depends on E, G of the glue and thickness

$$G \quad \mathbf{u}_1 - \mathbf{u}_2 \quad = \frac{1}{2} \hat{k}_s \quad \mathbf{u}_1 - \mathbf{u}_2 \quad ^2 - \frac{w_s}{\beta_s} \qquad w_S = \frac{1}{2} \frac{\bar{\sigma}_0^2}{E_S}$$

$k_{s,i}$ Depends on the interaction between the adhesive and the materials
The parameter has been fitted so that the gluing concrete surface exhibits the bond strength value reported by the adhesive supplier.

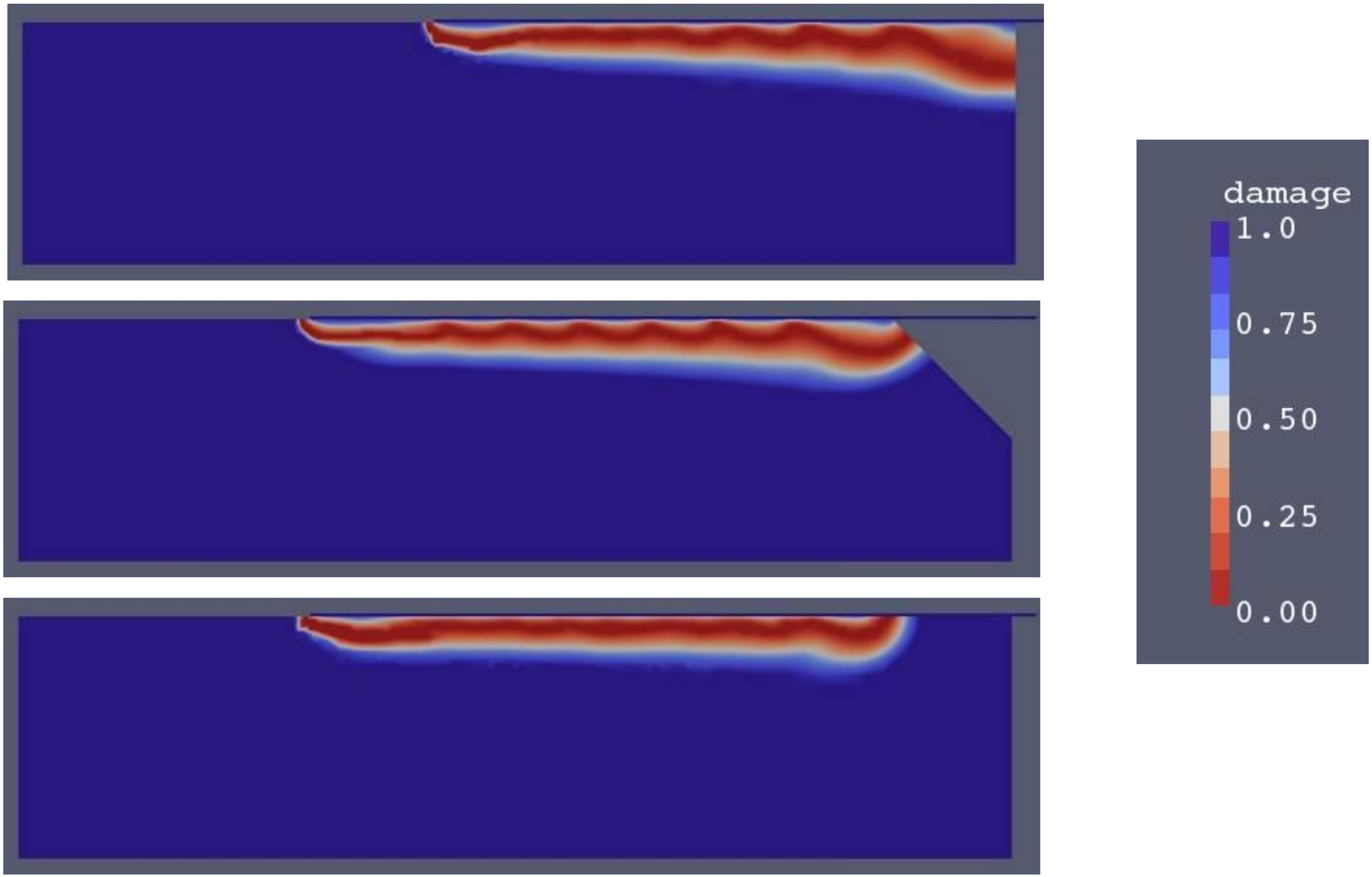
Numerical results

Tests: horizontal displacement



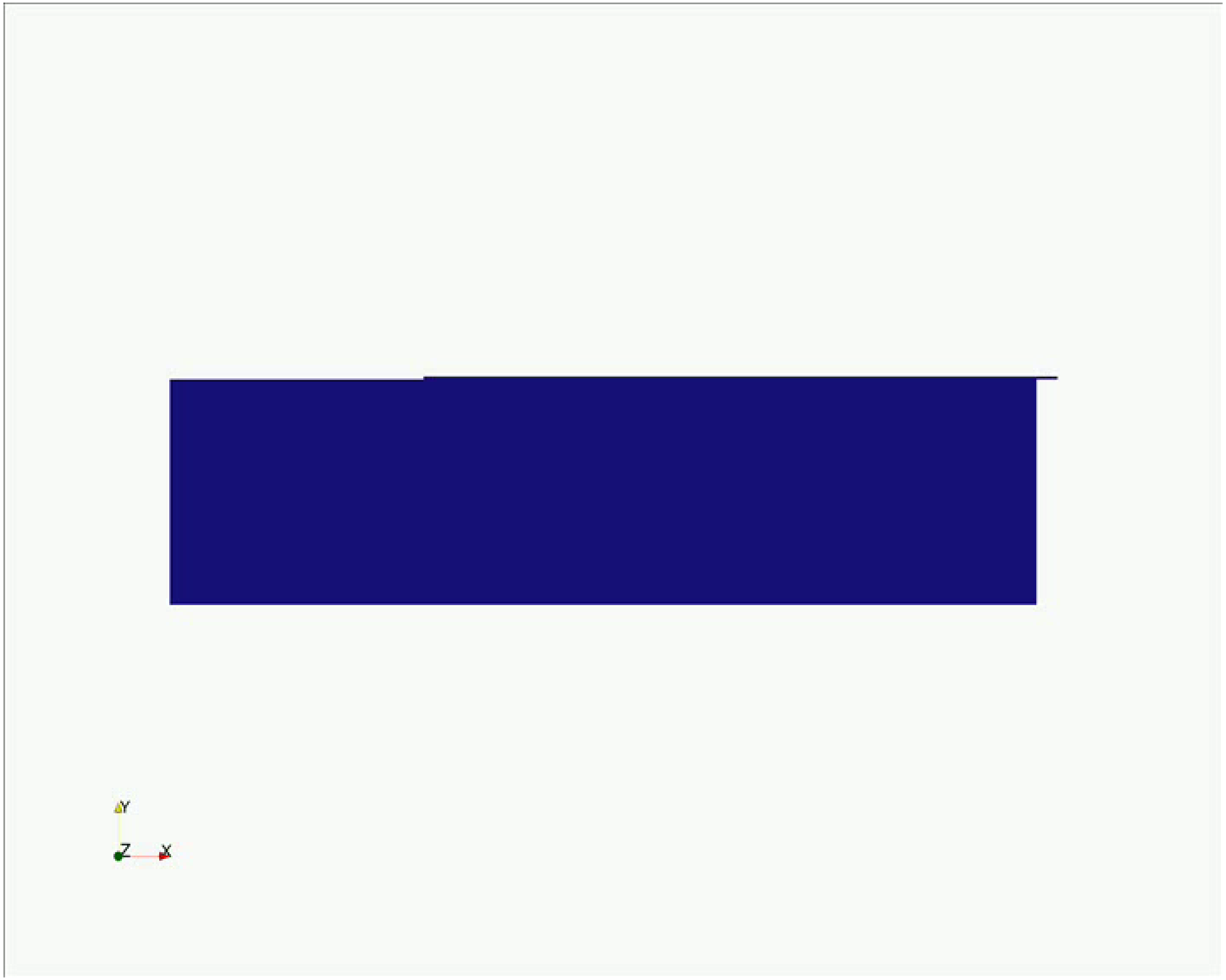
Numerical results

Damage: Coupled-model



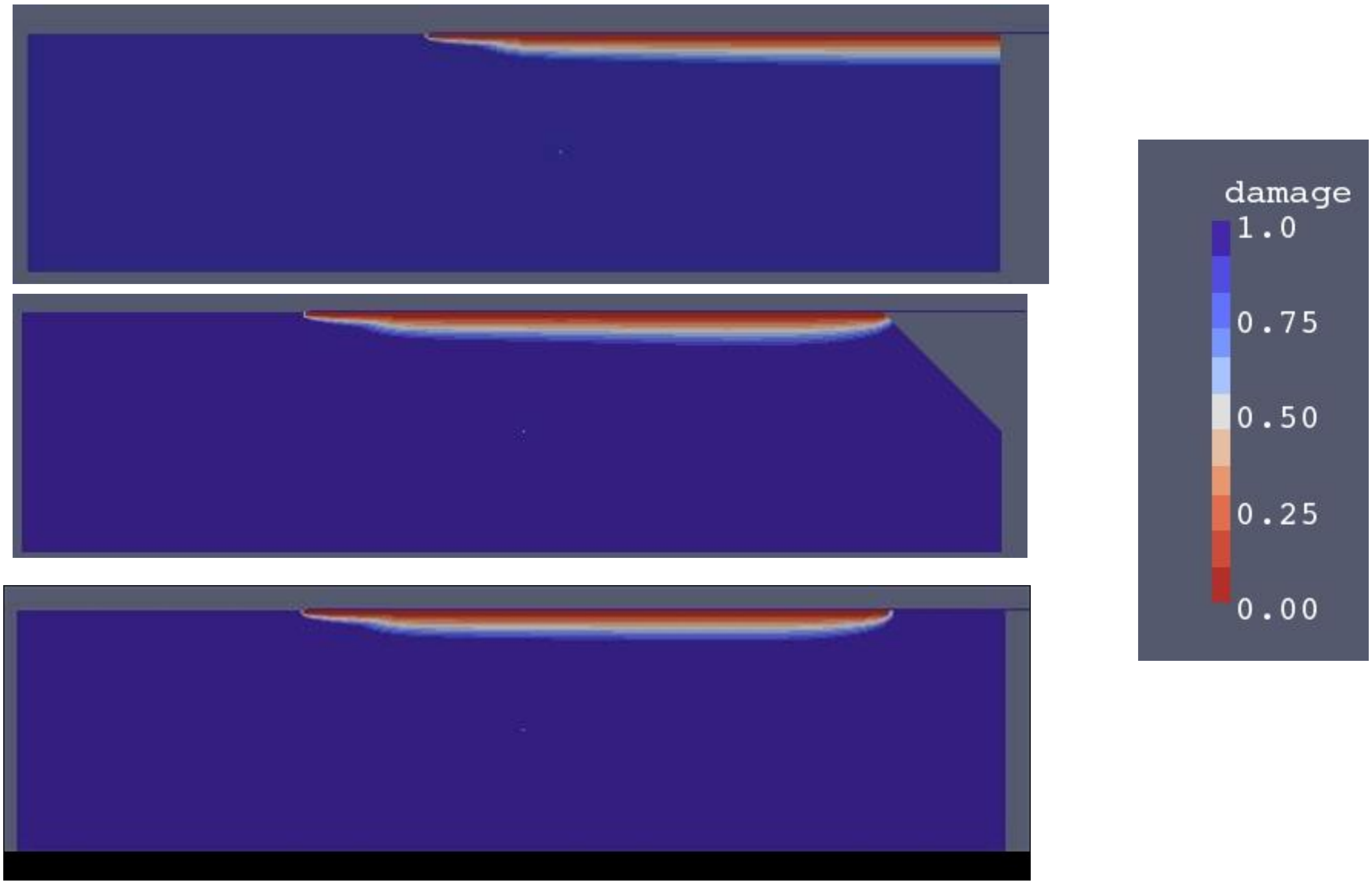
Numerical results

Test C



Numerical results

Damage: Uncoupled-model



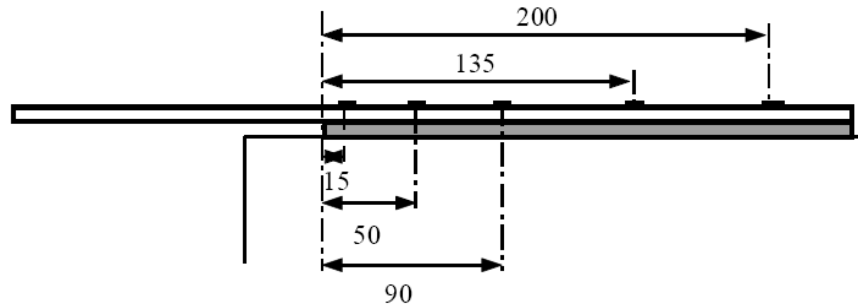
