Domain Decomposition methods for granular dynamics using discrete elements and application to railway ballast

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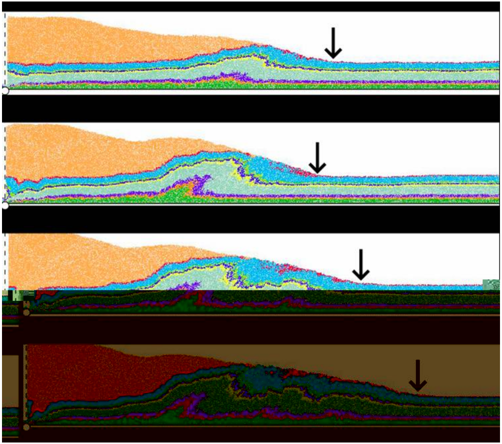
donner au train des idées d'avance

Outline

Why Domain Decomposition strategies ?

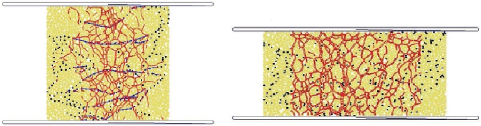
Increasing granular simulation size



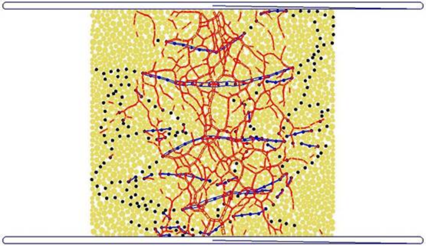
Forced-fold evolution on a numerical sandbox : 43 000 grains (2D)

Why to choose DDM among the parallel techniques ?

DDM may provide an automatic numerical homogenization procedure



TexSol/Sand comparison on a biaxial test



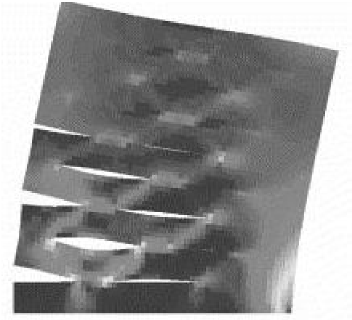
Domain Decomposition Method for granular dynamics

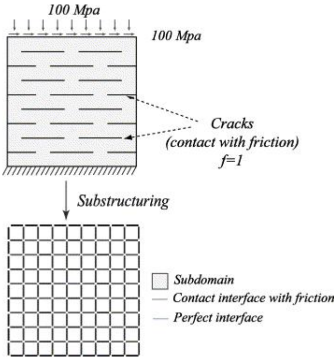
Specificities and difficulties of the topic

- Discrete system with an evolving connectivity between grains.
  - How to decompose such a discrete structure ? (further section)
  - How to update this decomposition ? (out of the present purpose)
- Non smooth contact dynamics within the whole domain.
  - The non smoothness cannot be isolated on some interfaces (next slide)
  - A velocity/impulse formulation has to be used (next section)

Generally the contact is located on the interface

Example of a micro-cracked domain [Ladeveze, Nouy, Loiseau 2002]
(Extreme case in continuum mechanics)



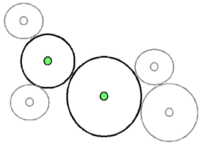


- Non smoothness (contact + friction) dealt with on interfaces.
- Smooth behavior within the subdomains.

