AN AXIOMATICS FOR ADHESIVE INTERFACES

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Unilateral Problems in Structural Analysis
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A unified model for adhesive interfaces with damage, viscosity, and friction

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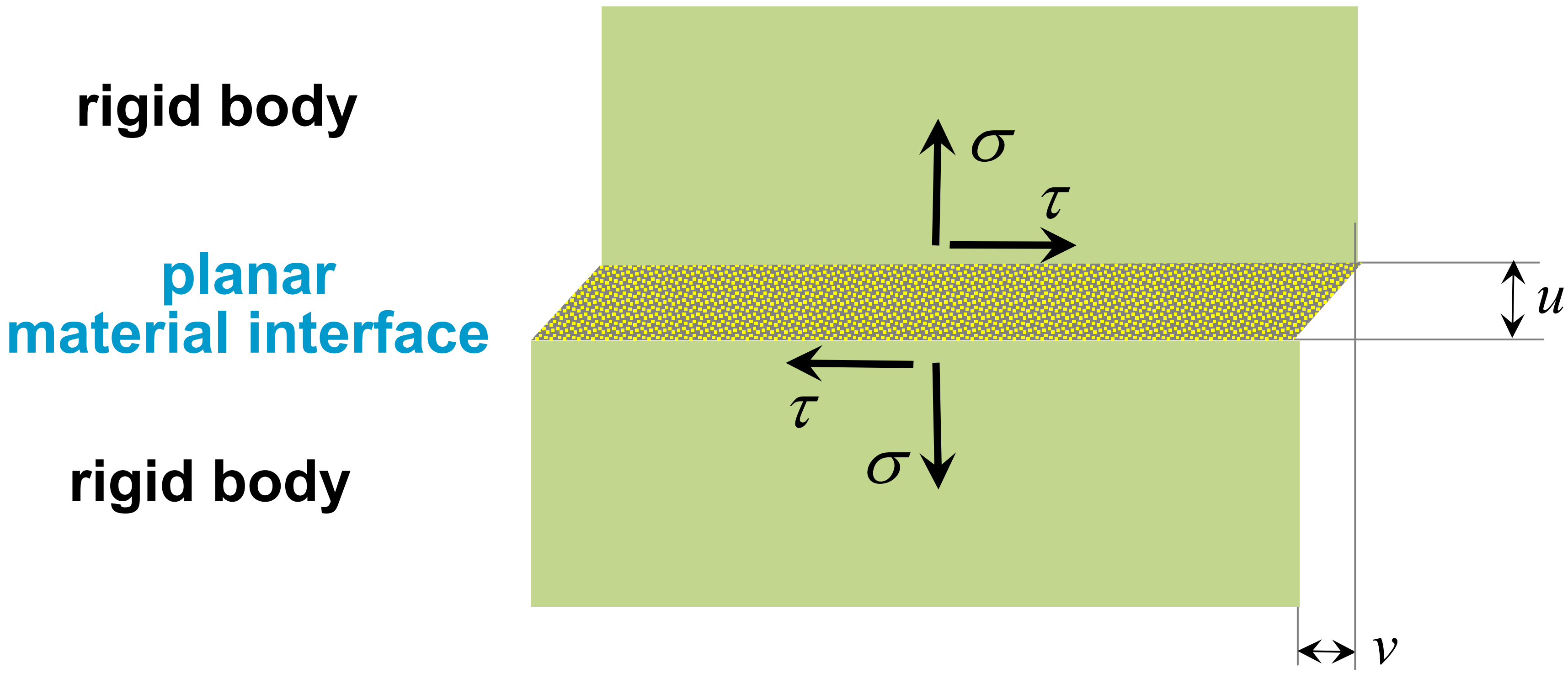
The purpose of this work is to model a complicated material response ...

... with the smallest possible number of variables

- general laws, typically, energy conservation and dissipation principle, that is, mechanical versions of the first two laws of thermodynamics,
- a set of state variables, that is, an array of independent variables which fully determine the response to all possible deformation processes,
- a set of elastic potentials and dissipation potentials, which are functions of state in terms of which the general laws take specific forms,
- a set of constitutive assumptions.

no energy minimization



u, v : normal and tangential relative displacements

σ, τ : normal and tangential forces

the material interface has negligible thickness

Problem

Given a deformation process $t \mapsto (u(t), v(t))$

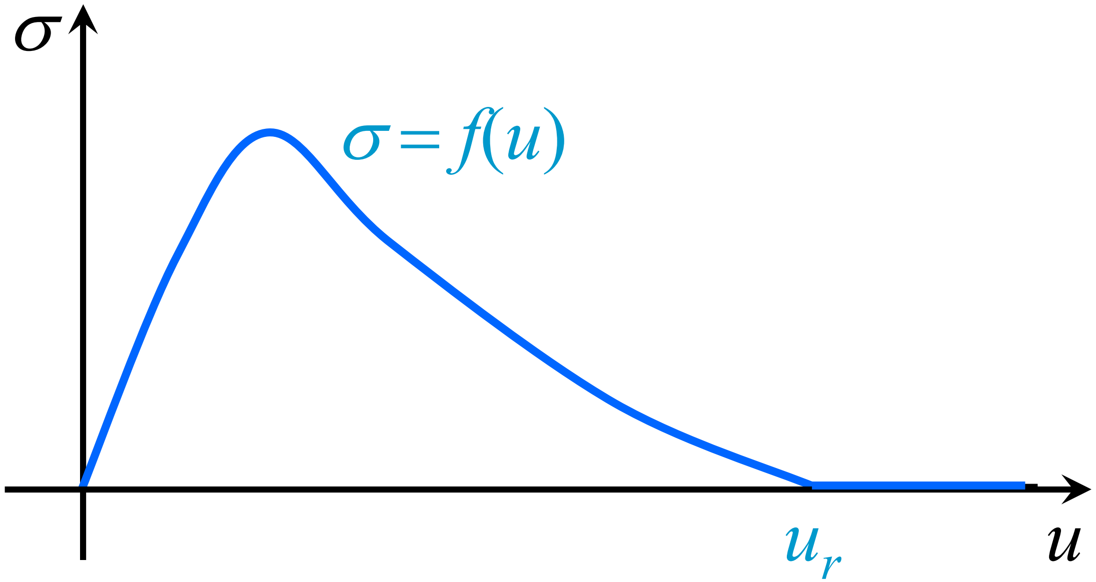
Find the response $t \mapsto (\sigma(t), \tau(t))$