Experiments vs Numerics

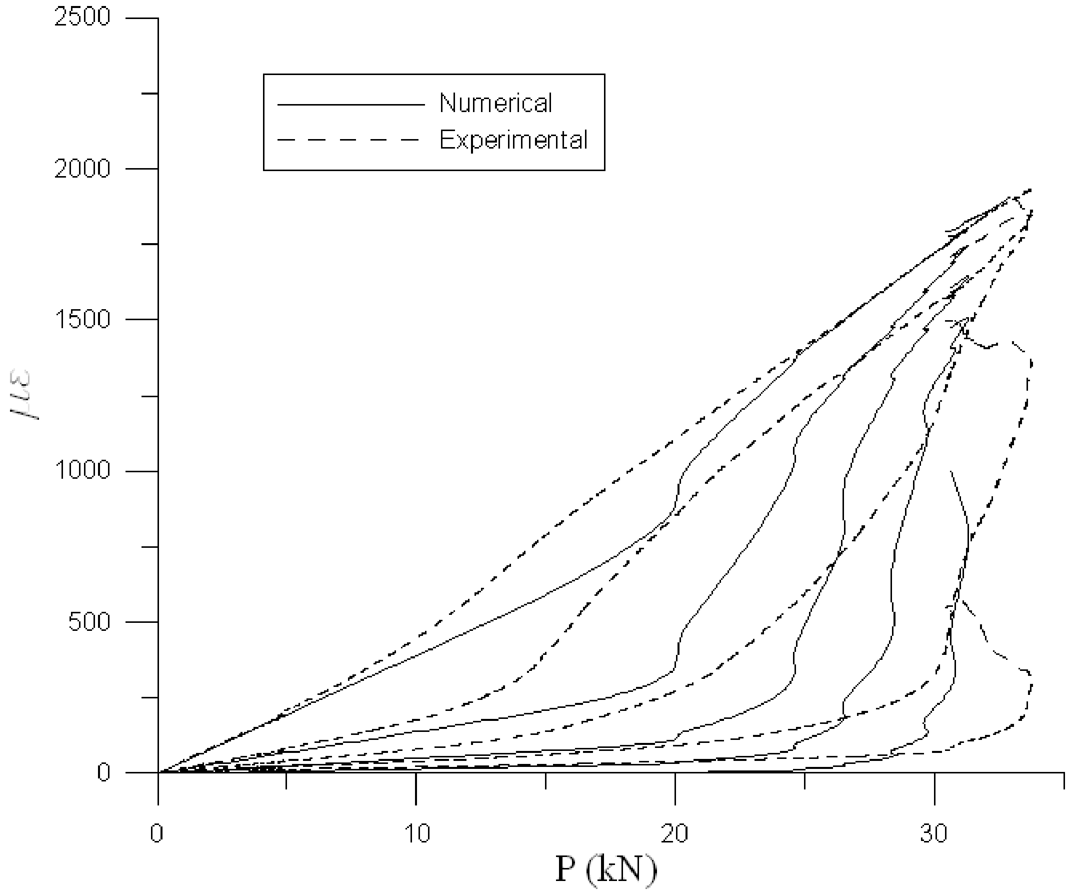
Maximum load for unit thickness

Series	Experimental (N/mm)	Numerical coupled (N/mm)	Numerical un-coupled (N/mm)
A	405	390	306
B	393	398	297
C	423	414	317

Experiments vs Numerics

Deformation Test A

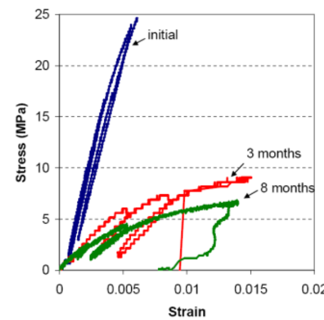




Preliminary results accelerated ageing tests

Macroscopic effects on the mechanical behavior of adhesive

- Loss of stiffness -> $\hat{k}_s$]
- Reduction of strength -> w_S]
- Lower interaction between the concrete and glue -> $k_{s,c}$]



Rupture in no longer in the concrete substrate
but propagates at the interface level

Numerical results

Test C: damage

Maximum transmitted force is almost constant

Numerical results

Test A: damage