Granular dynamics with a velocity/impulse formulation

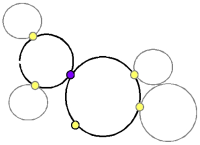
according to a time-stepping scheme

for a grain i



- V_i : grain velocity
- R_i : impulsion over $]t^-, t^+]$

for a contact α



- v_α **: relative velocity between 2 grains**
- r_α **: contact impulsion between 2 grains over** $]t^{--}, t^-]$

Ingredients

- **Dynamics** (grains)

$$M\ V = M\ V^{-} + R^{d} + R$$

- **Admissibility equations**

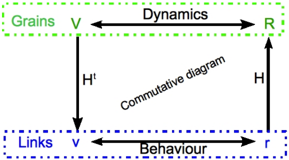
$$v = H^{T} V$$

$$R = Hr$$

- **Behaviour** (contacts)

$$\mathcal{R}(r_\alpha, v_\alpha) = 0$$

Implicit integration of dynamics (differential measures - Moreau 99)



Unilateral contact.
Explicit predictor of the gap g_α^p.

$$\begin{array}{ll} \text{if } g_\alpha^p > 0, & r_\alpha = 0 \\ \text{if } g_\alpha^p \leq 0, & 0 \leq r_\alpha \perp v_\alpha \geq 0 \end{array}.$$

Friction ... adhesion, capillarity...

Reference problem

- **(Reduced) dynamics at contacts** (indiced by α)
  (Dyn) + (Adm) lead to

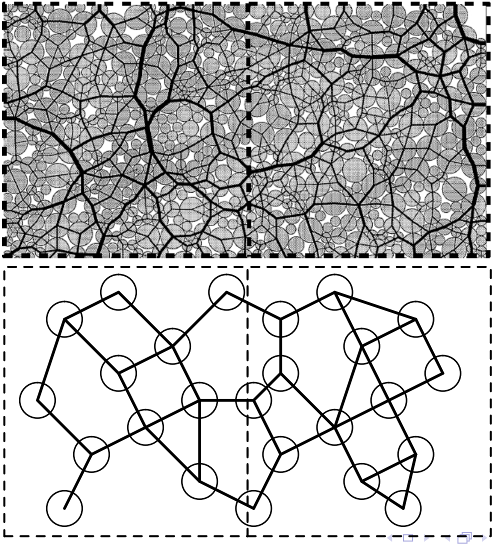
$$Wr - v = -v^d$$

  $W = H^t M^{-1} H$ Delassus operator
  $v^d = H^t M^{-1} R^d + H^t V$, "free" velocity.

- Reference problem

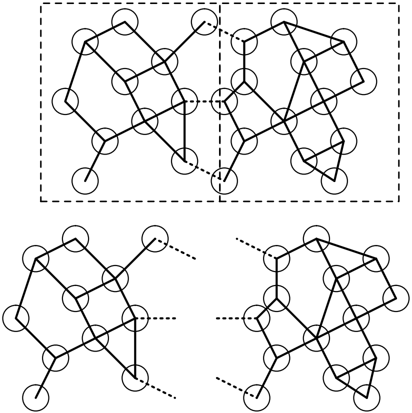
$$\boxed{\left\{\begin{array}{l} Wr - v = -v^d \\ \mathcal{R}(r_\alpha, v_\alpha) = 0, \alpha = 1, n_{contact} \end{array}\right. .}$$

"Geometrical box" approach for partitioning



Primal partitioning : distributing the **nodes** among substructures

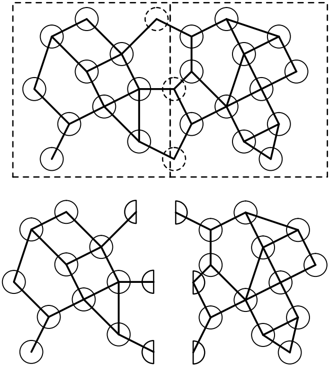
non smooth subdomains... and **non smooth** interface



Dual partitioning : distributing the **links** among substructures

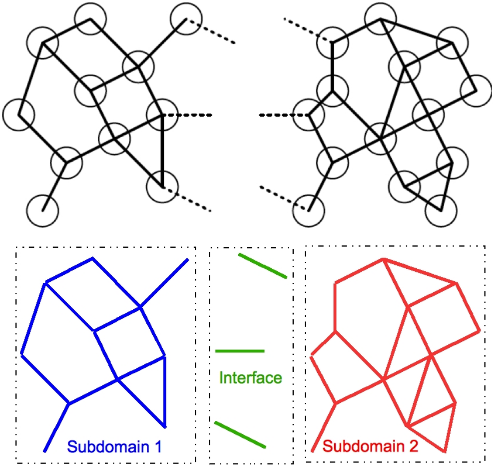
(as a remark)

non smooth subdomains... but **perfect** interface



... a "gluing" equation to add.