Consider first the *purely normal case* $v(t) = 0$

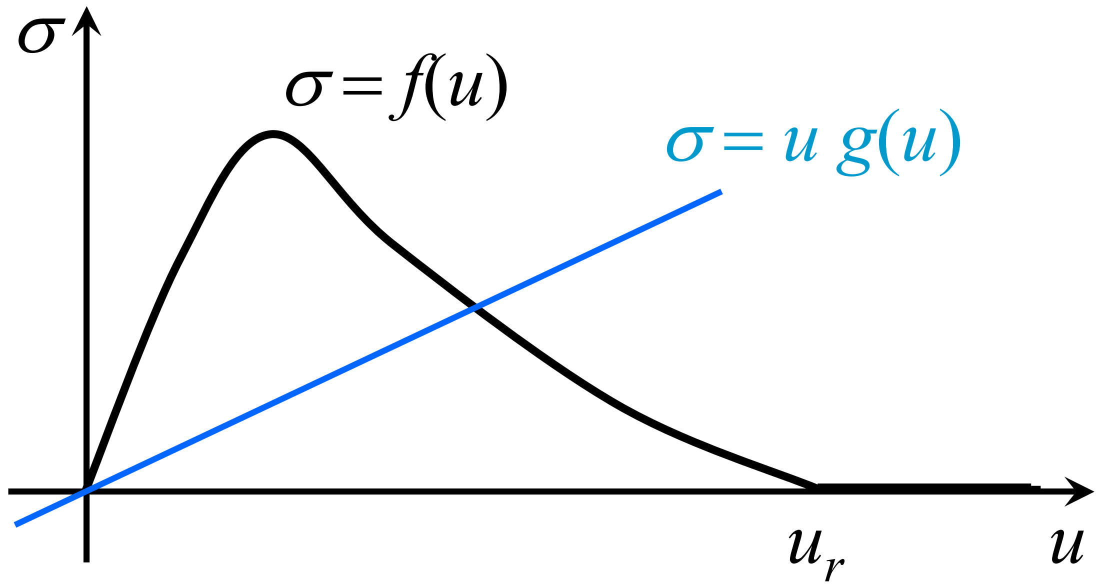
in the presence of *adhesion* and *damage*