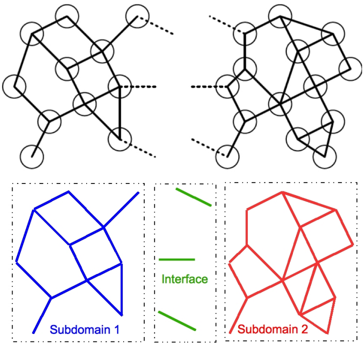
Primal partitioning $= n + 1$ substructures



Interface with (almost) non connected contacts

Primal partitioning : **non smooth** interface

Impulsions r = **internal** (subdomains) r_E et **interface** r_Γ



$$r^t = [\ldots, \quad r_E, \quad \ldots, \quad r_{E'}, \quad \ldots, \quad r_\Gamma]$$

Non smooth problem inside the subdomains E...

$$\begin{cases} W_E r_E - v_E + W_{E\Gamma} r_\Gamma = -v_E^d \\ \mathcal{R}(r_E, v_E) = 0 \end{cases}$$

... **and on the interface !**

$$\begin{cases} W_\Gamma r_\Gamma - v_\Gamma - \sum W_{\Gamma E} r_E = -v_\Gamma^d \\ \mathcal{R}(r_\Gamma, v_\Gamma) = 0 \end{cases}$$

Reduced dynamics over $n+1$ subdomains

The linear part is the reduced dynamics

$$\begin{bmatrix} W_E & 0 & W_{E\Gamma} \\ 0 & W_{E'} & W_{E'\Gamma} \\ W_{\Gamma E} & W_{\Gamma E'} & W_\Gamma \end{bmatrix} \begin{bmatrix} r_E \\ r_{E'} \\ r_\Gamma \end{bmatrix} - \begin{bmatrix} v_E \\ v_{E'} \\ v_\Gamma \end{bmatrix} = - \begin{bmatrix} v_E^d \\ v_{E'}^d \\ v_\Gamma^d \end{bmatrix}$$

Properties

- The problems per subdomains are uncoupled
- W_Γ is quasi-diagonal.

Gauss-Seidel like algorithm (with a particular renumbering)

- **Step 1** : GSNL solution per subdomain (parallel computing)
  Gauss-Seidel like partitioning per block
  W_E^L = diagonal and lower triangular parts of W_E

$$(W_E = W_E^L + (W_E - W_E^L)).$$

$$\begin{cases} W_E^L r_E^k - v_E^k = -v_E^d - W_{E\Gamma} r_\Gamma - \left(W_E - W_E^L\right) r_E^{k-1} \\ \mathcal{R}(r_E^k, v_E^k) = 0 \end{cases}$$

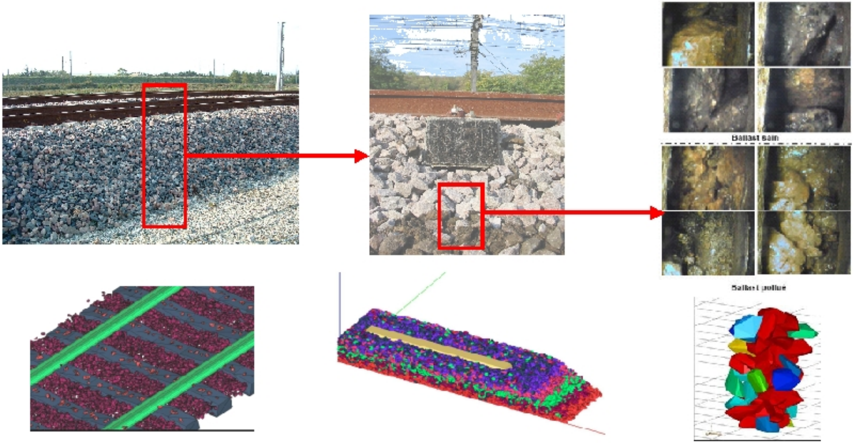
  k = number of Gauss-Seidel sweepings.

- **Step 2** : GSNL solution at the interface

$$\begin{cases} W_\Gamma^L r_\Gamma^k - v_\Gamma^k = -v_\Gamma^d - \sum W_{\Gamma E} r_E - \left(W_\Gamma - W_\Gamma^L\right) r_\Gamma^{k-1} \\ \mathcal{R}(r_\Gamma^k, v_\Gamma^k) = 0 \end{cases}$$

Railway ballast topic

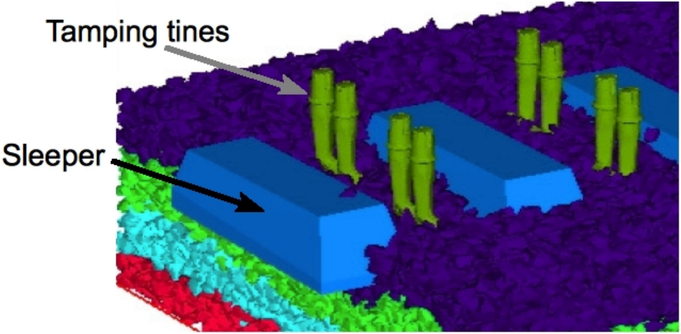
Complex medium - Polyhedral grains - Relevant discrete scale



A maintenance example : the tamping process

Three phases (with vibrating forces and after lifting of the sleeper)

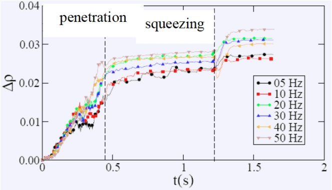
- Penetration of the tamping tines
- Squeezing of the ballast grains under the sleeper
- Lifting the tamping tines



Example of a parametric study on a single sleeper

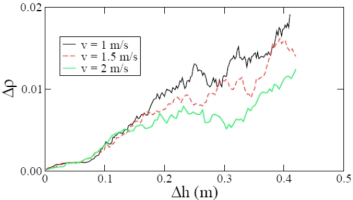
[Saussine et al., WCRR 2008]

- Evolution of the compaction under the sleeper



Penetration phase contributes to 50% of the final compaction gain

- The compaction with respect to the driven velocity



Low speeds improve the final compaction gain
→ Recommendation

Identification of relevant parameters

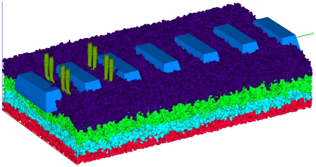
Evolutive nonsmooth problem $\Rightarrow$ **solution multiplicity**
$\Rightarrow$ one needs a range of qualitative **indicators** to qualify the computations

- **Volume error** (quantifying interpenetration)
- Compactness
- **Coordination number** (average neighbors number)
- Simple / double / triple contacts number
- Average / maximum velocity magnitude
- **Inertia parameter**

$$I = \dot{\epsilon}\sqrt{\frac{m}{p.d}}$$

$\dot{\epsilon}$: shear velocity m : average grain mass.
p : average pressure. d : average grain diameter.