experimental input

the loading curve



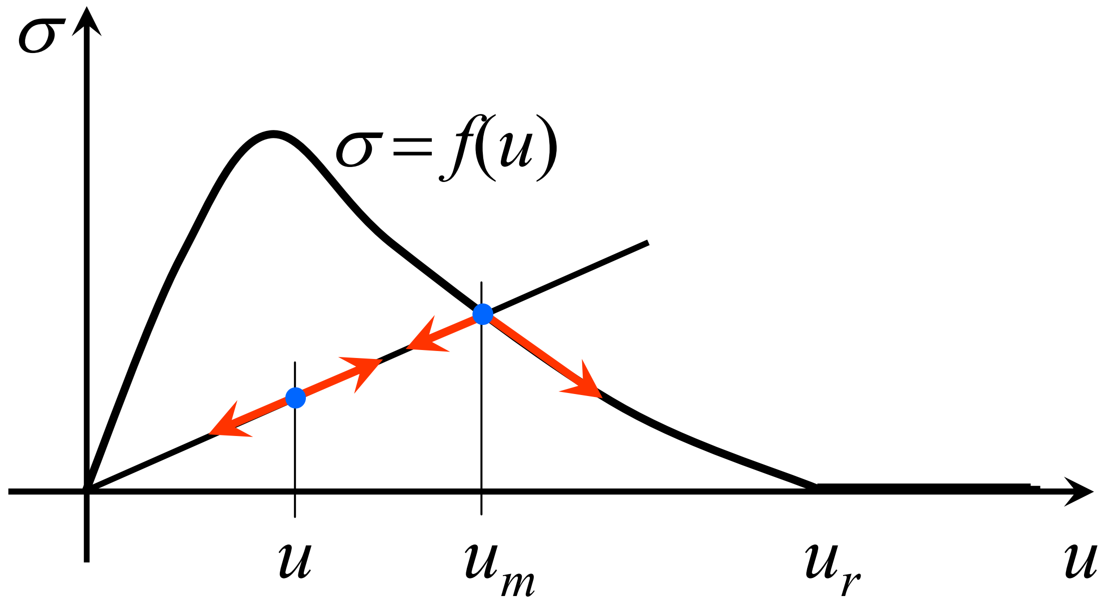
$u_r =$ **complete rupture**



f is ***star-shaped*** **with respect to the origin :**

$u \mapsto g(u)$ **is decreasing**

the desired loading-unloading response

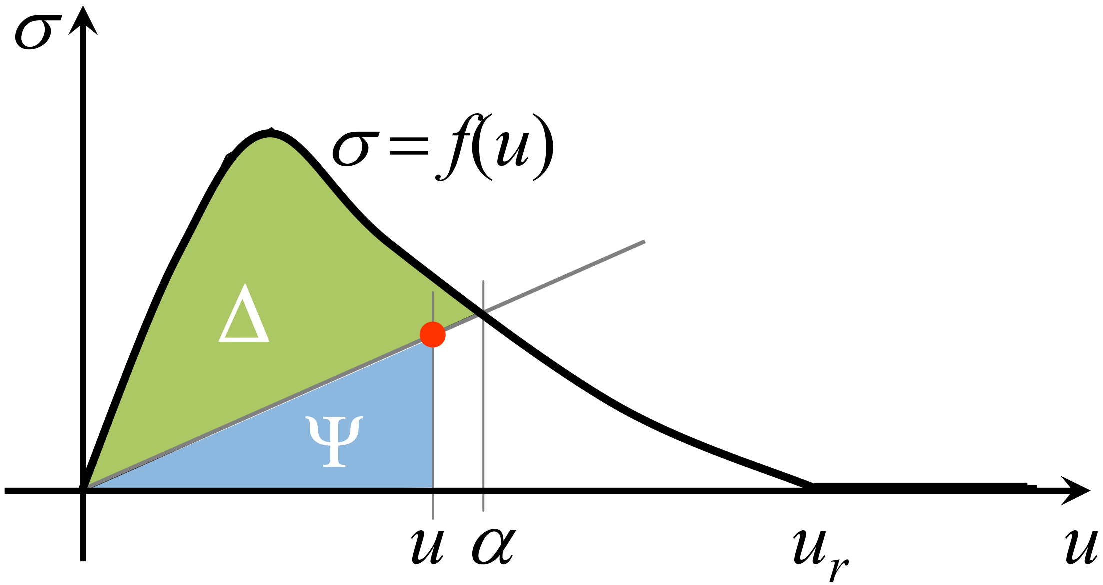


u_m = **the largest displacement reached in the past history**

$\alpha = u_m =$ **state variable**

construction of the model

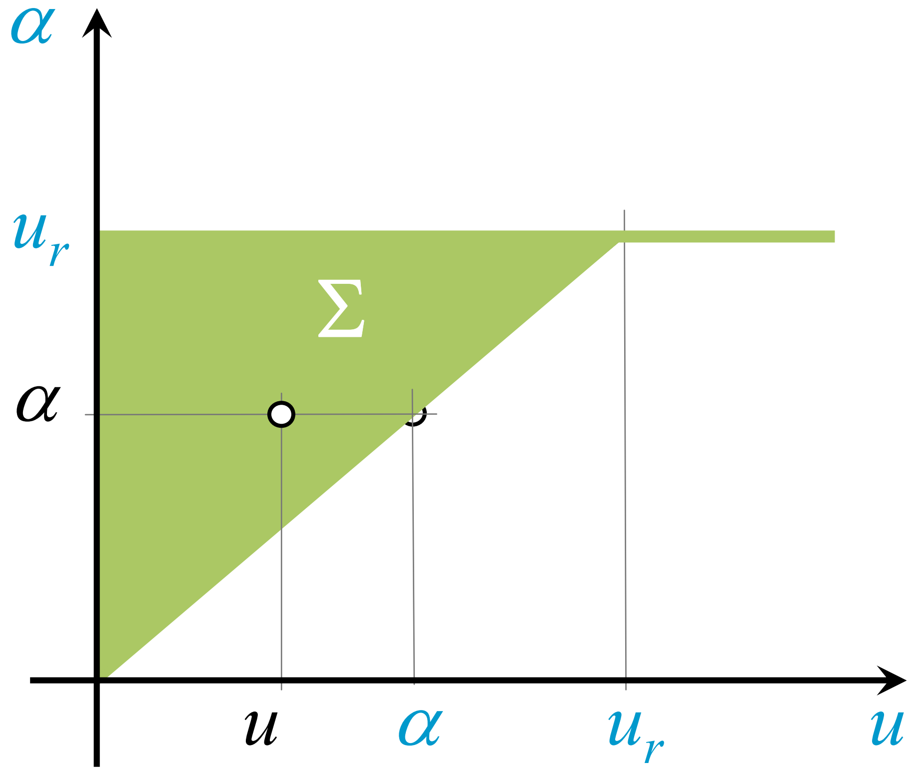
(i) strain energy and dissipation



strain energy $\Psi(u,\alpha) = \frac{1}{2} g(\alpha) u^2$

dissipation $\Delta(\alpha) = \int_0^{\alpha} f(s)\mathrm{d}s - \frac{1}{2} g(\alpha)\alpha^2$

(ii) state space



$$\alpha \in [u, u_r] \quad \text{if} \quad 0 \le u \le u_r,$$
$$\alpha = u_r \quad \text{if} \quad u > u_r.$$

(iii) general laws

energy conservation

$$W_{(t_1,t_2)} = \Psi_{(t_2)} - \Psi_{(t_1)} + \Delta_{(t_2)} - \Delta_{(t_1)}$$

dissipation principle

$$\dot{\Delta}_{(t)} \geq 0$$

(iv) choice of the potentials

dissipation potential

damage, rate-independent

$$\Phi_{\mathrm{d}}(\alpha, \dot{\alpha}) = -\tfrac{1}{2} g'(\alpha) \alpha^2 \dot{\alpha}$$

dissipation rate

$$\dot{\Delta}_{(\mathrm{t})} = D_{\mathrm{d}}(\alpha, \dot{\alpha}) = \dot{\alpha} \frac{\partial}{\partial \dot{\alpha}} \Phi_{\mathrm{d}}(\alpha, \dot{\alpha}) = -\tfrac{1}{2} g'(\alpha) \alpha^2 \dot{\alpha}$$

dissipation principle

$$\dot{\alpha} \geq 0$$

power equation

by differentiation of energy equation

$$P_{(t)} = \dot{\Psi}_{(t)} + \dot{\Delta}_{(t)}$$

external power

$$P_{(t)} = \sigma_{(t)} \; \dot{u}_{(t)}$$

internal power

$$\dot{\Psi}(u,\alpha) = \tfrac{1}{2} g'(\alpha) u^2 \dot{\alpha} + g(\alpha) u \dot{u}, \quad \dot{\Delta}_{(t)} = - \tfrac{1}{2} g'(\alpha) \alpha^2 \dot{\alpha}$$

power equation

$$(\sigma - g(\alpha) u) \dot{u} + \tfrac{1}{2} g'(\alpha)(\alpha^2 - u^2) \dot{\alpha} = 0.$$

power equation

$$(\sigma - g(\alpha)u)\dot{u} + \boxed{\frac{1}{2}g'(\alpha)(\alpha^2 - u^2)\dot{\alpha}} = 0$$

≤ 0

$$(\sigma - g(\alpha)u)\dot{u} \geq 0$$

$\dot{u}$ is free if $u > 0$

$$\boxed{\sigma = g(\alpha)u \geq 0 \quad \text{if} \quad u > 0}$$

(v) choice of the constitutive equation

$$\boxed{\sigma^{+} = g(\alpha)u}$$

$$\sigma = \sigma^{+} - \sigma^{-}$$

Signorini contact law

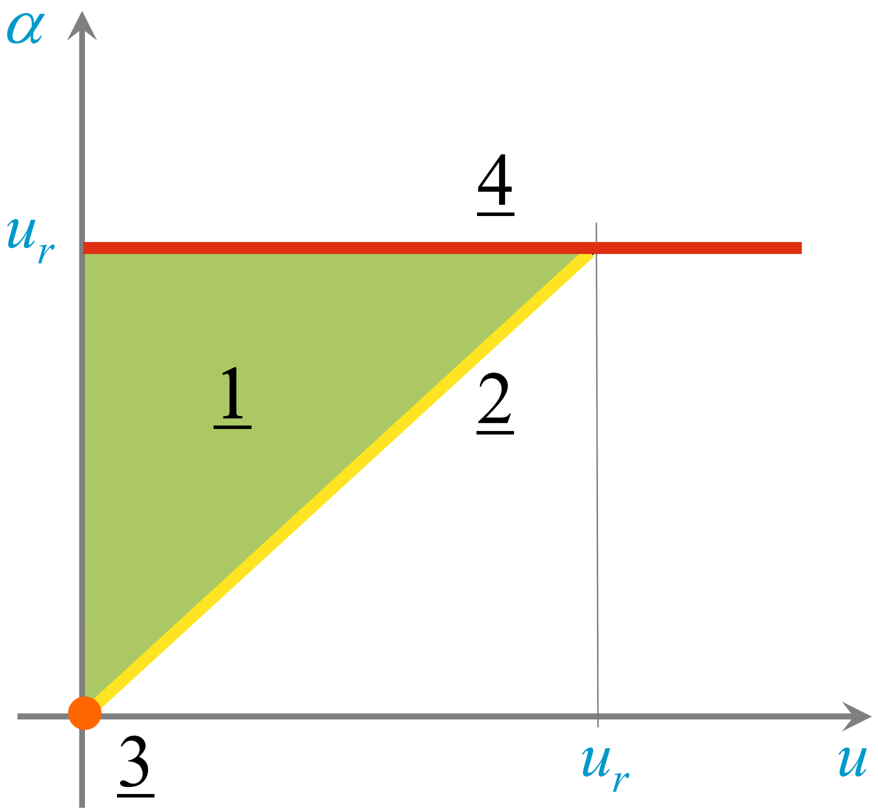
$$\sigma^{-} \geq 0, \quad u \geq 0, \quad \sigma^{-}u = 0$$

Signorini contact law for the rates at $u = 0$

$$\sigma^{-} \geq 0, \quad \dot{u} \geq 0, \quad \sigma^{-}\dot{u} = 0$$

note : $\sigma^{-} \neq 0$ only if $u = \dot{u} = 0$.

(vi) evolution equation for α



partition of the state space

power equation

$$(\sigma - g(\alpha)u)\dot{u} + \frac{1}{2}g'(\alpha)(\alpha^2 - u^2)\dot{\alpha} = 0$$

reduces to

$$\boxed{g'(\alpha)(\alpha^2 - u^2)\dot{\alpha} = 0}$$

in <u>1</u>, $g'(\alpha) \neq 0, \quad \alpha^2 - u^2 \neq 0 \;\Rightarrow\; \dot{\alpha} = 0$

<u>1</u> is the elastic region

in <u>2</u>, <u>3</u>, <u>4</u>, the power equation is identically satisfied

It is necessary to take the second derivative of the energy equation

$$g'(\alpha)\left(\alpha\dot{\alpha} - u\dot{u}\right)\dot{\alpha} = 0$$

in <u>2</u>, $\quad g'(\alpha) \neq 0, \quad \alpha = u \neq 0 \Rightarrow (\dot{\alpha} - \dot{u})\dot{\alpha} = 0$

$$\dot{u} \geq 0 \quad \Rightarrow \quad \dot{\alpha} = \dot{u} \qquad \text{damage}$$

$$\dot{u} < 0 \quad \Rightarrow \quad \dot{\alpha} = 0 \qquad \text{elastic unloading}$$