Penetration phase on a long railway slice :
90 000 grains, 310 000 contacts,
7 subdomains, 1 interface

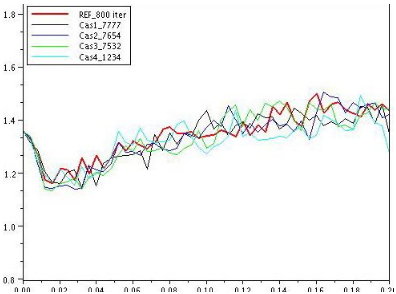


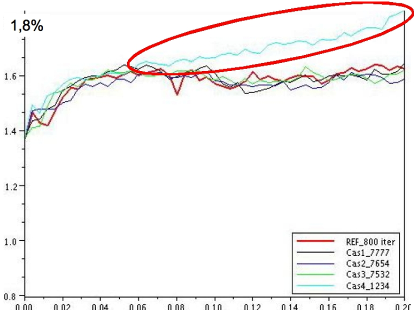
R	800	800	800	800	800	800	800
1	800	800	800	700	700	700	700
2	800	800	800	700	600	500	400
3	800	800	800	700	500	300	200
4	800	800	800	100	200	300	400

Number of sweepings for each SD in 5 runs
(Reference and 4 cases with different ♯ sweepings per SD)

Comparison of the volume (interpenetration) error ($< 2\%$)

Sleeper 3 / Sleeper 4

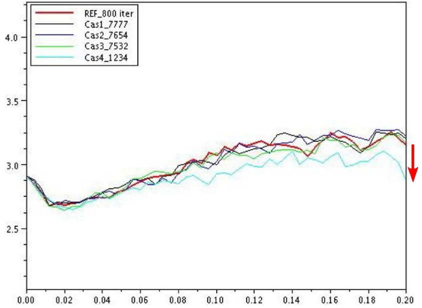


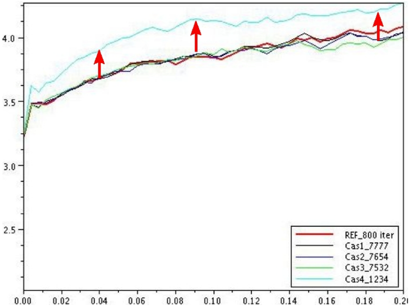


100 sweepings (run 4) for SD4 is not sufficient to avoid interpenetration...
but the error does not propagate to others SD.

Comparison of the coordination number (≈ compactness)

Sleeper 3 / Sleeper 4

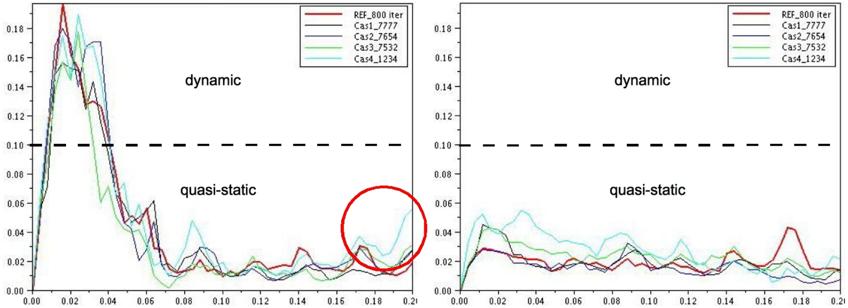




100 sweepings (run 4) for SD4 overestimates the coordination number...
and underestimates it on the neighboring SD3 !

Comparison of the inertia parameter

Sleeper 3 / Sleeper 4



100 sweepings (run 4) for SD4 modifies slightly the inertia parameter (quasi-static regime)...
and modifies it clearly at the end of the process on the neighboring SD3 !

First conclusions and forthcoming works

1. First conclusions
   - Easy to carry on (simple renumbering)
   - Difficult to optimize in term of task balancing (dynamic/static behaviour)
   - High performance computing for complex, non linear and evolutional systems first requires computational efforts with simple and generic algorithms (before developping sophisticated algorithms)
2. Forthcoming works
   - Developping parallel versions with OpenMP then MPI
   - Defining forerunner indicators of dynamic behaviour
   - Investigating the dual partitioning strategy (closer to classical DDM formulation for linear pb)