in <u>3</u>, <u>4</u>, the second-order power equation is identically satisfied

It is necessary to take the third derivative of the energy equation

$$\left(\dot{\alpha}^2 - \dot{u}^2\right)\dot{\alpha} = 0$$

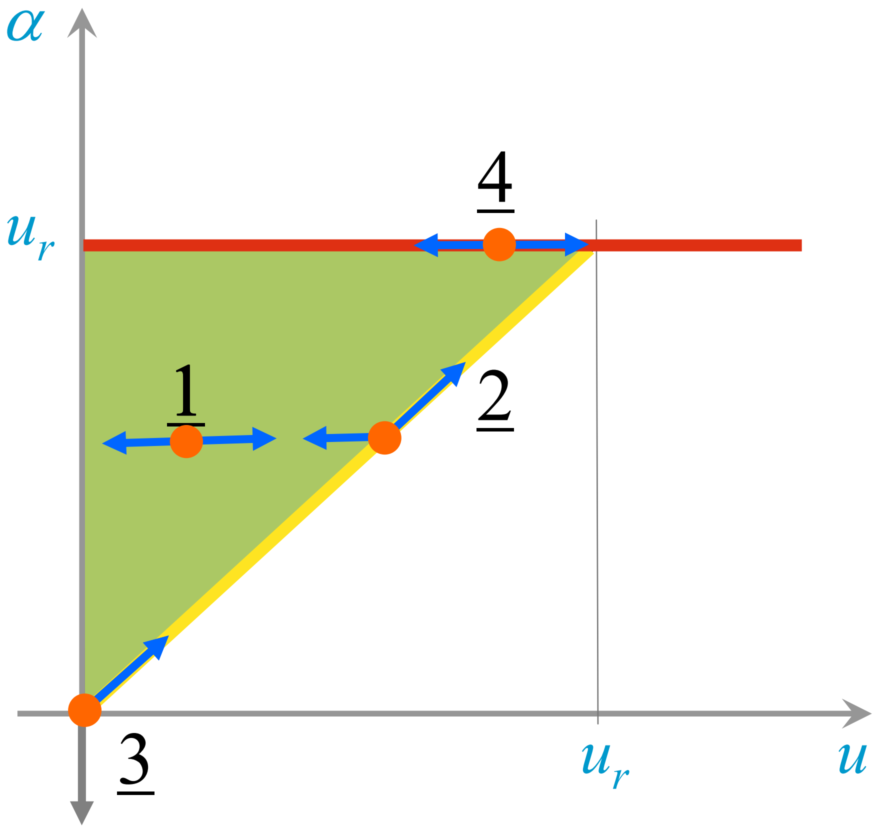
in **3**,

$\dot{u} > 0 \quad \Rightarrow \quad \dot{\alpha} = \dot{u} > 0$ damage

$\dot{u} = 0 \quad \Rightarrow \quad \dot{\alpha} = \dot{u} = 0$ unilateral contact

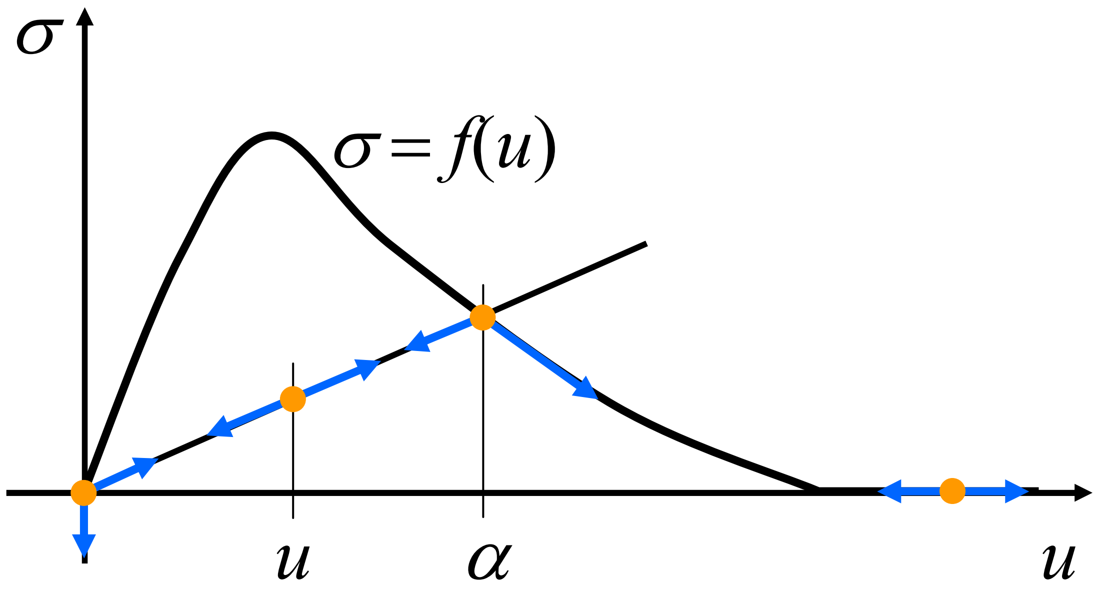
in **4**, $\alpha = u_r \quad \Rightarrow \quad \dot{\alpha} = 0$ complete rupture

the evolution of α



$$\dot{\alpha} = \begin{cases} \dot{u} & \text{if } \alpha = u < u_r \text{ and } \dot{u} > 0 \\ 0 & \text{otherwise,} \end{cases}$$

directions of evolution in the (σ, u) plane



the desired response has been obtained by a proper choice of the potentials, assuming energy conservation and dissipation principle

the purely normal case in the presence of *viscosity*

the viscous dissipation potential

$$\Phi_{\mathrm{v}}(\alpha, \dot{\alpha}) = \frac{1}{4} h(\alpha) \dot{\alpha}^2$$

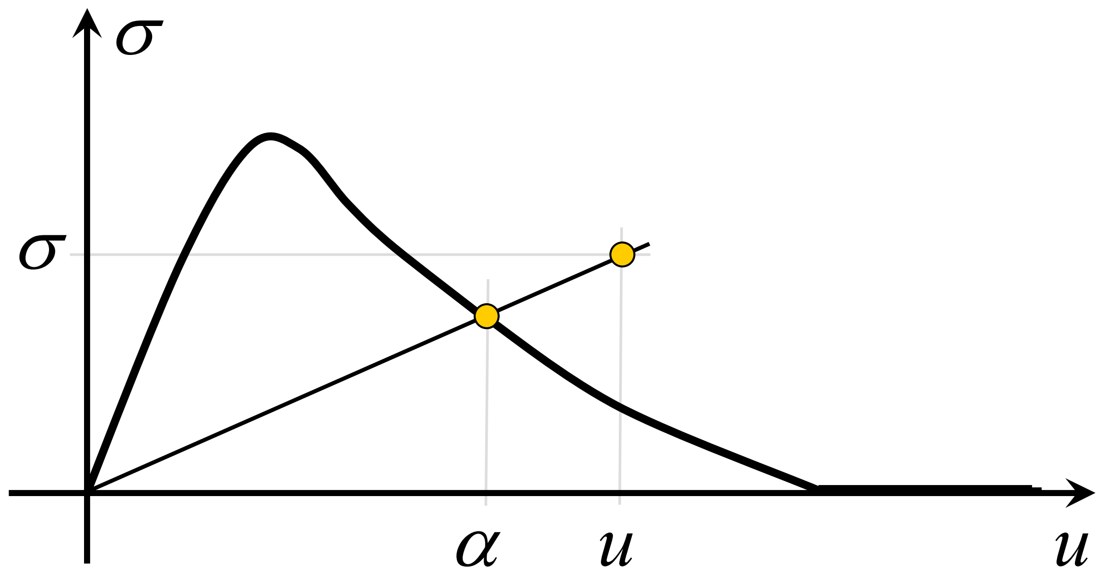
same state variable for dissipation and damage

the viscous dissipation rate

$$D_{\mathrm{v}}(\alpha, \dot{\alpha}) = \frac{\partial}{\partial \dot{\alpha}} \Phi_{\mathrm{v}}(\alpha, \dot{\alpha}) \dot{\alpha} = \frac{1}{2} h(\alpha) \dot{\alpha}^2$$

the curve $\sigma = f(u)$ is not anymore the loading curve,

the state variable α is not anymore the maximum displacement attained in the past history.



but $\sigma = f(u)$ is again the border of the elastic zone,

and α is again the intersection of the loading curve $\sigma = f(u)$ with the straight line joining the point (σ, u) and the origin

with the addition of the viscous term, the power equation becomes

$$(\sigma - g(\alpha)u)\dot{u} + \tfrac{1}{2}g'(\alpha)(\alpha^2 - u^2)\dot{\alpha} - \tfrac{1}{2}h(\alpha)\dot{\alpha}^2 = 0$$

and with the same constitutive equation as above it reduces to

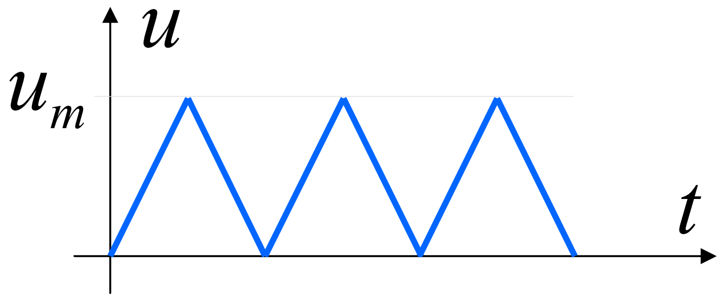
$$g'(\alpha)(\alpha^2 - u^2)\dot{\alpha} - h(\alpha)\dot{\alpha}^2 = 0$$

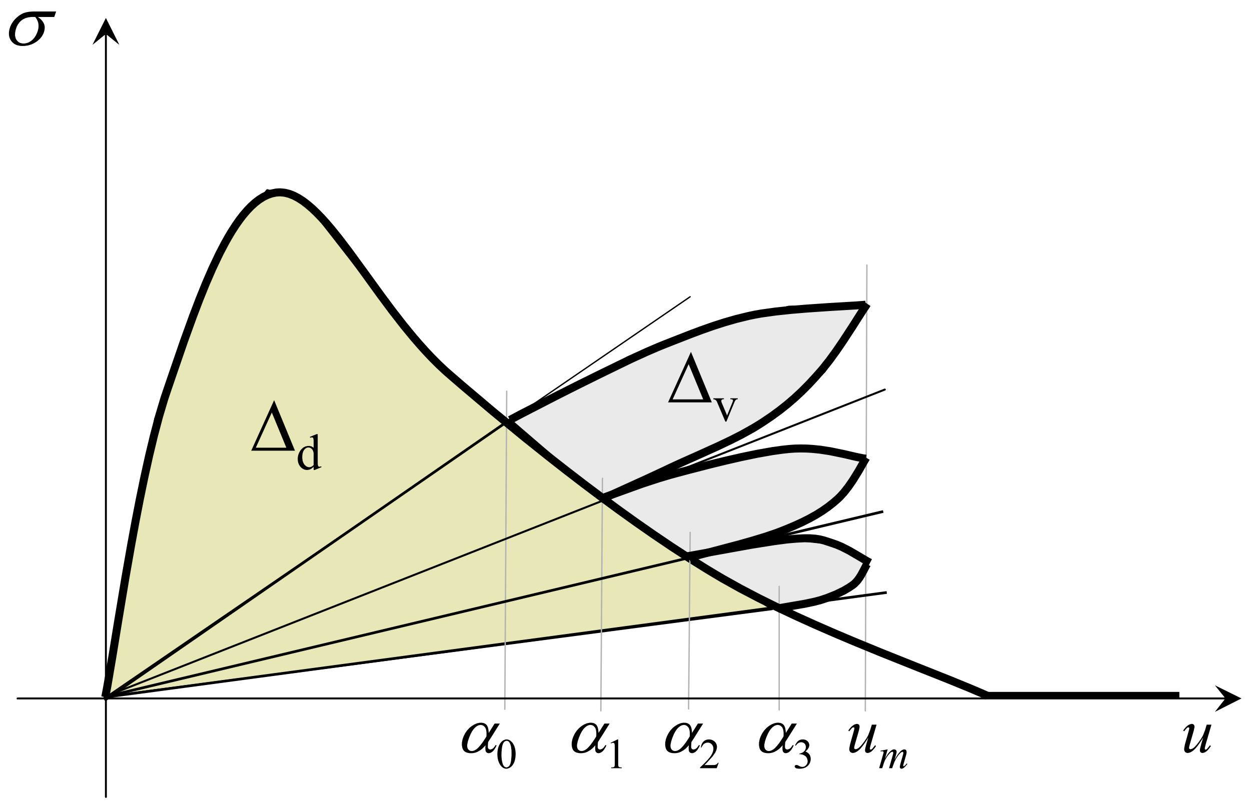
the new evolution law for the state variable is

$$\dot{\alpha} = \begin{cases} -\dfrac{g'(\alpha)}{h(\alpha)}\left(u^2 - \alpha^2\right) & \text{if } \alpha < u, \ \ 0 \leq \alpha < u_{\mathrm{r}}, \\ 0 & \text{otherwise.} \end{cases}$$

there is dissipation only if $u > \alpha$

the response to a cyclic process from $\alpha(0) = \alpha_0$





normal + tangential loading
adhesion + damage

normal response: $\sigma = f_N(u)$

tangential response: $\tau = f_T(v)$

strain energy

$$\Psi(u,v,\alpha) = \frac{1}{2} g_{\mathrm{N}}(\alpha) u^2 + \boxed{\frac{1}{2} g_{\mathrm{T}}(\alpha) v^2}$$

dissipation

$$\Delta(\alpha) = \int_0^\alpha (f_{\mathrm{N}}(s) + \boxed{f_{\mathrm{T}}(s))\mathrm{d}s} - \frac{1}{2}(g_{\mathrm{N}}(\alpha) + \boxed{g_{\mathrm{T}}(\alpha))\alpha^2}$$

the power equation becomes

$$\sigma\dot{u} + \tau\dot{v} = g_N(\alpha)u\dot{u} + g_T(\alpha)v\dot{v} + \tfrac{1}{2}g_N'(\alpha)(u^2 - \alpha^2)\dot{\alpha} + \tfrac{1}{2}g_T'(\alpha)(v^2 - \alpha^2)\dot{\alpha}.$$

and **with** the constitutive equations

$$\sigma^+ = g_N(\alpha)u, \quad \tau = g_T(\alpha)v$$

it reduces to

$$\left(g_N'(\alpha)\left(u^2 - \alpha^2\right) + g_T'(\alpha)\left(v^2 - \alpha^2\right)\right)\dot{\alpha} = 0$$

with the positions

$$\rho(\alpha) := \frac{g'_N(\alpha)}{g'_N(\alpha) + g'_T(\alpha)}, \quad \varphi(u, v, \alpha) := \rho(\alpha)u^2 + (1 - \rho(\alpha))v^2 - \alpha^2$$

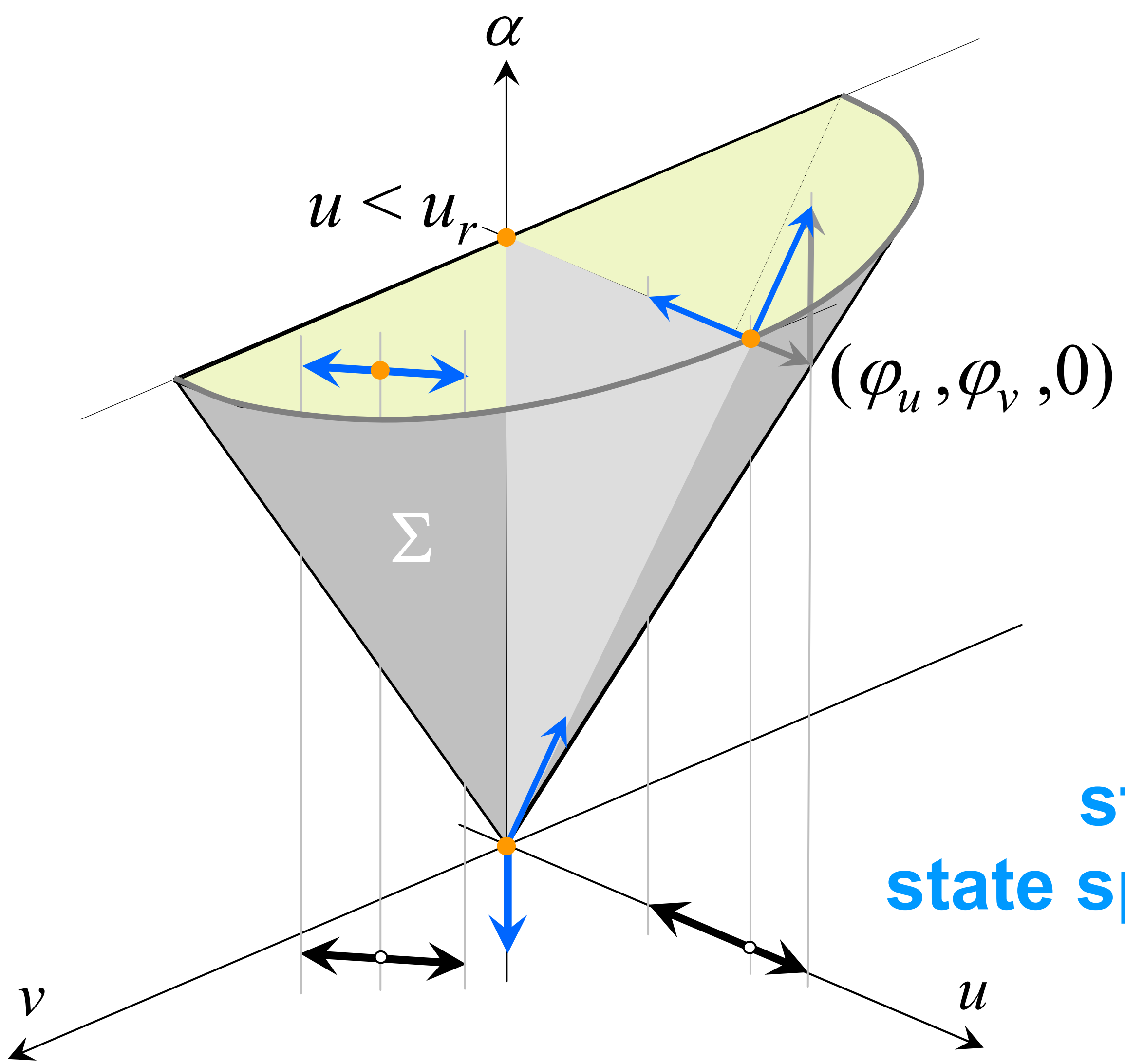
the power equation takes the form

$$\varphi(u, v, \alpha)\dot{\alpha} = 0$$

and the evolution equation for α becomes

$$\dot{\alpha} = \begin{cases} -\dfrac{\varphi_u(u,v,\alpha)\dot{u} + \varphi_v(u,v,\alpha)\dot{v}}{\varphi_\alpha(u,v,\alpha)} & \text{if } \varphi_u(u,v,\alpha)\dot{u} + \varphi_v(u,v,\alpha)\dot{v} > 0, \\ 0 & \text{otherwise.} \end{cases}$$

directions of evolution



state: (u, v, α)

state space: $\varphi(u, v, \alpha) \leq 0$

normal + tangential loading
adhesion + damage + *viscosity* + *friction*

dissipation power due to damage

$$D_{\mathrm{d}} = -\frac{1}{2}(g'_N(\alpha) + g'_T(\alpha))\,\alpha^2\dot{\alpha}$$

due to viscosity

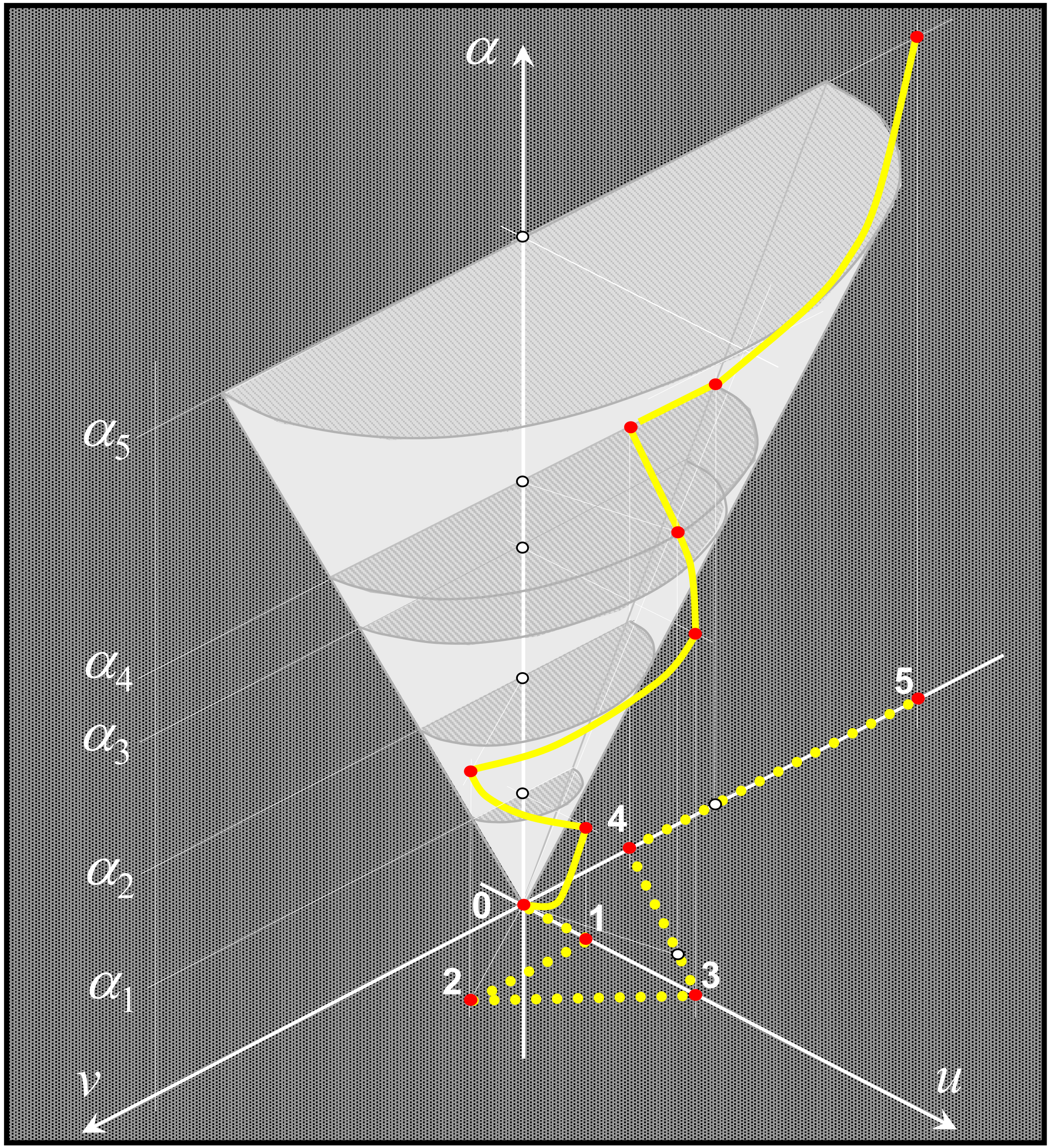
$$D_{\mathrm{v}} = \frac{1}{2}h(\alpha)\,\dot{\alpha}^2$$

due to friction

$$D_{\mathrm{f}} = \mu(\alpha)\,\sigma^{-}\,|\,\dot{v}\,|$$

μ = friction coefficient

response to a given process



conclusions

- a relatively simple and general model for the response of an adhesive interface has been constructed
- besides damage, viscosity and friction, other effects can be taken into account by introducing appropriate potentials
- more sophisticated responses can be obtained by introducing supplementary state variables
- the axiomatic frame developed here provides a flexible tool for describing a wide range of experimentally observed material behavior

THE END